University of Calgary
Winter semester 2007

PHYS 443: Quantum Mechanics I

Homework assignment 4

Due March 6, 2007

Problem 4.1. Write the matrix of the operator $\hat{\sigma}_x \otimes \hat{\sigma}_y$ in the canonical basis of the bipartite tensor product Hilbert space.

Problem 4.2. Consider the state $|\Theta\rangle = (3|HH\rangle + 4|VV\rangle)/5$ shared between Alice and Bob.

a) Alice performs a measurement on $|\Theta\rangle$ in the canonical basis. Using the Second Postulate (extension to multipartite measurements), find the probabilities $\text{pr}_{A,H}$ and $\text{pr}_{A,V}$ of the two possible measurement results. What state will be remotely prepared in Bob's Hilbert space in both cases?

b) Both Alice and Bob measure $|\Theta\rangle$ in the canonical basis. Using the original Second postulate, find the probabilities pr_{HH}, pr_{HV}, pr_{VH}, pr_{VV} of the four outcomes.

c) Verify that the results of parts (a) and (b) are consistent with each other, i.e. $\text{pr}_{A,H} = \text{pr}_{HH} + \text{pr}_{HV}$ and $\text{pr}_{A,V} = \text{pr}_{VH} + \text{pr}_{VV}$.

d) Repeat parts (a)–(c) for measurements in the diagonal basis.

Problem 4.3. Consider the *Greenberger-Horne-Zeilinger* state $|\Psi_{GHZ}\rangle = \frac{1}{\sqrt{2}}(|HHH\rangle + |VVV\rangle)$ distributed among Alice, Bob, and Charley.

a) Alice and Bob perform a joint measurement on $|\Psi_{GHZ}\rangle$. What is the probability for them to detect

   - $|\Phi^+\rangle$,
   - $|\Phi^-\rangle$,
   - $|\Psi^+\rangle$,
   - $(3|HH\rangle + 4|VV\rangle)/5$.

   and onto which state will Charley's particle project? For each of the above states, assume any measurement basis that contains the state in question.

b) Show that $|\Psi_{GHZ}\rangle$ is an eigenstate of the operators

   - $\hat{\sigma}_x \otimes \hat{\sigma}_y \otimes \hat{\sigma}_y$,

- $\hat{\sigma}_y \otimes \hat{\sigma}_x \otimes \hat{\sigma}_y$,
- $\hat{\sigma}_y \otimes \hat{\sigma}_y \otimes \hat{\sigma}_x$,
- $\hat{\sigma}_x \otimes \hat{\sigma}_x \otimes \hat{\sigma}_x$

with eigenvalues -1, -1, -1, $+1$, respectively.