University of Calgary
Winter semester 2007

PHYS 443: Quantum Mechanics I

Homework assignment 5

Due March 20, 2007

Problem 5.1. Verify that the quantum teleportation protocol will work if the initial entangled state shared between Alice and Bob is $|\Phi^+\rangle$. For each possible outcome of Alice's measurement in the Bell basis, determine the local operation Bob would need to perform on his photon after receiving a classical communication from Alice.

Problem 5.2. The tensor product Hilbert space of Alice's and Bob's photons evolves under a Hamiltonian

$$\hat{H} = \hbar\omega(\hat{\sigma}_x \otimes \hat{\sigma}_x + \hat{\sigma}_y \otimes \hat{\sigma}_y + \hat{\sigma}_z \otimes \hat{\sigma}_z).$$

a) Find the 4×4 matrix of the Hamiltonian in the canonical basis.

b) Find the matrix of the evolution operator $e^{-iHt/\hbar}$.

c) What is the final state of the system after the period $\omega t = \pi/4$ if the initial state is an arbitrary separable state $(a|H\rangle + b|V\rangle) \otimes (c|H\rangle + d|V\rangle)$?

Problem 5.3. Find the Fourier transform of the following functions.

a) $f(x) = \delta(x+a) + \delta(x-a)$.

b) $f(x) = \sin \kappa x$.

c) $f(x) = \sin^3 \kappa x$.

d) $f(x) = xe^{-x^2}$. (**Hint:** use the expression for the Fourier transform of a derivative)

e) $f(x) = \begin{cases} 1 & \text{if } -a \le x \le a \\ 0 & \text{otherwise} \end{cases}$

f) $f(x) = \begin{cases} 1 & \text{if } a \le x \le b \\ 0 & \text{otherwise} \end{cases}$. Calculate your answer (i) by direct integration and (ii) from part (e) using the expression (C.23) for the Fourier transform of a shifted function. Verify that the two answers are consistent.

g) $f(x) = \begin{cases} \sin \kappa x & \text{if } -a \le x \le a \\ 0 & \text{otherwise} \end{cases}$. Calculate your answer (i) by direct integration and (ii) noting that $f(x)$ is the product of the functions from parts (b) and (e) and using the fact that the Fourier transform of a product is a convolution. Verify that the two answers are consistent.

Problem 5.4.

a) Show that

$$e^{-i\hat{p}a/\hbar} |x\rangle = |x + a\rangle$$

b) Express the wavefunction of state $e^{-i\hat{p}a/\hbar} |\psi\rangle$ through the wavefunction $\psi(x)$ of state $|\psi\rangle$.