University of Calgary
Winter semester 2007

PHYS 443: Quantum Mechanics I

Final examination

April 20, 2006, 12.00–15.00 (3 hours)

Open books. Answer all questions. Calculators permitted but not needed.

Total points: 100.

Problem 1 (10 pts). A state of a spin-1/2 particle in the eigenbasis of the x-component of the spin has matrix $\frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

a) (7 pts) Express the state of this particle in the eigenbasis of the z-component of the spin.

b) (3 pts) Find the probabilities for each result of a Stern-Gerlach measurement with the magnetic field oriented along the x axis.

Problem 2 (20 pts). The trace $\mathrm{Tr}\hat{A}$ of operator $\hat{A}$ is the sum of the diagonal elements of its matrix in an orthonormal basis. In class, we have shown that the trace of an operator is basis independent.

a) (10 pts) For any two operators $\hat{A}$ and $\hat{B}$, show that $\mathrm{Tr}(\hat{A}\hat{B}) = \mathrm{Tr}(\hat{B}\hat{A})$.

b) (10 pts) Using the result of part (a), show that in a Hilbert space of finite dimension, a commutation relation $[\hat{A}, \hat{B}] = i\hbar\hat{\mathbf{1}}$ is not possible for any pair of operators.

Problem 3 (5 pts). A two-dimensional system has a Hamiltonian, which in some basis is given by a matrix

$$H = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

What can be said about the mathematical properties of complex numbers a, b, c and d?

EXAM CONTINUES ON THE OTHER SIDE

Problem 4 (20 pts). A two-dimensional system has a Hamiltonian, which in some basis is given by a matrix

$$H = \begin{pmatrix} h & g \\ g & h \end{pmatrix},$$

where h and g are real constants.

a) (10 pts) Find the eigenvalues and eigenvectors of this Hamiltonian

b) (10 pts) Suppose at $t = 0$ the system was in the state $|\psi(0)\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Find the matrix of the state $|\psi(t)\rangle$ at an arbitrary time t.

Problem 5 (25 pts). Consider a particle of mass m attached to a spring with spring constant k. The other end of the spring is attached to a massive wall. Neglect gravity, consider motion in one dimension only.

a) (4 pts) The particle is initially in the ground energy eigenstate. Find its wavefunction in the *non-rescaled* position basis.

b) (7 pts) At time $t = 0$, an additional, position-independent force F begins to act on the particle. Write the new Hamiltonian and find its ground state.

c) (7 pts) Find the probability to detect the particle in the ground state of the new potential.

d) (7 pts) Find the expectation values of the position $\langle x(t)\rangle$ and momentum $\langle p(t)\rangle$ of the particle as a function of time. **Hint:** you need not find the evolution of the wavefunction.

Problem 6 (20 pts). Show that Pauli operations $\hat{\sigma}_x$, $\hat{\sigma}_y$ and $\hat{\sigma}_z$ on a spin-1/2 particle can be implemented (up to a phase factor) by applying magnetic field in the x, y, and z directions, respectively, for a certain time interval. Find this interval. All the necessary parameters of the particle are known.