PHYS 443

Second Midterm

Solutions

1

$$\hat{A}|H\rangle \leftrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \leftrightarrow (|H\rangle + |V\rangle)/\sqrt{2}$$

$$\hat{A}|V\rangle \leftrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} \leftrightarrow (|H\rangle - |V\rangle)/\sqrt{2}$$

a) $|\Psi_{AB}\rangle = \hat{A}|H\rangle \otimes \hat{A}|H\rangle = \frac{1}{2}(|H\rangle + |V\rangle)(|H\rangle + |V\rangle) = \frac{1}{2}(|HH\rangle + |HV\rangle + |VH\rangle + |VV\rangle)$

b) $\hat{H} = 0|HH\rangle\langle HH| + 0|HV\rangle\langle HV| + 0|VH\rangle\langle VH| + \hbar\omega|VV\rangle\langle VV|$

$$e^{-i\frac{\hat{H}}{\hbar}t} = 1|HH\rangle\langle HH| + 1|HV\rangle\langle HV| + 1|VH\rangle\langle VH| + e^{-i\omega t}|VV\rangle\langle VV|$$

$$e^{-i\frac{\hat{H}}{\hbar}\frac{\pi}{\omega}}|\Psi_{AB}\rangle = \frac{1}{2}(|HH\rangle + |HV\rangle + |VH\rangle - |VV\rangle) \quad \text{(because } e^{-i\pi} = -1\text{)}$$

c) $$\hat{A}_B \frac{1}{2}(|HH\rangle + |HV\rangle + |VH\rangle - |VV\rangle) = \frac{1}{2\sqrt{2}}(|H\rangle\otimes(|H\rangle + |V\rangle) + |H\rangle\otimes(|H\rangle - |V\rangle)$$
$$+ |V\rangle\otimes(|H\rangle + |V\rangle) - |V\rangle\otimes(|H\rangle - |V\rangle)) = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle) = |\Phi^+\rangle$$

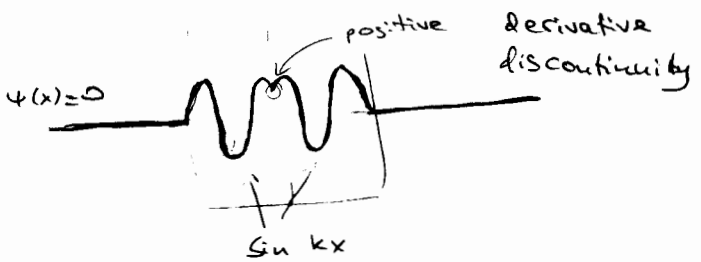
2

a) Find the sign of discontinuity at the delta-potential

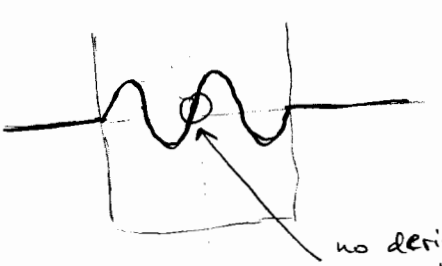
$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi(x) = E\psi(x)$$

($\frac{d^2\psi}{dx^2}$: derivative discontinuity is of the same sign as $\psi(x)$; $V(x)$: positive $\delta(x)$)

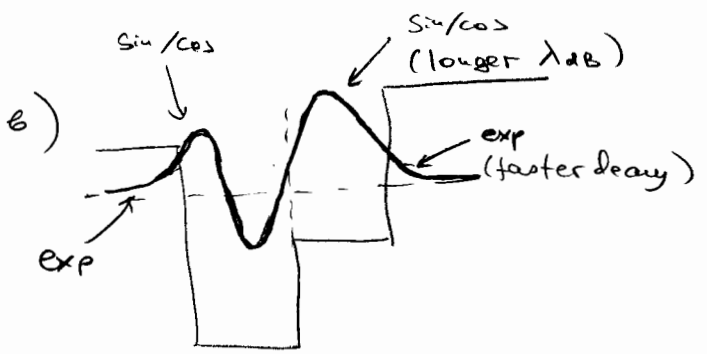
even solution



odd solution



b)



c)