PHYS 443

Midterm 1

Solutions

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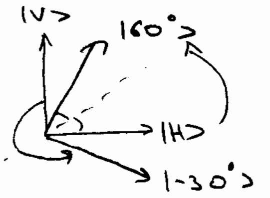

a) $\hat{A}: |H\rangle \to |60^\circ\rangle = \frac{1}{2}|H\rangle + \frac{\sqrt{3}}{2}|V\rangle$

$|V\rangle \to |-30^\circ\rangle = \frac{\sqrt{3}}{2}|H\rangle - \frac{1}{2}|V\rangle$

b) $\hat{A} \leftrightarrow \begin{pmatrix} 1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix}$

Note: Because a phase factor does not change a quantum state, this solution is not unique. For example, one can use

$|30^\circ\rangle = -\frac{\sqrt{3}}{2}|H\rangle + \frac{1}{2}|V\rangle$

Full credit is given for all correct solutions

c) $A^{+}A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow A$ is unitary

d) $\det(A - \lambda\hat{\mathbb{1}}) = 0$

$\lambda^2 - \frac{1}{4} - \frac{3}{4} = 0$

$\lambda_1 = 1 \to$ eigenvector $\begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix}$

$\lambda_2 = -1 \to$ eigenvector $\begin{pmatrix} -1/2 \\ +\sqrt{3}/2 \end{pmatrix}$

e) Notice that eigenvectors are $|30^\circ\rangle$ and $|-120^\circ\rangle$. They correspond to the waveplate's optical axis and its orthogonal. The waveplate does not affect these polarizations

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a) $\hat{A} = \hat{A}^{+} \Rightarrow \hat{A}$ is Hermitian

$\hat{A}^{+}\hat{A} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \neq \mathbb{1} \Rightarrow A$ is not unitary

b) Find eigenvalues and eigenstates

$\det(\hat{A} - \lambda\hat{\mathbb{1}}) = 0$

$(1/2 - \lambda)^2 = (1/2)^2$

$\lambda_1 = 1 \to$ eigenvector $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = |v_1\rangle$

$\lambda_2 = 0 \to$ eigenvector $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = |v_2\rangle$

Possible measurement outcomes: $A_1 = \lambda_1 = 1$, $A_2 = \lambda_2 = 0$

Probabilities: $pr_1 = |\langle\psi|v_1\rangle|^2 = \left|\frac{1}{\sqrt{10}}(1\ \ 2)\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right|^2 = \frac{9}{10}$

$pr_2 = |\langle\psi|v_2\rangle|^2 = \left|\frac{1}{\sqrt{10}}(1\ \ 2)\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right|^2 = \frac{1}{10}$

c) $\langle\psi|\hat{A}|\psi\rangle = \frac{1}{10}(1\ \ 2)\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \frac{1}{10}(1\ \ 2)\begin{pmatrix} 3 \\ 3 \end{pmatrix} = \frac{9}{10} = A_1 pr_1 + A_2 pr_2$

$\boxed{3}$ $\hat{H} = 0 \cdot |30^\circ\rangle\langle 30^\circ| + \hbar\omega\, |120^\circ\rangle\langle 120^\circ|$

$|\Psi(t)\rangle = e^{-\frac{i}{\hbar}\hat{H}t}\,|\Psi(0)\rangle = e^{0}\,|30^\circ\rangle\langle 30^\circ|V\rangle + e^{-i\omega t}\,|120^\circ\rangle\langle 120^\circ|V\rangle$

$|30^\circ\rangle = \frac{\sqrt{3}}{2}|H\rangle + \frac{1}{2}|V\rangle$

$|120^\circ\rangle = -\frac{1}{2}|H\rangle + \frac{\sqrt{3}}{2}|V\rangle$

$|\Psi(t)\rangle = \frac{1}{2}|30^\circ\rangle + \frac{\sqrt{3}}{2}e^{-i\omega t}|120^\circ\rangle = \frac{\sqrt{3}}{4}(1 - e^{-i\omega t})|H\rangle + \frac{1}{4}(1 + 3e^{-i\omega t})|V\rangle$