

PHYS 443: Quantum Mechanics I

Second midterm examination

March 30, 2006

Open books. Answer both questions.

Problem 1. Alice and Bob each have a photon whose polarization is initially horizontal. In order to entangle their photons, they subject them to the following sequence of operations:

- a) (15 pts) Both Alice and Bob act on their photon with the unitary operator  $\hat{A}$  whose matrix in the  $(H, V)$  basis is  $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ .
- b) (20 pts) The resulting state is subjected to the action of the Hamiltonian  $\hat{H} = \hbar\omega |VV\rangle\langle VV|$  for a time duration of  $\pi/\omega$ . [**Hint:** other Hamiltonian eigenstates ( $|HH\rangle$ ,  $|HV\rangle$ , and  $|VH\rangle$ ) correspond to zero energy values.]
- c) (15 pts) Bob again applies operator  $\hat{A}$  to his photon.

Find the state of the photon pair after each operation.

Problem 2. Sketch qualitative plots of the stationary wavefunctions of the potentials shown in the figure below with energy levels as shown. **Note:** full credit will be given to solutions with proper attention to details, such as: relations between de Broglie wavelengths in different areas of the plot, continuity conditions, sign of the derivative discontinuity at the delta function potential, etc.

