

University of Calgary  
Winter semester 2006

PHYS 443: Quantum Mechanics I  
First midterm examination

February 16, 2006

Open books. Answer any two of these questions.

Problem 1. Consider an operator  $\hat{A}$  whose action corresponds to that of a half-wave plate with optical axis oriented  $30^\circ$  degrees to horizontal.

- (10 pts) Onto which states does  $\hat{A}$  map  $|H\rangle$  and  $|V\rangle$ ?
- (10 pts) Find the matrix of  $\hat{A}$  in the canonical basis
- (10 pts) Is  $\hat{A}$  unitary?
- (20 pts) Find the eigenvalues and eigenvectors of  $\hat{A}$ .
- (extra credit: 20 pts) Verify that the physical states corresponding to these eigenvectors remain unchanged when transmitted through the wave plate.

Problem 2. Consider an operator described in the canonical basis by the following matrix:

$$\hat{A} \leftrightarrow \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}. \quad (1)$$

- (10 pts) Is  $\hat{A}$  unitary? Hermitian?
- (20 pts) Observable  $\hat{A}$  is measured in the quantum state  $|\psi\rangle = (|H\rangle + 2|V\rangle)/\sqrt{5}$ . What are the possible measurement outcomes  $A_1, A_2$ ? What is the probability  $\text{pr}_1, \text{pr}_2$  of each outcome?
- (20 pts) Find the expectation value  $\langle \psi | \hat{A} | \psi \rangle$ . Verify that this expectation value equals  $\text{pr}_1 A_1 + \text{pr}_2 A_2$ .

Problem 3. Consider a birefringent medium with the following properties:

- linear  $30^\circ$  polarized photon has a certain energy value 0;
- linear  $120^\circ$  polarized photon has a certain energy value  $\hbar\omega$ .

- (20 pts) Find the Hamiltonian.
- (30 pts) Find the polarization state  $|\psi(t)\rangle$  of the photon at time  $t$  if its initial state is  $|\psi(0)\rangle = |V\rangle$ .

**Note:**  $\sin 30^\circ = \frac{1}{2}$ ;  $\cos 30^\circ = \frac{\sqrt{3}}{2}$