University of Calgary
Winter semester 2006

PHYS 443: Quantum Mechanics I

Home assignment 2

Due January 31, 2006

Problem 2.1. Decompose $|H\rangle$ and $|V\rangle$ in the $(|R\rangle, |L\rangle)$ basis

a) by solving Eqs. (1.4) and (1.5);

b) by using Eq. (1.9).

Verify that the results are identical.

Problem 2.2.

a) Decompose polarization states $|a\rangle = |{+30^\circ} \text{ to horizontal}\rangle$ and $|b\rangle = |{-30^\circ} \text{ to horizontal}\rangle$ in the $(|H\rangle, |V\rangle)$, $(|+\rangle, |-\rangle)$, and $(|R\rangle, |L\rangle)$ bases. **Hint:** First use the analogy between classical and quantum fields to express $|a\rangle$ and $|b\rangle$ in the canonical basis. Then convert to other bases using one of the methods from the previous problem.

b) Find the probabilities associated with quantum measurements in each of the three bases.

c) Find the inner product $\langle a|b\rangle$ using the result of Ex. 1.15 in all three bases. Does it come out the same?

Problem 2.3. Perform the Gram-Schmidt procedure for three 3D vectors: $\vec{w}_1 = (4, 3, 0)$, $\vec{w}_2 = (-4, -3, 1)$, $\vec{w}_3 = (1, 1, 1)$. Verify that the basis obtained is indeed orthonormal[1].

Problem 2.4. Prove the Cauchy-Schwartz inequality for a two-dimensional Hilbert space. Show that equality is achieved if and only if $|a\rangle$ and $|b\rangle$ differ only by a scalar factor.

Problem 2.5. Ex. 1.34 from the lecture notes.

Problem 2.6. Linear operator $\hat{A}$ transforms $|R\rangle$ into $|{+30^\circ} \text{ to horizontal}\rangle$, and $|L\rangle$ into $|{+120^\circ} \text{ to horizontal}\rangle$.

a) Find the matrix of $\hat{A}$ in the canonical basis.

[1] Note a typo in Eq. (1.11) in the lecture notes. It should read $|v_{k+1}\rangle = \mathcal{N}\left[|w_{k+1}\rangle - \sum_{i=1}^{k} \langle v_i|w_{k+1}\rangle |v_i\rangle\right]$.

b) Propose an arrangement of waveplates that would implement this operator physically.

Problem 2.7. Ex. 1.44 from the lecture notes. Verify that the matrix of $\hat{A}$ has the same trace in the canonical and circularly polarized bases.

Problem 2.8. Ex. B.28 from the lecture notes (Appendix B).