University of Calgary
Winter semester 2006

PHYS 443: Quantum Mechanics I

Home assignment 3

Due February 14, 2006

Problem 3.1. Show that if operator $\hat{A}$ is Hermitian, then

a) $\forall \left|a\right\rangle$, $\left\langle a\middle|\hat{A}\middle|a\right\rangle$ is real;

b) $\forall \left|a\right\rangle, \left|b\right\rangle$, $\left\langle a\middle|\hat{A}\middle|b\right\rangle = \left\langle b\middle|\hat{A}\middle|a\right\rangle^*$.

Problem 3.1. Ex. 1.59 from the lecture notes.

Problem 3.2. Ex. 1.61 from the lecture notes.

Problem 3.3. Ex. 1.62 from the lecture notes. Verify your answers by calculating the operators' matrices from their eigenstates and eigenvalues using Eq. (1.25).

Problem 3.4. Observables $\hat{\sigma}_z$ and $\hat{\sigma}_x$ are measured in the state $\left|\psi\right\rangle = \frac{\sqrt{3}}{2}\left|H\right\rangle + \frac{i}{2}\left|V\right\rangle$.

a) Find the expectation values and uncertainties associated with these measurements.

b) Find the expectation value of the observable $i[\hat{\sigma}_z, \hat{\sigma}_x]$.

c) Verify that the uncertainty principle [Eq. (1.49)] holds.

Problem 3.5. Find the matrix of $\exp(i\hat{A})$ in the canonical basis if

$$\hat{A} \leftrightarrow \begin{pmatrix} 5 & -i \\ i & 5 \end{pmatrix}. \tag{1}$$

Verify that $\hat{A}$ is Hermitian, $\exp(i\hat{A})$ is unitary (*cf.* Ex. 1.83).

Problem 3.7 **(extra credit).** Ex. 1.78 from the lecture notes.

Problem 3.8. Ex. 1.87(c) from the lecture notes.