

University of Calgary
Winter semester 2006

PHYS 443: Quantum Mechanics I

Home assignment 6

Due March 28, 2006

Problem 6.1. Ex. 3.18 from the lecture notes.

Problem 6.2. Ex. 3.19 from the lecture notes.

Problem 6.3. Suppose each of the two observers, Alice and Bob, holds a one-dimensional point-like particle. The two particles are prepared in entangled state $|\Psi_{AB}\rangle$ whose wavefunction is

$$\Psi(x_A, x_B) = \frac{1}{\sqrt{\pi d D}} e^{-\frac{(x_A - x_B)^2}{4d^2}} e^{-\frac{(x_A + x_B)^2}{4D^2}}, \quad (1)$$

where $d \neq D$.

- a) Express the state of the two particles in the momentum representation.
Hint: the integration is simplified with the following variable replacement: $x_s = (x_A + x_B)/\sqrt{2}$; $x_d = (x_A - x_B)/\sqrt{2}$; $p_s = (p_A + p_B)/\sqrt{2}$; $p_d = (p_A - p_B)/\sqrt{2}$. Your answer should still be expressed through p_A and p_B .
- b) Suppose Alice performs a measurement of her particle's position and obtains some result x_0 . Onto which state $|\psi_{B1}\rangle$ will Bob's particle project?
- c) Suppose Alice instead performs a measurement of her particle's momentum and obtains some result p_0 . Onto which state $|\psi_{B2}\rangle$ will Bob's particle project?
- d) Using the uncertainty principle, show that the physical realities associated with states $|\psi_{B1}\rangle$ and $|\psi_{B2}\rangle$ are incompatible with each other.

Problem 6.4. Ex. 3.22 from the lecture notes. **Note:** because the wavefunction in the momentum/wavevector representation is an *inverse* Fourier transform of the wavefunction in the position representation, the rules obtained in Ex. C.11 must be revised. Mind a typo in the answer: $\langle \Delta p^2 \rangle = \hbar^2/(2d^2)$.

Problem 6.5. Ex. 3.33 from the lecture notes. Show that the probability current for the de Broglie wave is proportional to its momentum and the square absolute value of the amplitude.

Problem 6.6. Ex. 3.41 from the lecture notes.

Problem 6.7. Ex. 3.42(b,c) from the lecture notes.

Problem 6.8. Ex. 3.43 from the lecture notes.

Problem 6.9 (Extra credit: 40 pts.). Ex. 3.45(b,c) from the lecture notes