University of Calgary
Winter semester 2006

PHYS 443: Quantum Mechanics I

Home assignment 7

Due April 11, 2006

Problem 7.1. Ex. 3.52 from the lecture notes

Problem 7.2. Calculate $\langle X\rangle$, $\langle\Delta X^2\rangle$, $\langle P\rangle$, $\langle\Delta P^2\rangle$ and verify the uncertainty principle:

a) for an arbitrary Fock state $|n\rangle$

b) for an arbitrary coherent state $|\alpha\rangle$

of a harmonic oscillator. **Hint:** do *not* use wavefunctions.

Problem 7.3. Find the evolution of $\langle X\rangle$ and $\langle P\rangle$ of a coherent state as a function of time. **Hint:** instead of using the straightforward method (finding the time evolution of the coherent state), you may instead wish to try Eq. (3.42) from the notes.

Problem 7.4. Verify the following commutation properties of the angular momentum operator:

a) $[\hat{L}_j, \hat{r}_k] = i\hbar\epsilon_{jkl}\hat{r}_l$;

b) $[\hat{L}_j, \hat{p}_k] = i\hbar\epsilon_{jkl}\hat{p}_l$;

c) $[\hat{L}_j, \hat{L}_k] = i\hbar\epsilon_{jkl}\hat{L}_l$;

d) $[\hat{L}_j, \hat{r}^2] = 0$;

e) $[\hat{L}_j, \hat{p}^2] = 0$;

f) $[\hat{L}_j, \hat{L}^2] = 0$.

Problem 7.5. Show that:

a) $[\hat{L}^2, \hat{L}_\pm] = 0$; $[\hat{L}_+, \hat{L}_-] = 2\hbar L_z$;

b) $\hat{L}^2 = \hat{L}_+\hat{L}_- + \hat{L}_z^2 - \hbar\hat{L}_z = \hat{L}_-\hat{L}_+ + \hat{L}_z^2 + \hbar\hat{L}_z$.

Problem 7.6. Write the matrices of $\hat{L}_x$, $\hat{L}_y$ and $\hat{L}_z$ in the $|lm\rangle$ basis for $l = 3/2$. **Note:** traditionally, the angular momentum basis eigenvectors are listed in the order of *decreasing* m.