University of Calgary
Winter semester 2006

PHYS 443: Quantum Mechanics I

Home assignment 1

Due January 17, 2006

Problem 1.1. Using only the definition of the linear space, show the following:

a) $-\left|\text{zero}\right\rangle = \left|\text{zero}\right\rangle$;

b) $\forall \left|a\right\rangle \in \mathbb{V} \qquad -(-\left|a\right\rangle) = \left|a\right\rangle$;

c) $\left|a\right\rangle = \left|b\right\rangle$ if and only if $\left|a\right\rangle + (-\left|b\right\rangle) = \left|\text{zero}\right\rangle$;

d) if, for some $\left|a\right\rangle, \left|b\right\rangle \in \mathbb{V}$, $\left|a\right\rangle + \left|b\right\rangle = \left|a\right\rangle$, then $\left|b\right\rangle = \left|\text{zero}\right\rangle$.

In every step of your proof, indicate which axioms you are using.

Problem 1.2. Among three vectors $\left|a\right\rangle$, $\left|b\right\rangle$, $\left|c\right\rangle$, any two are linearly independent. Does this mean that all three are linearly independent? Provide a proof or show a counterexample.

Problem 1.3. Consider two elements of the linear space of geometric vectors in a plane: $\vec{v}_1 = (1,2)$ $\vec{v}_2 = (-3,1)$.

a) Show that these vectors form a basis;

b) decompose vector $\vec{a} = (-4,-3)$ into these basis vectors and write the decomposition in the matrix form.

Problem 1.4. Ex. 1.23 from the lecture notes (you can use the Cauchy-Schwarz inequality[1]).

[1] Note a typo in Eq. (1.12) in the lecture notes. The left-hand side of the inequality should read $|\left\langle a|b\right\rangle|$.