University of Calgary
Winter semester 2006

PHYS 443: Quantum Mechanics I

Final examination

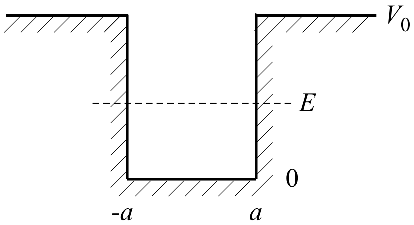
April 20, 2006, 12.00-15.00 (3 hours)

Open books. Answer all questions. Calculators permitted but not needed

Total points: 100

Problem 1 (10 pts) A photon polarized at 30° is measured in the circular basis. Find the probability of each possible measurement result.

Problem 2 (10 pts) Make qualitative plots of the even and odd real solutions of the time-independent Schrödinger equation for the potential shown in the figure. The energy E, such that $0 < E < V_0$, is given.



Problem 3 (15 pts) An atom is prepared in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|l=2, m=1\rangle + |l=2, m=2\rangle).$$

Find the expectation value for the measurement of the x-component of the angular momentum.

Problem 4 (10 pts) A harmonic oscillator is prepared in a *photon added coherent state* $\hat{a}^\dagger |\alpha\rangle$ (where $|\alpha\rangle$ is a coherent state) and subjected to a photon number measurement. Find the probability to detect vacuum (zero photons).

Problem 5 (15 pts) Alice and Bob share a pair of electrons prepared in an entangled state

$$|\Psi_{AB}\rangle = \frac{1}{3}\left[2|\uparrow\uparrow\rangle - 2|\uparrow\downarrow\rangle + |\downarrow\downarrow\rangle\right],$$

where $|\uparrow\rangle = |m = 1/2\rangle$, $|\downarrow\rangle = |m = -1/2\rangle$ Alice performs a Stern-Gerlach measurement on her electron, with the magnetic field oriented along the x-axis.

Find the probability of each possible outcome. For each outcome, indicate the state onto which Bob's photon will project.

Problem 6 (10 pts) For two arbitrary states $|u\rangle$ and $|v\rangle$ show that $\mathrm{Tr}(|u\rangle\langle v|) = \langle v|u\rangle$.

Problem 7 An atom with angular momentum $l = 1$ is initially prepared in an eigenstate of $\hat{L}_x$ with eigenvalue $\hbar$. At time $t = 0$, a magnetic field B oriented along the z axis is turned on.

a) (10 pts) Express the initial state of the atom in the martix form in the canonical ($|lm\rangle$) basis.

b) (15 pts) Calculate the Schrödinger evolution of this state.

c) (5 pts) Verify that at $\Omega t = \pi/2$ (where Ω is the precession frequency), the atom will be in an eigenstate of $\hat{L}_y$.

Useful expressions

$$\sin 30^\circ = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$e^{i\pi/2} = i$$

$$e^{i\pi} = -1$$

$$\hat{L}_x \leftrightarrow \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \qquad \hat{L}_y \leftrightarrow \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \qquad \hat{L}_z \leftrightarrow \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

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