



UNIVERSITY OF  
CALGARY

FACULTY OF SCIENCE

FINAL EXAMINATION

PHYSICS 443

April 25, 2005, 8.00-11.00

Time: 3 hrs.

TOTAL MARKS: 100

**Notes:**     *Attempt All Questions*  
                  *Open books*  
                  *Calculators Permitted*

1. A quantum system can be found in one of three orthogonal states  $|a\rangle, |b\rangle, |c\rangle$ . These three states form a basis.  $\hat{R}$  is an operator which cyclically permutes the states, i.e.  $\hat{R}|a\rangle = |b\rangle, \hat{R}|b\rangle = |c\rangle, \hat{R}|c\rangle = |a\rangle$ . The Hamiltonian is

$$\hat{H} = \hbar\omega(\hat{R} + \hat{R}^\dagger)$$

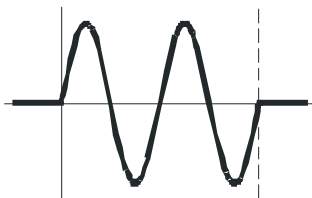
- a) [5 pts] Is  $\hat{R}$  an observable? Explain why or why not.
- b) [15 pts] Find the energy eigenvalues and eigenstates of the system.

2. [20 pts] A particle is in a stationary state of a potential well  $V(x) = -V_0\delta(x)$ . At time  $t = 0$ , the depth of the well instantaneously doubles. What is the probability to detect a finite value of particle's position at  $t = \infty$ ?

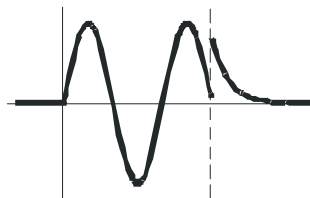
Note. You may use the result of Ex. 3.43 in the lecture notes.

3. Make a qualitative sketch of a potential that would generate the stationary real wavefunction for each case shown below. Mark the energy level on your plots. Explain your answers. If you believe there is no such potential explain why.

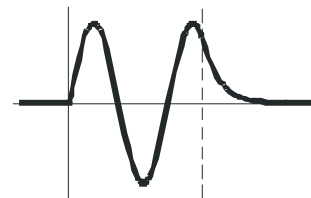
a) [5 pts]



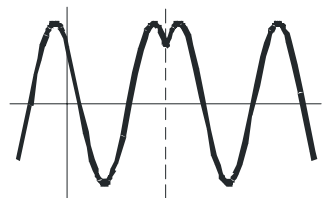
b) [5pts]



c) [5pts]



d) [5pts]



4. Two spin-1/2 particles interact via the Hamiltonian

$$\hat{H} = J \vec{S}_A \cdot \vec{S}_B$$

- a) [10 pts] Find the 4x4 matrix of the Hamiltonian in the canonical ( $|m_s = \pm 1/2\rangle$ ) basis.
- b) [20 pts] Find the matrix of the evolution operator  $\exp(-i\hat{H}t / \hbar)$
- c) [10 pts] Show that the evolution over the period  $J\hbar t = \pi$  will swap the states of the particles, i.e. convert any state  $|\psi\rangle_A \otimes |\varphi\rangle_B$  into  $|\varphi\rangle_A \otimes |\psi\rangle_B$ .

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