UNIVERSITY OF CALGARY

Physics 443: Quantum Mechanics I
Second midterm examination

March 24, 2005

Open books

1. Three photons are prepared in an entangled state

$$|\Psi_{ABC}\rangle = \frac{1}{6}\left(|HHH\rangle - 3i|HHV\rangle + 4i|HVH\rangle - |VHV\rangle + 3|VVH\rangle\right),$$

which is distributed among three observers, Alice, Bob, and Charley ($|H\rangle$ and $|V\rangle$ denote horizontal and vertical polarizations, respectively). Alice and Bob perform a joint measurement of their photons in the Bell basis.

a) [20 pts] On which state will Charley's photon project if Alice and Bob detect the state $|\Phi^+\rangle = \frac{1}{\sqrt{2}}\left(|HH\rangle + |VV\rangle\right)$?

b) [20 pts] What is the probability of this event?

2 [*original formulation of the Einstein-Podolsky-Rosen paradox*]. Each of the two observers, Alice and Bob, hold a one-dimensional point-like particle. The two particles are prepared in an entangled state

$$|\Psi_{AB}\rangle = \Psi_x(x_A, x_B)|x_A, x_B\rangle,$$

with $\Psi_x(x_A, x_B) = \delta(x_A - x_B)$.

a) [20 pts] Express the state of the two particles in the momentum representation. Neglect normalization.

b) [10 pts] Suppose Alice performs a measurement of her particle's *position* and obtains some result X_0. How will this event affect the quantum state of Bob's particle?

c) [10 pts] Suppose Alice performs a measurement of her particle's *momentum* on a newly prepared state $|\Psi_{AB}\rangle$ and obtains some result P_0. How will this event affect the quantum state of Bob's particle?

d) [10 pts] Suppose Alice and Bob perform simultaneous measurements of their particles' positions or momenta. Sketch the probability densities $\mathrm{pr}(x_A, x_B)$ and $\mathrm{pr}(p_A, p_B)$.

e) [10 pts] Use the results (b) and (c) and the uncertainty principle to argue that quantum mechanics contradicts the locality principle.