Kinetic Theory and Thermodynamics: Problems

Example Problems

Questions to be answered during lecture 20.

8.1 An ideal gas at temperature T and pressure P is confined in a container within an evacuated space. The gas molecules can escape through a small hole of area S in a plane wall of the container. The number of molecules per unit volume within the container with speeds between v and $v + dv$ is $N_v dv$. The rate at which molecules with speeds between v and $v + dv$ travelling at angles between θ and $\theta + d\theta$ to the normal to the container wall pass through the hole is $n(v, \theta)dv\, d\theta$. Show that

$$n(v, \theta)dv\, d\theta = \frac{1}{2} S N_v v\, dv \sin\theta \cos\theta d\theta.$$

A plane circular disk of radius R is located outside the gas container with its plane parallel to the container wall and with its axis passing through the small hole. The plane of the disk is a distance R from the container wall and all molecules that strike the disk adhere to it. Given that the total number of molecules per unit volume in the container is N, derive an expression for the force exerted on the disk by the molecules that strike it after passing through the hole.

8.2 Use simple kinetic theory of gases to show that the viscosity η of a gas is proportional to $T^{1/2}$, where T is the absolute temperature. What are the conditions for η to be independent of pressure?

The viscosity of carbon dioxide at various temperatures and at atmospheric pressure is as follows:

T (K)	252	283	455	575
$\eta \times 10^6$ (Nm^{-2}s)	12.9	15.8	22.2	26.8

Estimate the collision diameter of carbon dioxide for the four different temperatures. Comment on any variation that you find.

8.3 An ideal air conditioner operating on a Carnot cycle absorbs heat Q_2 from a house at temperature T_2 and discharges Q_1 to the outside at temperature T_1, consuming electrical energy E. Heat leakage into the house follows Newton's Law,

$$Q = A[T_1 - T_2]$$

where A is a constant. Derive an expression for T_2 in terms of T_1, E and A for continuous operation when the steady state has been reached.

The air conditioner is controlled by a thermostat. The system is designed so that with the thermostat set at 20°C and outside temperature 30°C the system operates at 30% of the maximum electrical energy input. Find the highest outside temperature for which the house may be maintained inside at 20°C.

8.4 An equation of state for one mole of CO_2 is

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT.$$

Calculate the change in temperature when one mole of CO_2 expands freely from a volume of $10^{-3}\,\mathrm{m}^3$ to $3 \times 10^{-3}\,\mathrm{m}^3$. Given that the initial temperature is 320 K, what is the corresponding change in the entropy of the gas?

[For one mole of CO_2, $C_V = 28.2\,\mathrm{J\,K^{-1}}$, $a = 0.26\,\mathrm{J\,m^3}$, $b = 3 \times 10^{-5}\,\mathrm{m^3}$.]

8.5 One mole of an ideal monatomic gas is confined in a cylinder by a piston and is maintained at a constant temperature T_0 by contact with a heat reservoir. The gas expands its volume from V_1 to V_2 reversibly and isothermally. Obtain expressions for the work done by the gas and the changes in the internal energy U and the entropy S of (a) the gas and (b) the reservoir.

One mole of another gas obeys the equation of state

$$PV = RT\left(1 + \frac{A}{V}\right),$$

where A is a constant. The gas undergoes a reversible expansion in volume from V_1 to V_2 at a fixed temperature T_0. Obtain expressions for the work done by the gas and the changes in its internal energy and entropy during the expansion.

8.6 A spherical satellite of radius 1.2 m is in a circular orbit about the Sun at a distance of 1 astronomical unit. The emissive power per unit area of the satellite relative to that of a black body at the same temperature is independent of wavelength and has a value of 0.08. The temperature of the satellite is uniform. Estimate the equilibrium temperature of the satellite.

When the power supply for the instruments on the satellite is turned on, heat is generated at a uniform rate of $14\,\mathrm{W\,m^{-3}}$. Calculate the new equilibrium temperature of the satellite.

[Effective black-body temperature of the Sun's surface = 5800 K.]

8.7 The equilibrium vapour pressure p of water as a function of temperature is given in the following table:

T (°C)	0	10	20	30	40	50
p (Pa)	611	1 228	2 339	4 246	7 384	12 349

Deduce a value for the latent heat of evaporation L_v of water. State clearly any simplifying assumptions that you make.

Estimate the pressure at which ice and water are in equilibrium at -2°C given that ice cubes float with 4/5 of their volume submerged in water at the triple point (0.01°C, 612 Pa).

[Latent heat of sublimation of ice at the triple point, $L_s = 2776 \times 10^3\,\mathrm{J\,kg^{-1}}$.]

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