Electric Fields in Matter

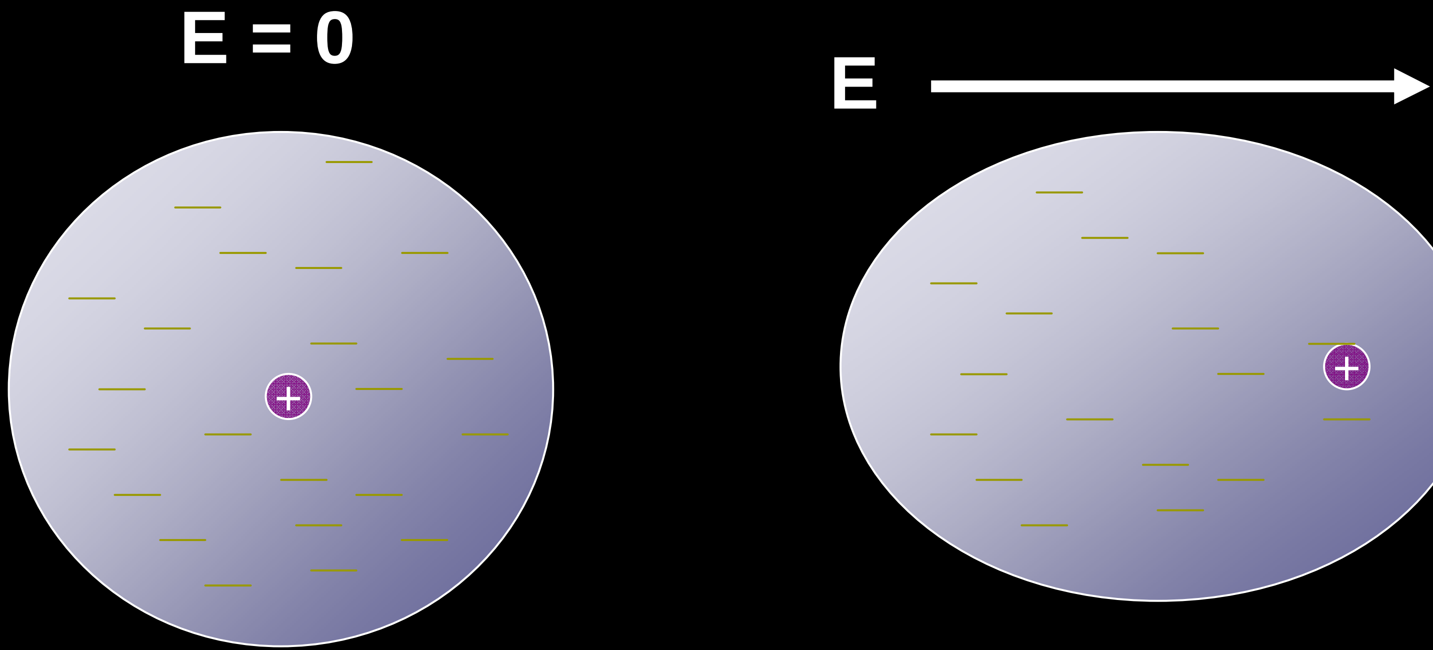
Linear Dielectrics

E, ***P***, and ***D***

Insulators

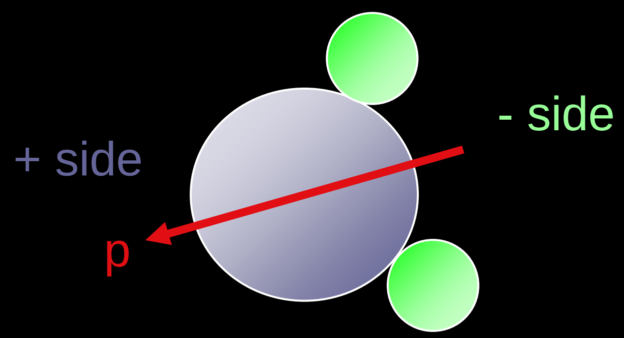
- We have seen how conductors behave.
  - What about materials where the charge is not free to move?
- The electrons are so tightly bound to the atomic nucleus that normal-sized electric fields cannot pull them away.
  - What happens?
  - Do they move at all?
  - Will there be an effect on electrostatics?

A Simple Atom in an Electric Field



Molecules are not so simple.

- But they often form natural dipoles all the same.
  - Case in point is the water molecule
    - This natural polar molecule is why water is such a good solvent.



In a Gas

- We think we understand a classical-type gas though!
  - Similar to our plasma and metal problems
  - There is a restoring force, obviously, holding the electron in place.
- $F = qE_x - m\omega_0^2 x = m\,dv_x/dt$
  - *Surely our current density method works again?*

The total final Electric field must include a contribution from these dipoles.

In a Gas

* **Gas is thin.** So we can apply our usual rule that any e-field generated by a gas molecule itself will die out by the time it gets to the next gas molecule – on average.

$$qE_0 e^{i(\omega t - kz)} = m\frac{dv_x}{dt} + m\omega_0^2 x$$

Fundamentally though, now the equation of motion of the electrons depends on position…not just velocity of the electrons!
A new language would be better than trying to force this model to generate currents for us.

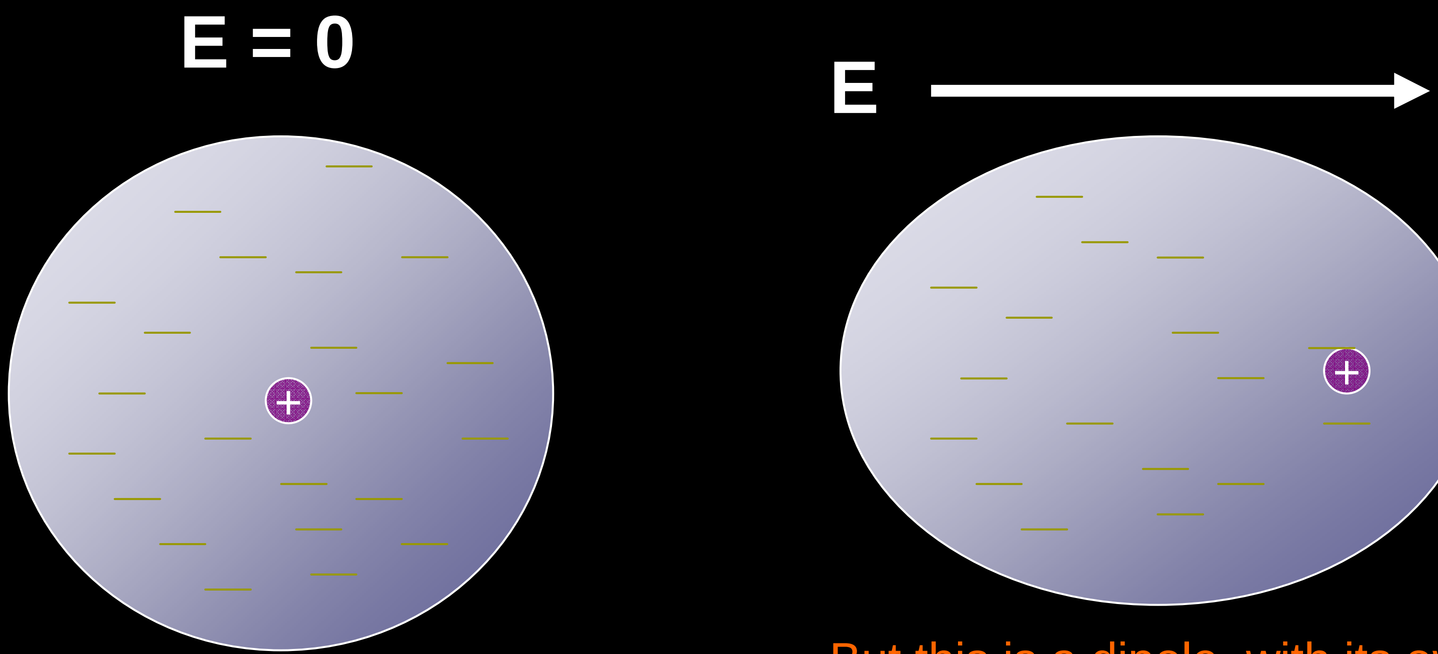
In a Gas

* **The Equation of Motion for an electron in a Gas atom.**
Is approximated thusly.

$$qE_0 e^{i(\omega t - kz)} = m\frac{d^2x}{dt^2} + m\omega_0^2 x$$

A note of caution is in order just as before.
An atom is a quantum bound state.
We are borrowing from Classical Physics…some things are bound to be wrong if we get away from the classical realm.

More Charge Centroid motion.



But this is a dipole, with its own electric field!
We invent a ‘polarization’ field to account for this.

In a Gas

- In a Gas with all the atoms sort of evenly distributed. We would expect to get a dipole moment per unit volume.
- $\mathbf{P}=n\mathbf{p}=N\alpha\mathbf{E}$
  - Where '*N*' is the number of atoms per unit volume.
  - p = qx → polarization depends on 'x'!

The total final Electric field must include a contribution from these dipoles.

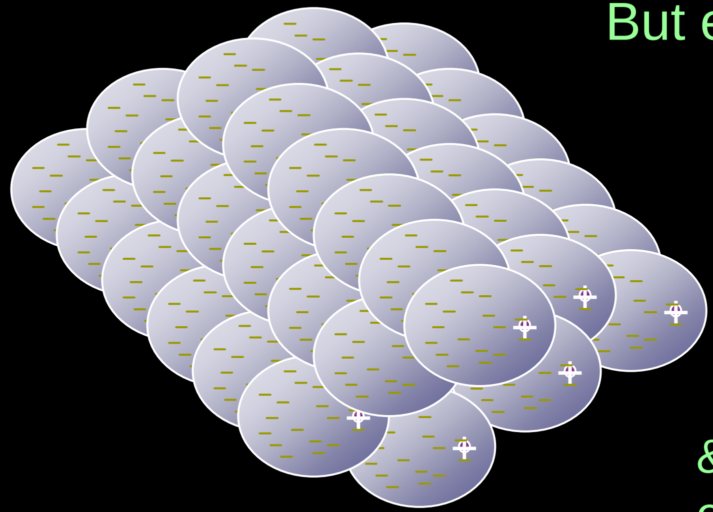
Electric Fields in Matter – Part B

Linear Dielectrics

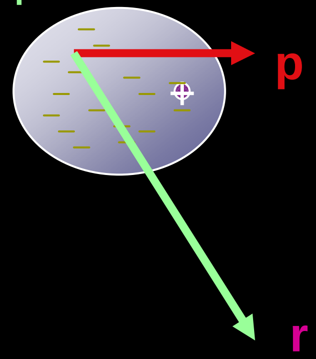
E, ***P***, and ***D***

Externally applied E-field

One is pretty small



But each has a tiny dipole moment.



& Every dipole has an electric field and electric potential.

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\hat{r}\cdot\vec{p}}{r^2}$$

They all add up to a Polarization Field we designate → **P**

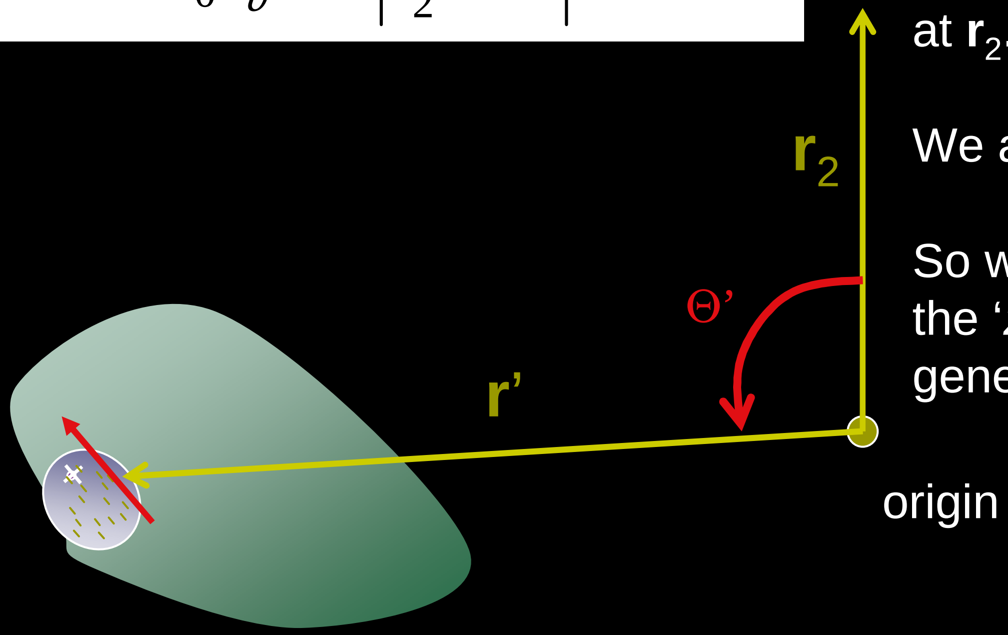
Integrate to add up all the dipoles

$$V(\vec{r}_2) = \frac{1}{4\pi\varepsilon_0}\int_{\upsilon}\frac{(\vec{r}_2 - \vec{r}\,')\cdot\vec{p}(\vec{r}\,')}{\left|\vec{r}_2 - \vec{r}\,'\right|^3}d\tau'$$

Each Dipole gives us a potential at $\mathbf{r}_2$.

We are not moving $\mathbf{r}_2$ around.

So we can take $\mathbf{r}_2$ to be along the '*z*' axis without any loss of generality.



Math tricks on integral over all the dipoles

Some board work

Electric Displacement Field

$$\nabla \cdot \varepsilon_0 \vec{E} = \rho_b + \rho_f$$

We cannot do much about the bound charges

Manipulating this expression a bit

$$\nabla \cdot \varepsilon_0 \vec{E} = -\nabla \cdot \vec{P} + \rho_f$$

$$\nabla \cdot \left(\varepsilon_0 \vec{E} + \vec{P}\right) = \rho_f$$

"Free" charges are the ones we can place where ever we want…

as long as you are prepared to do work.

Define: $\vec{D} \equiv \varepsilon_0 \vec{E} + \vec{P}$

Linear Dielectrics

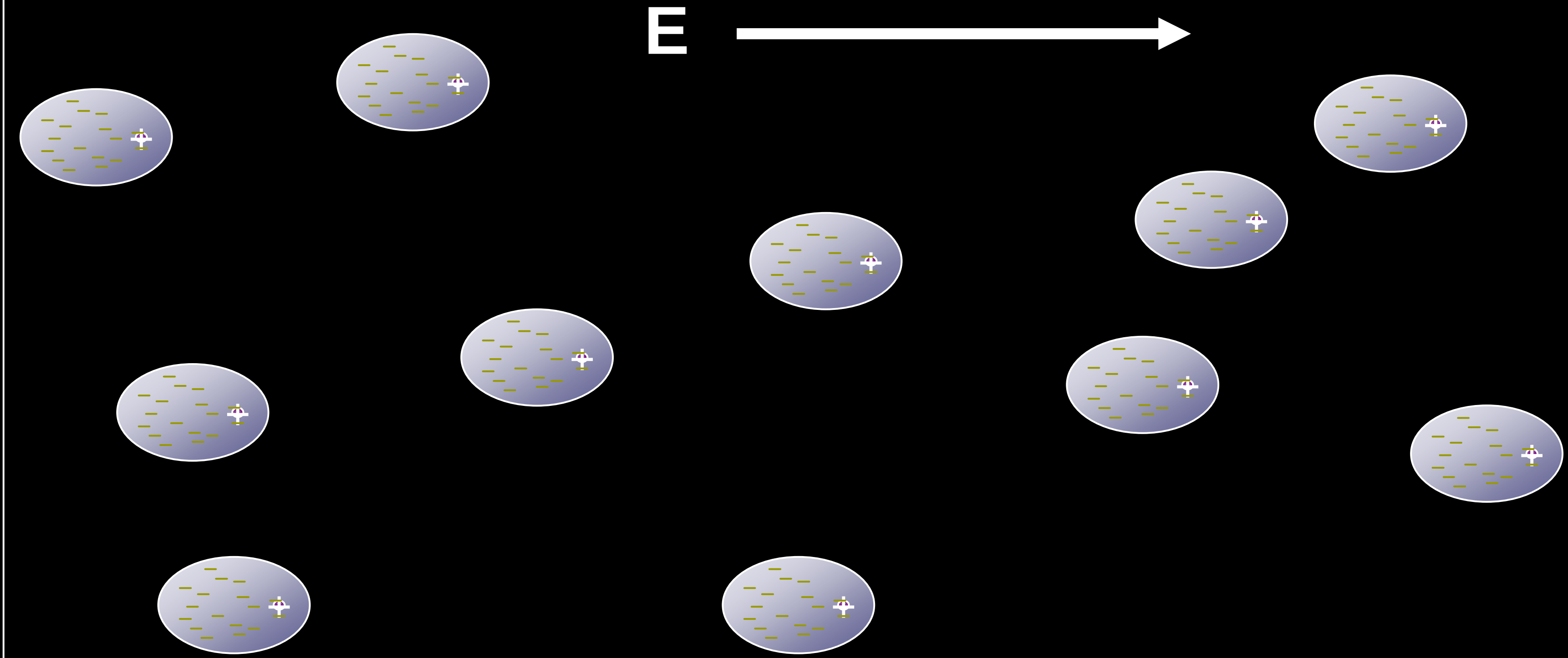
- In many materials at low field strengths:
  - $\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E}$ where χ_e is a constant.
  - $\mathbf{D} = \varepsilon_0 \mathbf{E} + \varepsilon_0 \chi_e \mathbf{E}$
  - $\mathbf{D} = \varepsilon_0 \mathbf{E}(1 + \chi_e)$
  - The expression $\varepsilon = \varepsilon_0(1+\chi_e)$ is called the "permittivity" of the dielectric
- Also Define $\varepsilon_r \equiv (1 + \chi_e)$ as → "relative permittivity" or also "dielectric constant".

D and **E** are vectors

Dielectric Constants of some materials

- Vacuum → 1.0
- Dry Air → 1.00054
- Benzene → 2.28
- Diamond → 5.7
- Water → 80.1
- $KTaNbO_3$ → 34,000
    - Wide range of values!

Back to our Gas problem



Using this idea we will predict some properties of the index of refraction in tenuous gasses.

Electric Fields in Matter – Part c

Dense Dielectrics

E, ***P***, and ***D***

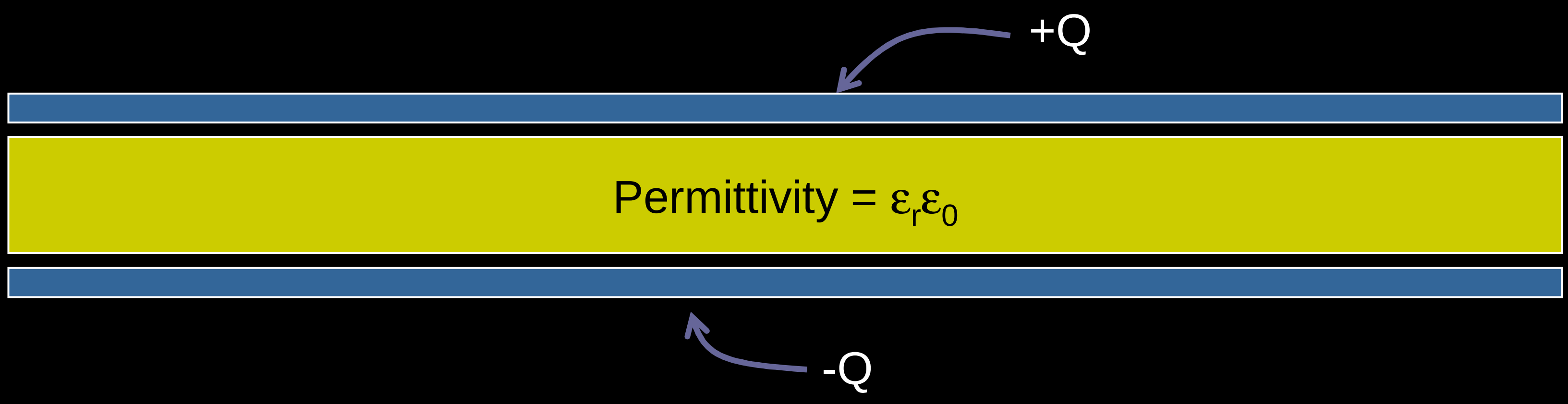
Since ε_r is Real

- Weak frequency dependence.
  - So let's ignore a lot of what we previously have done and just assume that we have a constant, or nearly constant, index of refraction.
- We can now solve problems in Electrostatics and Electrodynamics using this fact.
  - Much like what we did with **B** and **H** fields.
  - We now have **E** and **D** fields.

Write On Board

Use of Dielectrics

Two parallel metal plates with a dielectric in between them



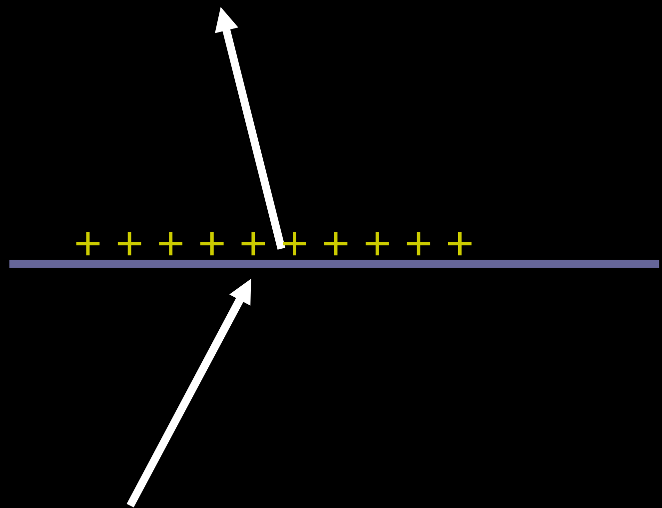
Here we can see the usefulness of **D**. We can find **D** and ignore the dielectric, at least initially.

Dielectric Effects Summary

- The dielectric *REDUCED* the electric field between the plates
  - We put the same charge on the plates though.
- The Capacitance increased.
  - All physical parameters of the capacitor stayed the same.
    - Highly desirable, since Q/C = V; we can put a lot of charge on the capacitor and keep the fields low.

D, *E*, and *P* boundary conditions

A Warning!



$D^{\perp}_{above}$ - $D^{\perp}_{below}$ = σ_f

The perpendicular component of the electric displacement vector is discontinuous to the value of the *free* surface charge.

But $E\|_{above}$ - $E\|_{below}$ = 0

The component of the electric field vector parallel to the surface is continuous.

Implies $D\|_{above}$ - $D\|_{below}$ = $P\|_{above}$ - $P\|_{below}$

A Big Assumption

- Any e-field generated by polarization of a gas molecule will die out by the time it gets to the next gas molecule – on average.
  - **Gas is rarefied. Much space between atoms**
  - **Glass is not. Atoms are packed tight.**
    - **There are quite a few things called ‘solids’ where this is true.**
    - **You might have encountered a few of them.**
- Let’s eliminate that assumption!

Lecture Summary

- All dielectrics respond to external electric fields by forming small dipoles.
  - Some are already polar molecules
- Define a Polarization field to quantify this.
- Many dielectrics respond linearly
    - At least in THIS course they do!
- This polarization reduces the total E-field in the material
  - Boundary conditions elucidated.
- Now ready to go back to boundary value problems and include dielectrics in our solutions.