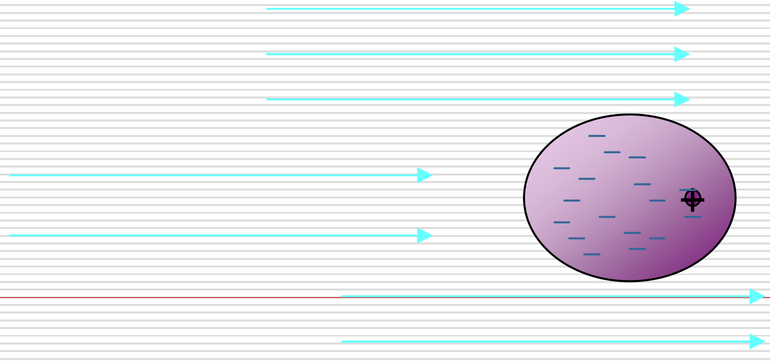
EM Waves in Plasmas

On the way to more dense materials

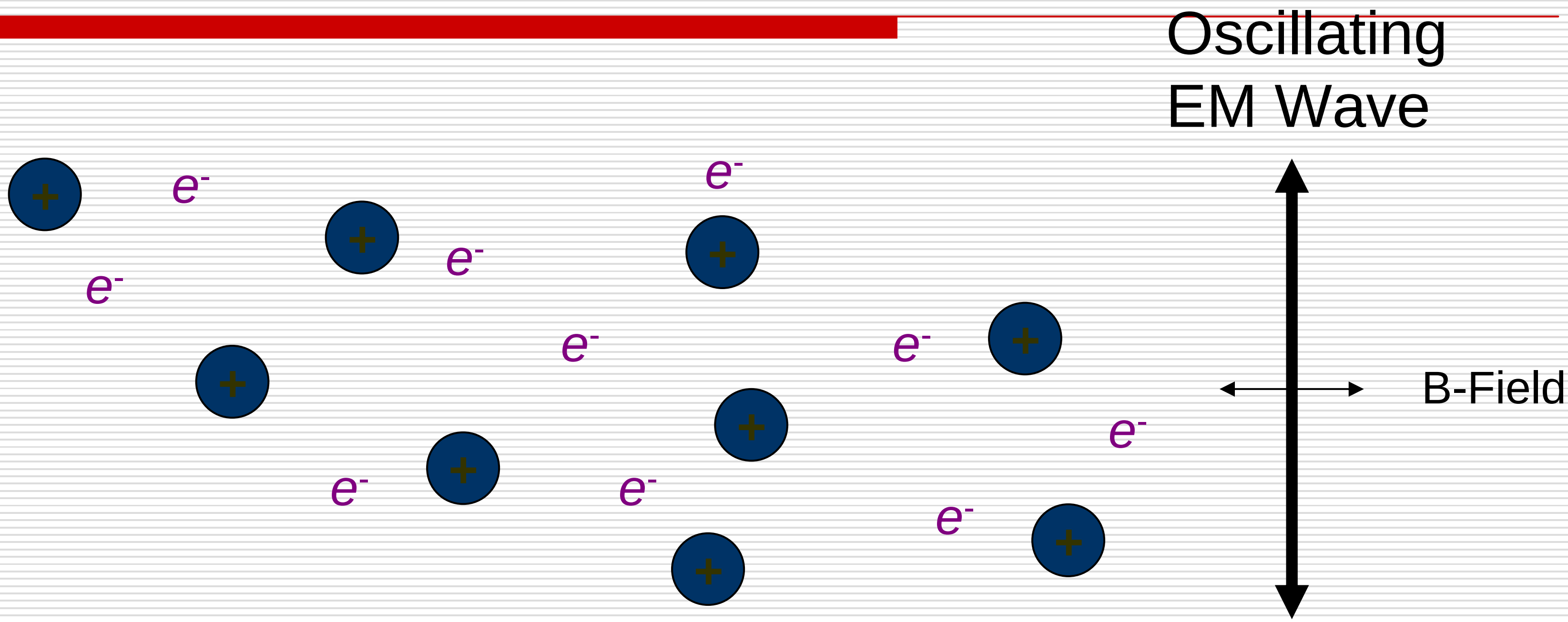
EM waves

- Thus far we have only done Maxwell’s equations in Free space.
- What happens when an electromagnetic wave encounters free charges?



In an electric Field,
We expect the charges to shift around.

Free Charges



This is like a plasma. The total charge is neutral. But the charges are free move.

The electrons move more quickly than any positive ions because of their small mass relative to the ions (or protons).
The magnetic field is not going to have a big influence. → E/c = B

Motion of Charges

- We know the classical law for motion of charges in Electric and Magnetic fields
- Mass of electron is 2000 times less than even the proton
  - So it's the electrons we worry about most.

$$m_e\vec{a} = q\vec{E} + q\vec{v} \times \vec{B}$$

Motion of Charges

- Have a look at one of Maxwell’s equations again though.

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

- Notice that this equation depends on the current density.
  - A “current” means that charges are moving around!

Time-Varying Fields

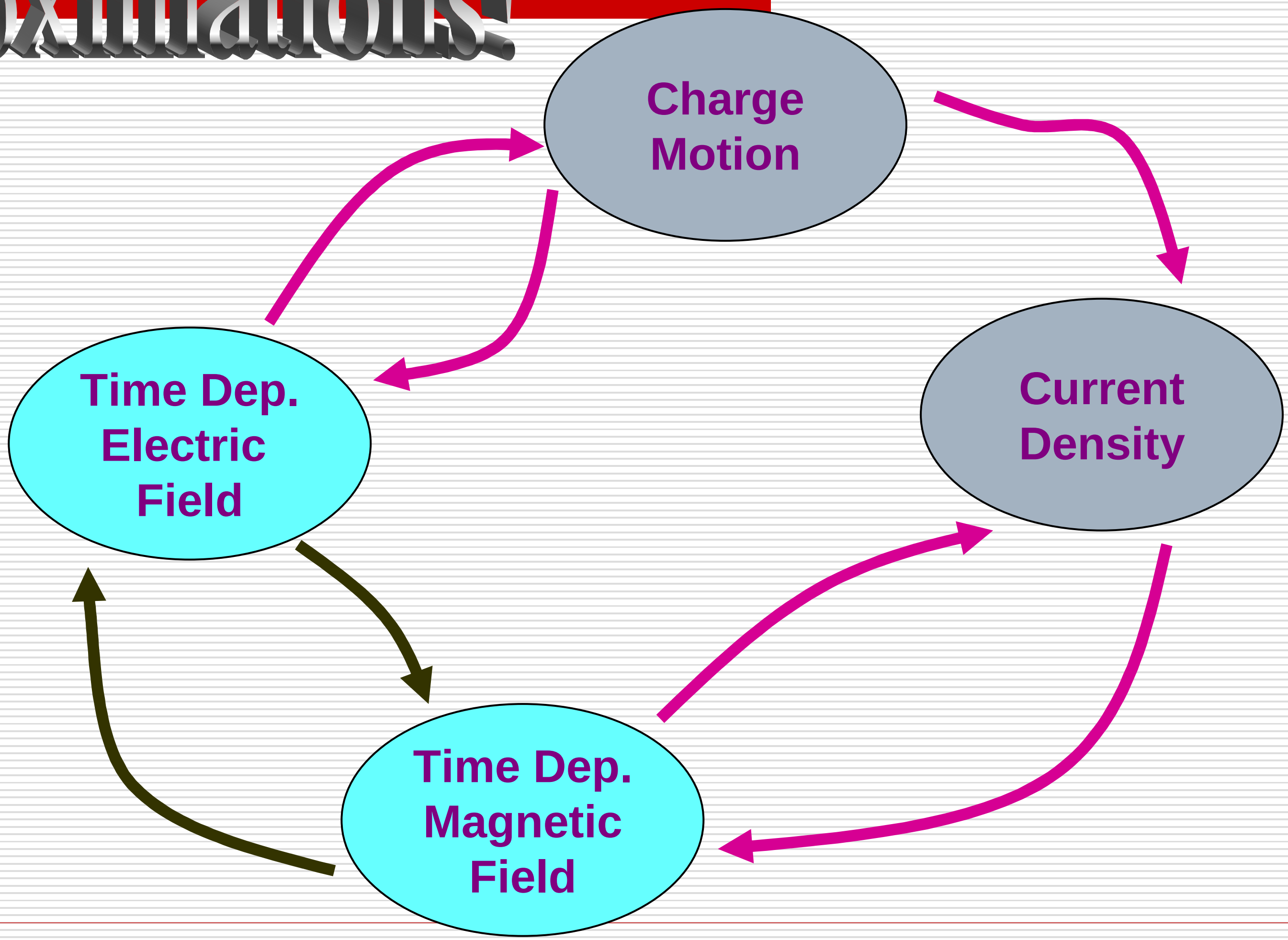
- The B-field depends on the current and on the E-field.
- But Maxwell's equations couple the E-field to the B-field as well!
  - Cannot forget this!

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

State of Play

Approximations!

Bit tough to get one's mind around it. Where to start?



Approximations!

- First
  - We do not yet know what the E-field is. But we do know about what happens in a vacuum with a plane-wave.
    - E/c = B
    - C → PBN (Pretty Big Number)
  - So we first need to start off close to a vacuum.

Approximations!

- Very Low Charge concentration.
  - Let ‘N’ = number of charges/volume
  - Charges far apart; will not effect each other.
- Magnetic Fields will not move charges very much.
  - Only the Electric Fields will supply the force for charge motion.
- Since one type of particle carrying a charge is much lighter than the other, the current is due to the lighter charge only.
  - The heavy ions do not move at all.

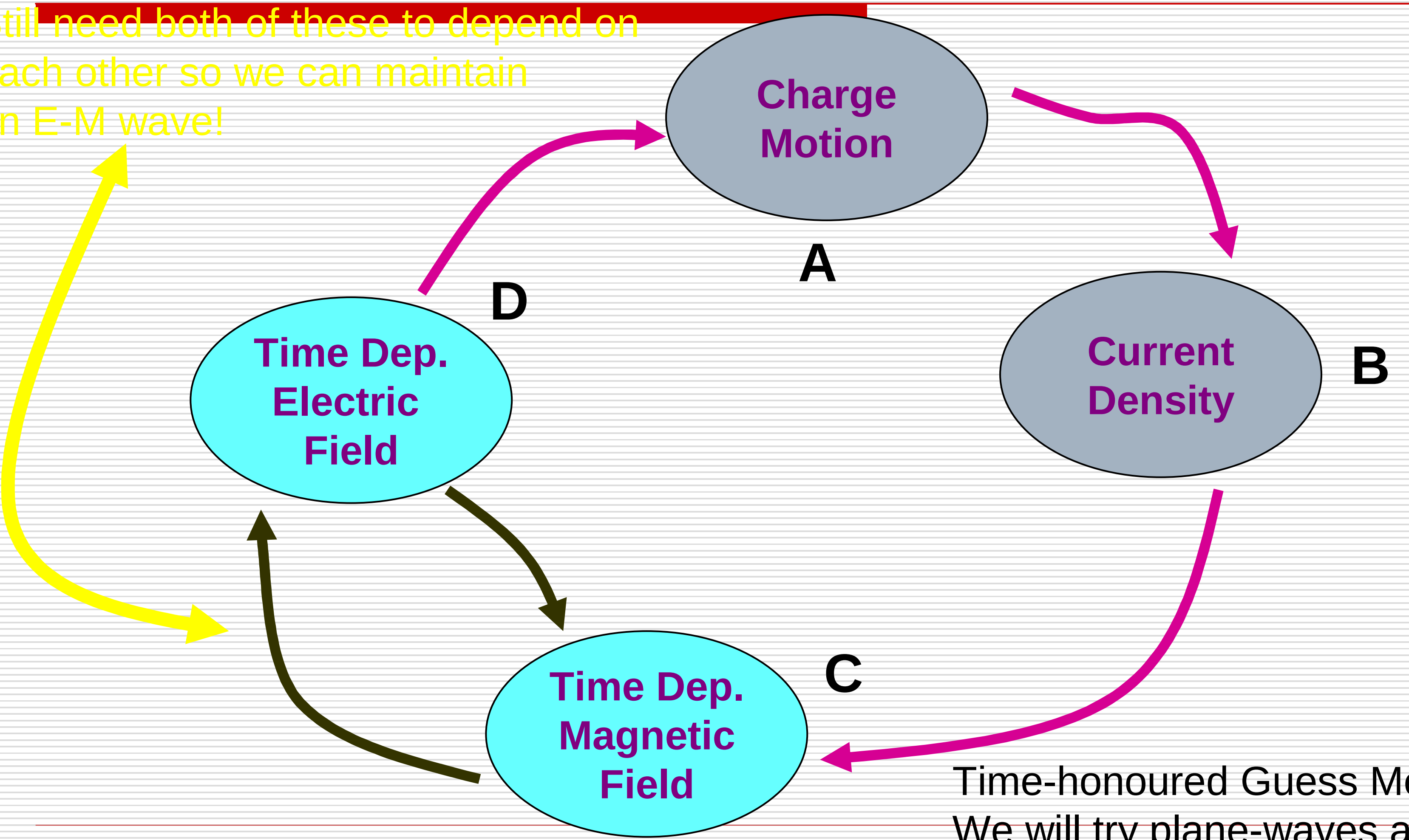
How does this alter our state of play?

State of Play

But this is easier…

Don't Panic!

Still need both of these to depend on each other so we can maintain an E-M wave!



Time-honoured Guess Method
We will try plane-waves and see if that will work!

To the Board!

We will start with one charge moving in an Electric Field and then work our way up.

Easiest thing I can think to do!

Current Density

- Current of single charge →

$$j_x = qv_x = i\frac{q^2}{\omega m_e}E_x$$

- Put in the number density 'N' to get the final current density. →

$$J_x = i\frac{Nq^2}{\omega m_e}E_x$$

Maxwell's Equations

- We will <u>not</u> assume **J**=0 this time.

$$\nabla \times \nabla \times \vec{E} = -\frac{\partial}{\partial t}\left(\nabla \times \vec{B}\right)$$

$$\nabla\left(\nabla \cdot \vec{E}\right) - \nabla^2 \vec{E} = -\frac{\partial}{\partial t}\left(\nabla \times \vec{B}\right)$$

0

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

To Board!

Parameters for the Ionisphere

- Charge Density (N)
  - At most 10^6 elec/cm^3
- Mass of electron (m_e)
  - 511 KeV/c^2 = 9.11×10^{-31} kg
- These imply a plasma frequency of:
  - 9 MHz → for Earth's ionisphere

Summary

- We now have two cases
  - $n \rightarrow$ real
  - $n \rightarrow$ imaginary
- Next time:
  - Explore how an EM wave will propagate inside a metal and then a low-pressure gas.
  - Both are cases where our approximation that the charges are 'far apart' can hold.