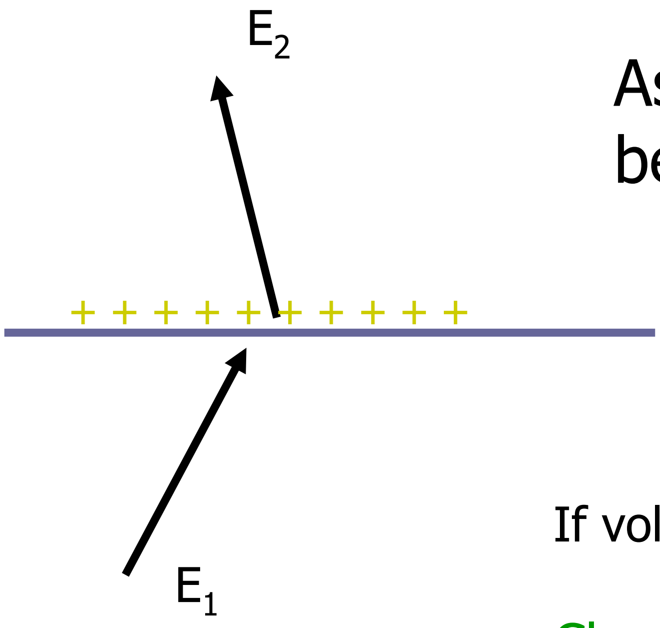
More details

A. Boundary Conditions
B. Electric Potential Solutions with spherical or Cylindrical symmetry

Boundary Conditions



Assume no charge in the space above or below the surface shown.

If volume 1 were a metal, E1 = 0.

Clearly there must be some kind of discontinuous behaviour of electric fields at a charged boundary.

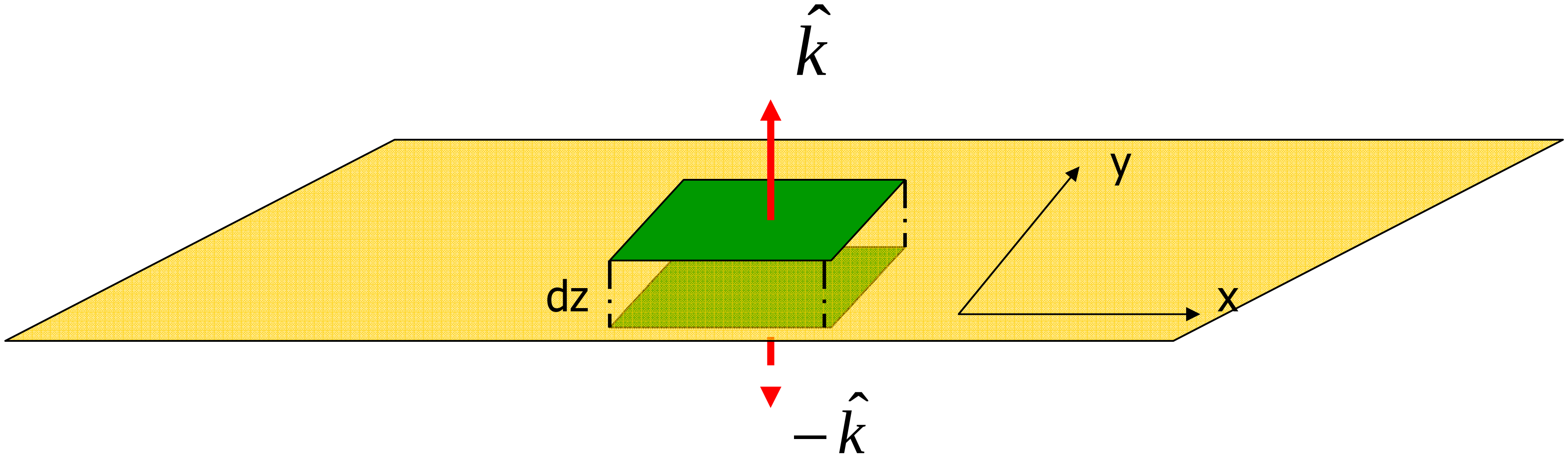
Boundary Conditions

- The differential and integral forms of Gauss's Law must always hold.

$$\nabla \cdot \vec{E} = \rho(\vec{r}) \big/ \varepsilon_0$$

$$\oint_s \vec{E} \cdot d\vec{l} = \int_V \frac{\rho(\vec{r})}{\varepsilon_0} d\vec{r}$$

Gauss's Law on small region of the boundary.



$$\oint_s \vec{E} \cdot d\vec{l} = \int_V \frac{\rho(\vec{r})}{\varepsilon_0} d\vec{r}$$

Separation in Cylindrical Coordinates

$$\frac{1}{Z(z)}\frac{\partial^2 Z}{\partial z^2} = k^2$$

$$\frac{1}{\Phi(\phi)}\frac{\partial^2 \Phi}{\partial \phi^2} = -m^2$$

m must be either zero or an integer.

$$\rho^2 \frac{\partial^2 R}{\partial \rho^2} + \rho \frac{\partial R}{\partial \rho} + \left(k^2 \rho^2 - m^2\right) R(\rho) = 0$$

Bessel's Equation

Separation in Spherical Coordinates

$$\frac{1}{\Phi(\phi)}\frac{\partial^2\Phi}{\partial\phi^2} = -m^2$$

m must be either zero or an integer.

$$\frac{1}{\Theta\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Theta}{\partial\theta}\right) = -l(l+1)$$

Legendre's Equation

$$r^2\frac{\partial^2 R}{\partial r^2} + 2r\frac{\partial R}{\partial r} - l(l+1)R = 0$$

Example in Spherical coordinates

- A sphere of radius ‘b’.
- There is a θ dependent potential on the sphere.