Second Year Lectures on Electromagnetism.
Problem Set 6

Hilary Term 2006.

1. Discuss the basis of the 'Telegraph Equations' for a transmission line

$$\frac{\partial V}{\partial x} = -L\frac{\partial I}{\partial t} - RI, \quad \frac{\partial I}{\partial x} = -C\frac{\partial V}{\partial t} - GV$$

   where the symbols have their conventional meanings, defined per unit length of line.

   For a loss-free line (G=R=0) show that V and I satisfy the wave equation. Determine the wave velocity v.

   Find the corresponding solution for I if counter-propagating waves are present and $V(x,t) = f(x - vt) + g(x + vt)$.

2. a) A transmission line consists of a pair of cylindrical conductors of radius r and separation d in air. Derive the capacitance per unit length C, and the inductance per unit length, assuming that $r \ll d$. Determine the wave velocity and characteristic impedance.

   b) A transmission line consists of a cylindrical conductor of radius a at a distance h from an infinite conducting ground plane in air Deduce C and L per unit length.

   c) A strip line comprises a thin copper strip of width w separated from an infinite conducting ground plane by a thin insulating layer of thickness t and relative dielectric permittivity ϵ_r. estimate C and L per unit length and the characteristic impedance assuming that $t \ll w$.

3. A coaxial cable which is filled with a medium of dielectric permittivity $\epsilon = 2$, has characteristic impedance 50 ohms. Calculate the ratio of the inner and outer conductor diameters.

4. If Z_1 and Z_2 are the input impedances of a given length ℓ of transmission line when terminated by an open and closed circuit respectively, show that $Z_1 Z_2 = Z_0^2$, independent of ℓ, where Z_0 is the characteristic impedance of the line.

5. Find the input impedance of a composite transmission line comprising a quarter wavelength line of impedance Z_2 followed by a terminated line of impedance Z_3.

   Show how this result may be used to match a transmission line of characteristic impedance Z_1 to another of impedance Z_3, i.e. a connection is required which will not generate a reflected wave. Show that this is possible if the two are joined by a quarter wavelength section of line of impedance Z_2, given by $Z_2 = \sqrt{Z_1 Z_3}$.

6. A plane electromagnetic wave travelling in a medium ($\epsilon = \epsilon_1, \mu = 1$) is incident normally on the plane surface of a second medium ($\epsilon = \epsilon_2, \mu = 1$). Show that the reflected wave can be eliminated by coating the surface with a quarter-wavelength thick layer of a medium for which $\epsilon = \sqrt{\epsilon_1 \epsilon_2}, \mu = 1$. Hint: Handle the problem by determining the characteristic impedance of transverse waves in each medium and drawing an analog with transverse waves in a transmission line whose length is a quarter-wavelength. (cf problem 4:4.)

7. A section of loss-free transmission line has length 1.0m and characteristic impedance 600 Ω. A terminating resistor of 100 Ω is connected across one end and the input impedance across the other end is measured. It is found that the input impedance is real and 100 Ω at 100 MHz, 200 MHz and 300 MHz, and is real and larger at 50 MHz and 150 MHz. Give an interpretation of these observations and deduce the input impedance at 50 MHz. What is the relative permittivity of the non-magnetic dielectric between the conductors?

8. Show that a 1/8 wavelength open circuited transmission line and a 3/8 wavelength short circuited transmission line both have input impedances equivalent to a pure capacitance, $C = 1/\omega Z_0$.

9. (*Beyond the syllabus*). Draw sketches to show the angular dependence of electric and magnetic field strengths in the radiation from an electric dipole aerial. Show also the angular distribution and direction of the Poynting vector and the magnetic vector potential $\mathbf{A}$.