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2nd year electronics

Lecture notes

Contents

Reminder of 1st year material

Passive sign convention

The passive sign convention is the standard definition of power in electric circuits. It defines electric power flowing from the circuit into an electrical component as positive, and power flowing into the circuit out of a component as negative. A passive component which consumes power, such as a resistor, will have positive power dissipation. Active components, sources of power such as electric generators or batteries, can have positive or negative power dissipation if there are more than one of them in a circuit, but if there is only one, it will have negative power dissipation.

The practical application of this principle of power for passive circuit elements is that one must label the positive terminal of the passive element as the one in which current flows. One can choose whether to first label the positive voltage terminal or to choose the direction of current flow, but once one of these two options is chosen, the other must be set accordingly so that the current flows into the positive terminal.

Kirchhoff's laws

There are two circuit laws first described by the German physicist Gustav Kirchhoff in 1845, which are extremely useful to understand any electrical circuit. The first one deals with the currents in the circuit:

> *At any point in the circuit the sum of currents flowing in is equal to the sum of currents flowing out.*

This is also called 'Kirchhoff's current law' or KCL. For example see figure 1. Here the KCL at the node gives

$$\sum_n I_n = I_1 + I_2 - I_3 - I_4 = 0.$$

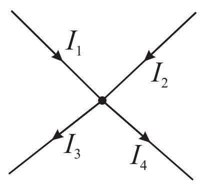


Figure 1: Example for Kirchhoff's current law.

For physicists this is an obvious consequence of the charge carrier densities following a continuity equation, and ultimately local conservation of charge.

The second of Kirchhoff's laws is Kirchhoff's voltage law (KVL):

> *The directed sum of the electrical potential differences (voltage) around any closed network is zero.*

The direction of the potential drops across resistances is given by the direction of the currents flowing through the resistor. For an example see figure 2. For this circuit

$$\sum_n V_n = -V_0 + IR_1 + IR_2 + IR_3 = 0.$$

The KVL is a consequence of the conservative character of the electromagnetic force.

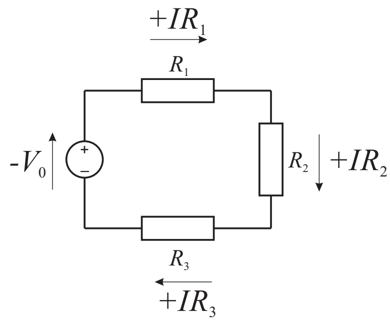


Figure 2: Example for Kirchhoff's voltage law.

Network replacement theorems

Any linear electrical network with voltage and current sources and resistances can be replaced by an equivalent source and an equivalent resistor. Two types of equivalences are possible:

These theorems can be extended to capacitances and inductances when they are expressed by their complex impedances (see next section below).

- Thevenin equivalent: The circuit can be replaced by an equivalent voltage source in series with an equivalent resistance. The equivalent voltage is the voltage obtained at the terminals of the network when they are not connected. The equivalent resistance is the resistance between the terminals if all voltage sources in the circuit are replaced by a short circuit and all current sources are replaced by an open circuit.

A voltage source is an electrical component which generates and maintains a difference in the electrical potential between its terminals, independent of the load (current).

- Norton equivalent: The circuit can also be replaced by an equivalent current source in parallel with an equivalent resistance. The equivalent current is the current obtained if the terminals of the network are short-circuited. The equivalent resistance is again the resistance obtained between the terminals of the network when all its voltage sources are shorted and all its current sources open circuit.

A current source is a component which provides a defined current, independent of the voltage between its terminals.

The equivalent resistances for the two cases are the same, and the equivalent voltage and current sources are related as $V_{\text{Thevenin}} = R_{\text{eq}} I_{\text{Norton}}$ with the equivalent resistance R_{eq}. The equivalent resistance can therefore be found from the ratio of the voltage between the terminals with no load connected (V_{Thevenin}) divided by the current flowing when the terminals are shorted (I_{Norton}).

AC circuit theory

Alternating currents (AC) can be described by a harmonic time dependence $V(t) = V_0 \cos(\omega t)$. More complicated time dependencies can be expressed by the superposition of harmonic components with different frequencies using a Fourier series. In general the Fourier series is complex and we therefore describe the voltage by $V = V_0 e^{j\omega t}$. The complex phase introduced by this generalization is relevant as different parameters in a circuit can have a different complex phase, equivalent to a phase shift of their harmonic development.

Electronics engineers prefer the use of the letter 'j' for the imaginary unit, to distinguish it from 'i', which is often used to denote a current. We will follow this convention.

In passive circuits we can use a generalized form of Ohm's law

$$V = ZI, \tag{1}$$

where the impedance of a pure resistor is $Z = R$. For a pure inductor $Z = j\omega L$ (can be easily seen from the definition of the self-inductance

$V = L\frac{\mathrm{d}I}{\mathrm{d}t}$), and for a pure capacitance $Z = (j\omega C)^{-1}$ (from $Q = VC$ and $I = \frac{\mathrm{d}Q}{\mathrm{d}t}$). These can be combined, and the overall impedance of a passive network can be written as

$$Z = |Z|e^{j\phi}. \qquad (2)$$

The current is then given by

$$I = \frac{V}{Z} = \frac{V_0 e^{j\omega t}}{|Z|e^{j\phi}} = \frac{V_0}{|Z|}e^{j(\omega t - \phi)}.$$

$|Z|$ gives the ratio of magnitudes of V and I, and ϕ gives the phase difference by which the current lags the voltage.

Notice that the time-dependent part is a common factor for voltage and current, so $e^{j\omega t}$ can be omitted, but it is understood to be present when returning to the time domain.

Warning: this is only true for circuits with *linear* behaviour.

Ideal op-amps

An ideal op-amp is a differential amplifier: it's output is $V_{\text{out}} = A(V_+ - V_-)$, with A the open-loop gain. Ideally, the open-loop gain is very large ($A \to \infty$), and the inputs have infinite input impedance (no current is flowing into the '+' and '-' inputs). The output impedance of the ideal op-amp is 0. Often op-amps are used in circuits with negative feedback (for examples see figures 3 and 4). In circuits with negative feedback the voltages at the two inputs adjust until they are equal, so that $V_+ = V_-$. This equality is sometimes referred to as a 'virtual' short.

Negative feedback circuits are circuits where the output is connected (through a resistor) back to the input, reducing the input (here connecting back to the '-' input).

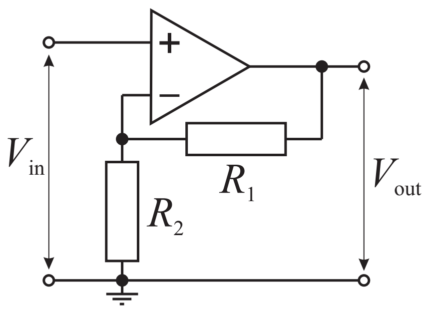


Figure 3: Non-inverting amplifier. The ideal gain of this circuit is $V_{\text{out}}/V_{\text{in}} = (R_1 + R_2)/R_2$ (You can see that from $V_+ = V_-$ and the voltage divider R_1 and R_2).

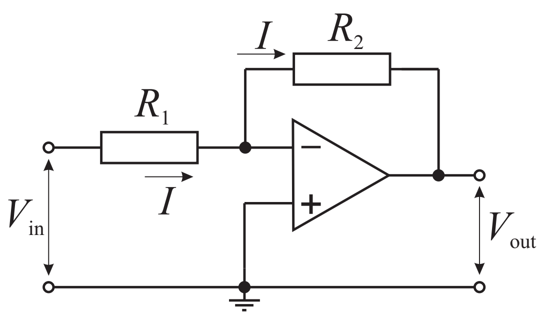


Figure 4: Inverting amplifier. The ideal gain of this circuit is $V_{\text{out}}/V_{\text{in}} = -R_2/R_1$ (You can see that from $V_- = 0$ and the currents through the resistors must be equal as there is no current flowing into the op-amp).

More realistic op-amps

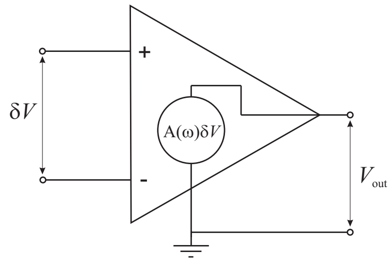


Figure 5: More realistic op-amp replacement diagram.

A more realistic model of the op-amp will encompass a finite and frequency-dependent open-loop gain. Commonly the frequency response will be similar to a simple low-pass RC filter. Stray capacitances within the circuit would cause such a behaviour, but often it is achieved by design and the deliberate use of capacitances in the circuit, to prevent instabilities at high frequencies. Typical DC gains for real op-amps are about 10^6, and the gain starts to decrease above a frequency around $\omega = 1$ rad/s (see figure 6 for an example).

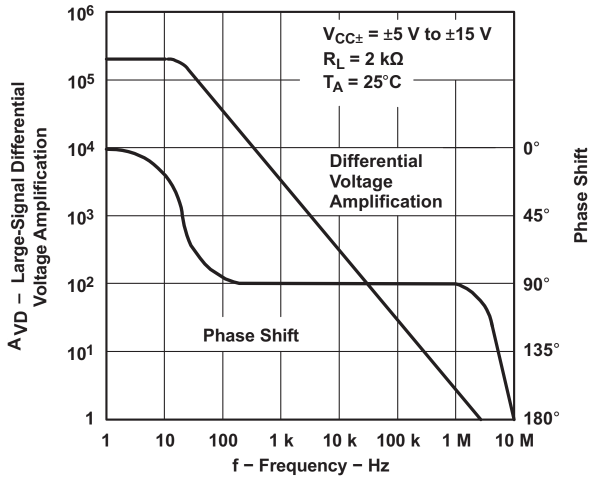


Figure 6: Frequency response of the TL081, a widely used operational amplifier from Texas Instruments (datasheet from http://www.ti.com/lit/ds/symlink/tl081.pdf). This specific op-amp has a slightly lower gain ($\sim 2 \times 10^5$) and cuts-off at a slightly higher frequency ($f_{\text{cut-off}} \simeq 20$ Hz) than described in the text.

A simple replacement circuit reproducing the behaviour around the roll-off and up to reasonably high frequencies ($\sim$ 1 MHz) is shown in fig. 7.

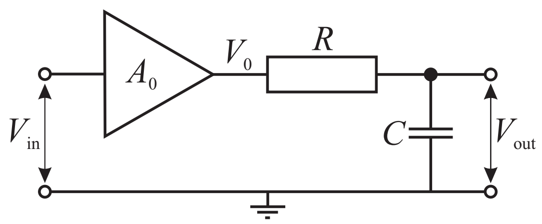


Figure 7: Simple replacement circuit to model a real op-amp.

The amplifier in this circuit is again an ideal op-amp with frequency-independent open-loop gain A_0, so that $V_0 = A_0 V_{\text{in}}$. Here we assume that the load which we will connect to this circuit has infinite input impedance, so that due to the KCL the current the resistor equals the current through the capacitor

$$\frac{V_{\text{out}} - V_0}{R} + j\omega C V_{\text{out}} = 0.$$

Substituting V_0 yields

$$A(\omega) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{A_0}{1 + j\omega CR} = \frac{A_0}{1 + j\frac{\omega}{\omega_0}}, \tag{3}$$

with the cut-off frequency ω_0.

What effect does this frequency characteristics have on the gain in a negative feedback circuit? We will study this here for the example of an inverting amplifier (figure 4). We start with the KVL for the input and the feedback legs:

$$V_{R_1} - \delta V - V_{\text{in}} = 0$$
$$V_{R_2} + \delta V + V_{\text{out}} = 0.$$

Note that we do not assume $\delta V = 0$ anymore.

We still assume that no current is flowing into the op-amp and therefore, using the KCL,

$$\frac{V_{R_1}}{R_1} = \frac{V_{R_2}}{R_2}.$$

Combining these equations yields

$$\frac{V_{\text{in}} + \delta V}{R_1} = -\frac{V_{\text{out}} + \delta V}{R_2},$$

or

$$\frac{V_{\text{in}}}{R_1} = -\frac{V_{\text{out}}}{R_2} - \delta V \left(\frac{1}{R_2} + \frac{1}{R_1}\right).$$

We can now use the gain characteristics described before

$$V_{\text{out}} = A(\omega)\delta V,$$

and we get

$$V_{\text{in}} = -\frac{R_1}{R_2} V_{\text{out}} - \frac{V_{\text{out}}}{A(\omega)} R_1 \left(\frac{1}{R_2} + \frac{1}{R_1}\right),$$

and

$$\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{R_2}{R_1 + \frac{R_1 + R_2}{A(\omega)}}.$$

With the parametrization in eq. (3) $A(\omega) = A_0(1 + j\omega RC)^{-1}$ this yields

$$\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{R_2}{R_1 + \frac{(R_1 + R_2)(1 + j\omega/\omega_0)}{A_0}}.$$

A further approximation is that to achieve some gain $R_2 \gg R_1$ and therefore

$$\frac{V_{\text{out}}}{V_{\text{in}}} \simeq -\frac{R_2}{R_1 + \frac{R_2(1 + j\omega/\omega_0)}{A_0}}.$$

This equation is demonstrated in figure 8. Note that despite the strong frequency dependency of the open loop gain the gain of the feedback circuit is constant up to much higher frequencies (10^4 rad/s compared to $\omega_0 = 1$ rad/s in this example). This is because the amplifier only needs to provide the gain to uphold the feedback, which is much lower than the open loop gain it can provide at low frequency.

The frequency response for the non-inverting amplifier has a similar behaviour.

You will see this in the EL16 practical.

The next level of op-amp imperfection which could be considered for an even more realistic op-amp model would be the finite input and output impedances.

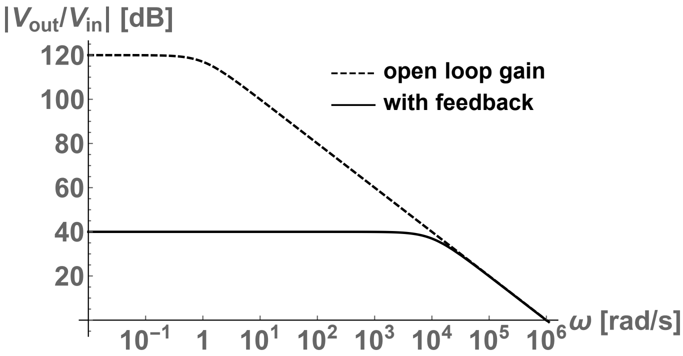


Figure 8: Frequency response of inverting amplifier ($A_0 = 10^6$, $\omega_0 = 1$ rad/s, $R_2 = 10\,\mathrm{k\Omega}$ and $R_1 = 100\,\Omega$).

dB is a unit often used in electronics. It is defined as $20\log_{10}(V_{out}/V_{in})$.

Semiconductors

Let's now investigate the components which allow us to build active circuits like operational amplifiers. Today almost all these components are made from semiconductors, typically silicon or germanium.

In this course we will use very simple models for the behaviour of semiconductor devices like the diode or the transistor in electrical circuits which do not require a detailed understanding of the quantum mechanics and the solid state physics in the semiconductor. If you are interested in these topics there is a slightly more detailed discussion in the appendix of this document and further material can be found in the Practical Course Electronics Manual.

Diodes

Due to the limited time available we will not discuss diodes in the lectures, but this section is part of the notes to give useful background material.

A circuit diagram representation of a diode is shown in figure 9.

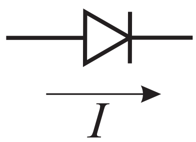


Figure 9: Circuit diagram representation of a diode.

In a diode there is one direction in which a current will easily flow (indicated by the arrow in figure 9). A current in the opposite direction will be blocked.

The number of charge carriers which are capable of entering the valence band at a temperature T is given by the Boltzmann distribution and therefore the current-voltage relation for a diode is given by

$$I = I_0 \left(e^{\frac{eV_D}{k_B T}} - 1 \right), \tag{4}$$

This equation is also known as the Shockley diode equation.

A more general version of this equation includes an ideality factor n (typically varies from 1 to 2) in the denominator of the exponential, to account for imperfections of junctions in real devices.

where I is the current through the diode and V_D the voltage across the diode. At room temperature $k_B T/e \simeq 25$ mV. I_0 is the reverse bias saturation current. Its exact value depends on the doping concentrations and the temperature, but is typically of the order $I_0 \simeq 10^{-10}$ mA for silicon diodes. The resulting characteristics is shown in figure 10. Note that the second term in the bracket quickly becomes negligible against the exponential for positive bias voltages, and can be omitted in that case.

It should be obvious that a diode is a highly non-linear device. Ohm's law does not apply. Other tricks that don't work are

1. the Norton equivalent,
2. the Thevenin equivalent, and

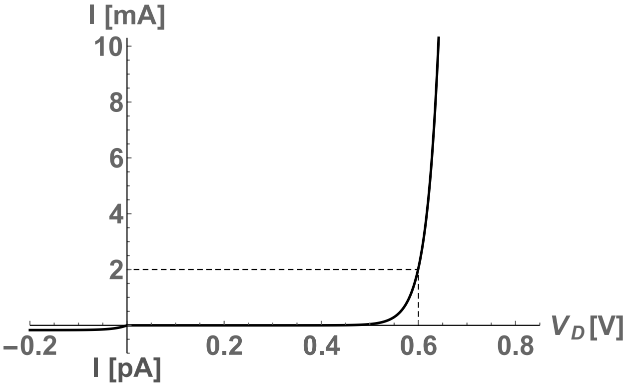


Figure 10: Diode current-voltage relation. The scale on the positive and negative y-axis differs by a factor 10^9.

For a real diode there is a breakdown voltage at which the diode becomes conductive in reverse bias (not shown in this figure). Usually this breakdown is avoided in circuit design, because its properties are badly defined, but there is a special class of diodes, so called Zener diodes, which exploit this breakdown.

3. superposition.

Still valid are KCL and KVL and the relation for power ($P = VI$, but not the $P = I^2R$ or $P = V^2/R$ variants).

A simple circuit with a diode is shown in figure 11. How can we find the operating point of this circuit?

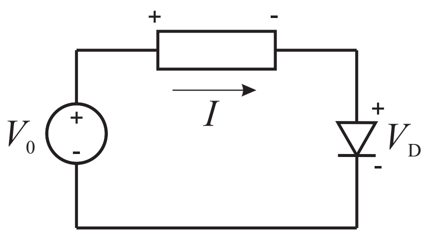


Figure 11: A simple circuit with a diode.

Note that the diode is not an active component and therefore not a power source, which justifies the signs in the figure.

We have

$$V_0 = V_R + V_D$$

and

$$\begin{aligned} I &= I_0\left(e^{\frac{eV_D}{k_BT}} - 1\right), \text{ and} \\ I &= \frac{V_R}{R} = \frac{V_0 - V_D}{R}. \end{aligned} \tag{5}$$

The easiest way to solve these two equations for I and V_D is by graphical or numerical means. Each of these equations can be plotted in a current-voltage diagram for the diode and where the two intersect is the operating point of the diode (figure 12).

This is very nice, and also correct, but difficult to do for more complicated circuits. Let's try a different approach.

We start with noting that

$$\frac{dI}{dV_D} = \frac{I_0 e}{k_BT} e^{\frac{eV_D}{k_BT}} \equiv \frac{1}{r_D},$$

where r_D is the small signal equivalent resistance of the diode. We have seen that at room temperature the forward bias is approximately 0.6 V. We can use $I \simeq I_0 e^{\frac{eV_D}{k_BT}}$ for forward bias to get

$$\frac{dV_D}{dI_D} = r_D \simeq \frac{25\,\text{mV}}{I_D} = \frac{25\,\Omega}{I_D\,[\text{mA}]}. \tag{6}$$

We will use the following notation convention: DC currents and voltages defining the working point are capitalized (e.g. V_0, I_D etc.). Small signals around that working point (and corresponding resistances) are written in small cursive script (e.g. v_{in}, i_D, r_D, etc.). These are often, but not necessarily, AC.

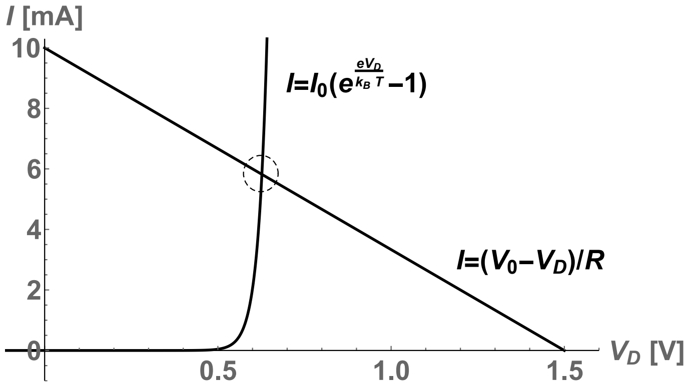


Figure 12: Finding the operating point for the diode ($V_0 = 1.5$ V, $R = 150\ \Omega$). Note that the voltage across the diode is always close to 0.6 V as long as the current is reasonably large. This voltage is sometimes referred to as the 'knee' voltage of the diode.

For small deviations from the operating point this gives a linear behaviour and we can use an equivalent linear circuit, which will work as long as we have "small signals" (figure 13). However, we should be aware that this equivalent resistance is not a constant, but depends on the operating point (current).

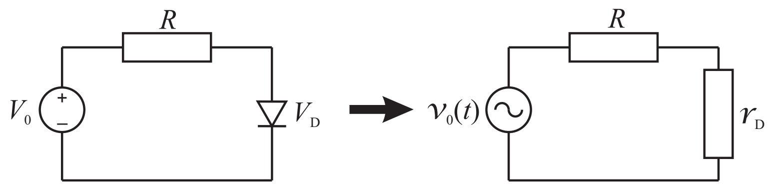


Figure 13: Equivalent small signal replacement for the diode.

For example, if $R = 1\ \mathrm{k}\Omega$ and $V_0 = 1.6$ V, then $V_R = V_0 - 0.6\ \mathrm{V} = 1$ V and $I = 1$ mA, implying that $r_\mathrm{D} = 25\ \Omega$.

We can use the same approach also for more complex circuits. This allows us to use all the tricks for linear circuits, but for small signals only.

Transistors

A bipolar junction transistor (BJT) consists of three regions with different doping (*pnp* or *npn*). The central doping region is called the base and the outer to regions are emitter and the collector. The two junctions form two back-to-back diodes, but with two important geometrical details (You cannot build a transistor from two discrete diodes). First, the central region (the base) is very thin so that minority charge carriers in this region (electrons in an *npn* transistor) can diffuse through to the collector. The second feature of a standard BJT is that its geometry is asymmetric, so that the collector completely surrounds the base to efficiently collect all charge carriers. In normal operation (active region) the emitter-base diode is forward biased (all discussions from the previous section apply), and the base-collector diode is reverse biased.

In this course we will focus on the BJT as one of the two most widely used types of transistors. The other important family of transistors are field effect transistors (FETs).
Here and in the following we will discuss the *npn* transistor. For *pnp* reverse all arrows (currents or diodes).

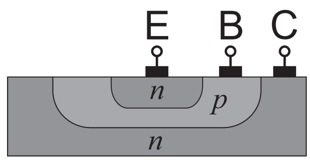


Figure 14: Cross-section of an *npn* BJT.

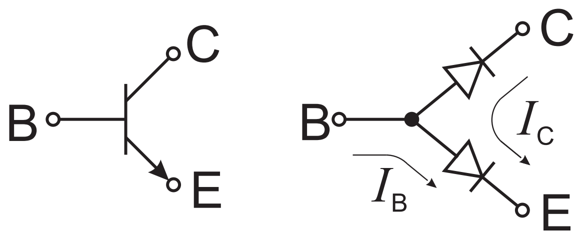


Figure 15: BJT circuit diagram symbol (left) and junction diodes (right). Note that as described in the text the actual junction geometry is different than the right schematics suggests.

In a very simple picture the transistor can be seen as a valve. You can use the valve as a switch (flow is on or off), and in between you can regulate the flow by the amount you open the valve. What makes the transistor work like a valve is the fact that the base current I_b flowing through the forward-biased base-emitter junction controls the current I_c from the collector to the emitter.

We will modify this argument slightly when we will discuss a slightly more quantitative model of the BJT.

This can be seen in the characteristic curves for the collector current of a BJT (figure 16). When the valve is closed $I_\mathrm{c} = 0$, and when it's open a large current can flow. If we take the analogy with a valve to the extreme then there is no pressure drop over a fully open valve, which is equivalent to $V_\mathrm{ce} \simeq 0$. In this case both junctions become forward biased and the transistor behaves almost ohmic (I_c increases with V_ce). This is referred to as "saturated region" (close to the y-axis in the left plot of fig. 16).

For us the most interesting region in figure 16 is the region with $V_{ce} > 0.6V$, which is also referred to as "active region". There the collector current is (to good approximation) proportional to the base current

$$I_\mathrm{c} = \beta I_\mathrm{b},$$

with β typically a very few 100. This proportionality is what makes the

Depending on the model used to describe the transistor behaviour you will sometimes see the proportionality factor written as h_FE. For our purposes these two parameters are identical.

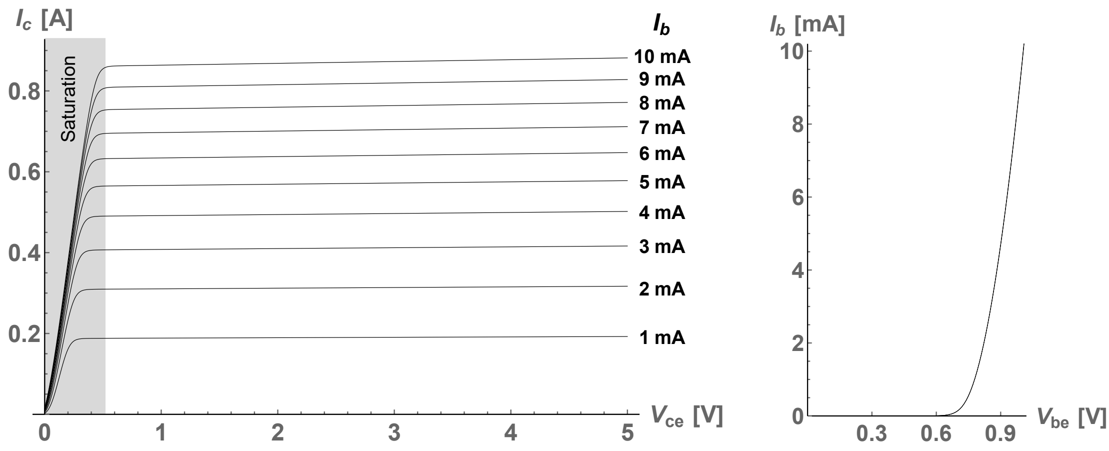


Figure 16: Characteristics of a BC337 BJT. Collector current versus V_{ce} (left) and base current versus V_{be} (right). The data has been obtained using MULTISIM, which uses the SPICE electronics simulation software.

transistor such a useful device. It is important, though, to appreciate that the exact value for β in real devices varies strongly (by several 100%) and the art of using transistors in electronics is to use them in circuits where the transistor provides amplification, but the exact gain is defined by other, more reproducible components (typically resistors). This typically entails a trade-off between gain and other desirable properties (linearity, stability, etc.).

One should be cautious about the analogy to the mechanical valve, though. Figure 16 also works the other way round. In a circuit where the collector current is set by external means the base current will adjust accordingly. Adjustment of the valve position in response to the flow is usually not a feature of mechanical flow valves.

To allow us to study transistor circuits in some more detail we will rely on a commonly used model of transistor behaviour, the Ebers-Moll model. In this model it is actually the base-emitter voltage V_{be} which controls the collector current,

$$I_{\text{c}} = I_0 \left(e^{\frac{eV_{\text{be}}}{k_{\text{B}}T}} - 1 \right). \tag{7}$$

The actual Ebers-Moll model is more detailed but we are using a simplified version, which is sufficient in the active region and at low frequencies.

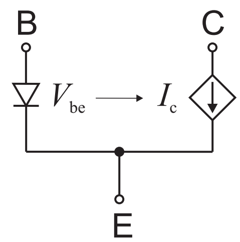


Figure 17: Simplified Ebers-Moll transistor model.

This equation is rather similar to the Shockley equation for the diode characteristics discussed before (eq. (4)). Again, the behaviour is highly non-linear. Significant currents will flow at $V_{\text{be}} \simeq 0.6$ V, and significant changes of I_{c} will correlate with tiny changes of V_{be}.

So, to effectively control the collector current, the base will have to be DC biased to 0.6 V. Small variations around that bias level will then result in sizable changes in the collector current. Again, we can focus on the analysis of these small fluctuations using a small signal analysis. For this we

1. assume $V_{\text{be}} = 0.6$ V,
2. assume $V_{\text{ce}} > 1$ V (transistor in active region),
3. assume $\beta \simeq 100$ (although, as discussed above, the circuit should not rely on a specific value for β),

4. solve the circuit for a linear version for small signals using KVL, KCL and Ohm's law, and re-think if an inconsistency is found.

Note that the Ebers-Moll equation describes a transconductance amplifier: a small change in input voltage results in a large change of current through the collector. Or, if we look again at small signals,

$$i_c = g_m v_{be}$$

with the transconductance g_m in units of Ω^{-1} or S (Siemens). In the Ebers-Moll model the transconductance is

In the past also the Mho was used as a unit, which is Ohm spelled backwards.

$$g_m = \frac{dI_c}{dV_{be}} = \frac{e}{k_B T} I_c, \tag{8}$$

where the derivative and I_c are evaluated at the working point. At room temperature again a good approximation is $g_m \simeq I_c/(25\text{ mV})$. The small signal equivalent circuit diagram for the transistor is shown in figure 18.

The working point is sometimes called the quiescent point, and the current I_c the quiescent current.

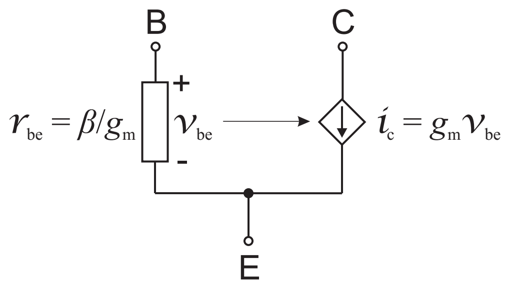


Figure 18: Equivalent small signal circuit diagram for a transistor.

The small signal resistance r_{be} can be found from

$$r_{be} = \frac{dv_{be}}{di_b} \simeq \frac{dv_{be}}{\frac{1}{\beta} di_c} = \frac{\beta}{g_m}. \tag{9}$$

A final addition to our phenomenological description of a transistor in the active region stems from the observation that I_c is not entirely independent of V_{ce} for a given base voltage (you can see this also in fig. 16). This is called the Early effect. It can be parametrized as

$$\frac{dV_{be}}{dV_{ce}} = -\alpha$$

for a fixed collector current, with $\alpha \simeq 10^{-4}$.

As this coefficient is small we will ignore this for most applications.

A first transistor amplifier circuit

A first transistor amplifier circuit is shown in figure 19.

As we will discuss later this circuit has some serious shortcomings and it should never be implemented like this, but it is instructive to look at how the assumptions above can be achieved, and the small signal gain found.

For simplicity we assume $V_{CC} = 10.6$ V and we will aim for a quiescent collector current of 2 mA. The first thing we need to do is to bias the circuit correctly. For this we assume $\beta = 100$, so that $I_b = I_c/100 = 20\ \mu$A.

It is a widely used practice to identify the power supply line which is close to the collector with V_{CC}. (Similarly a supply close to the emitter, but different than ground, is labeled as V_{EE}.)

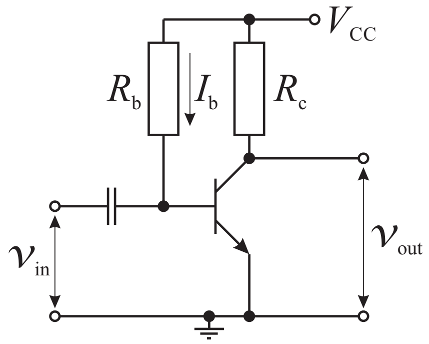


Figure 19: A first transistor amplifier circuit.

The DC current going into the base is the same current as is going through R_b, so that we need

$$R_b = \frac{V_{CC} - 0.6\,\text{V}}{20\,\mu\text{A}} = 500\,\text{k}\Omega.$$

480 kΩ or 560 kΩ is what you can buy as a single component, which would be perfectly adequate.

To maximize the possible output swing we choose a quiescent collector voltage in the middle of the available range, for example $V_c = 5$ V. This also puts us well into the active region of the transistor ($V_{ce} > 1$ V). This is achieved for

$$R_c = \frac{5\,\text{V}}{2\,\text{mA}} = 2.5\,\text{k}\Omega.$$

To obtain the small signal gain of the circuit we can assume that the input is AC. All bias is DC and is therefore like ground for the small signal analysis. The equivalent small signal circuit is therefore

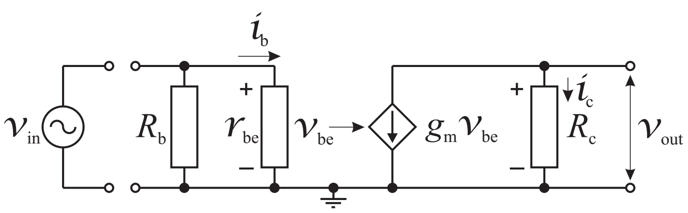


Figure 20: Small signal equivalent of the circuit in fig. 19.

From the left side of the equivalent circuit we can see that $v_{be} = v_{in}$. On the output side $v_{out} = i_c R_c$ (for now we will assume that any load connected to the output of this circuit has infinite impedance) and therefore

$$i_c = -g_m v_{be} = -g_m v_{in}$$

and

$$v_{out} = -R_c g_m v_{in}.$$

For our example we chose a working point which has $R_c = 2.5\,\text{k}\Omega$ and $g_m = 2\,\text{mA}/25\,\text{mV} = 0.08\,\Omega^{-1}$, and therefore the small signal gain for this circuit is

$$G = \frac{v_{out}}{v_{in}} = -R_c \frac{e}{k_B T} I_c = -200.$$

The gain is negative, it is an inverting amplifier.

However, this circuit has some serious short-comings:

1. For the biasing we have relied on $\beta = 100$. We have already said that this factor is highly device-dependent (even for different transistors of the same type). It is quite common for a different transistor of the same type to have $\beta = 400$, or $\beta = 50$. As we have programmed the base current by R_b, this would mean that the quiescent parameters (collector current and voltage) are all over the place.

2. This circuit is highly non-linear. If we have a signal with $\Delta v_\mathrm{in} = 5$ mV, then $\Delta v_\mathrm{out} = 200 \times 5$ mV = 1 V, so $\Delta I_\mathrm{c} = 0.5$ mA and the transconductance and with it the gain of the circuit will change by 50% over a cycle.

In practice this can be tolerated if the amplifier is used in a larger circuit with negative feedback.

3. The gain will depend on the impedance of the load. If a load with a resistance R_l is connected (this is just a resistance connecting the output to ground) the current through R_c is now shared between the transistor and the load and

$$g_\mathrm{m} v_\mathrm{in} = -i_\mathrm{c} - i_\mathrm{l} = -\frac{v_\mathrm{out}}{R_\mathrm{c}} - \frac{v_\mathrm{out}}{R_\mathrm{l}},$$

and therefore

$$G = \frac{v_\mathrm{out}}{v_\mathrm{in}} = g_\mathrm{m} \frac{R_\mathrm{c} R_\mathrm{l}}{R_\mathrm{c} + R_\mathrm{l}}.$$

If R_l is large the gain will be reasonable, but if it is small the gain will suffer.

The common-emitter amplifier

Technically the amplifier in the previous section is already a (grounded) common-emitter amplifier but we will use this name here for the more general and also capable circuit.

The amplifier circuit in figure 21 overcomes most of the issues observed in the previous section. It has its name because the input connects to the base and the output to the collector, and the emitter is shared between the two.

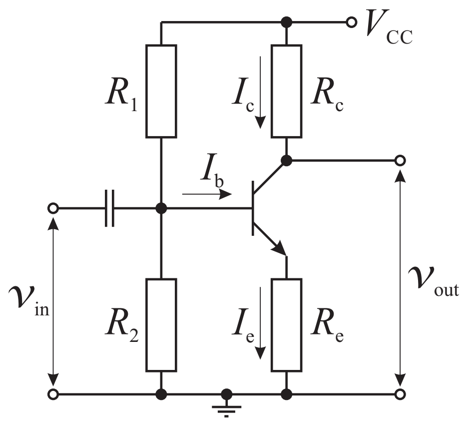


Figure 21: The common-emitter amplifier circuit.

We start again with our assumptions that $I_\mathrm{c} \simeq \beta I_\mathrm{b}$ and $V_\mathrm{be} \simeq 0.6$ V. From the first of these we can see that for a collector current around 2 mA the base current is about 20 μA. Therefore, as long as the resistors in the voltage divider R_1 and R_2 are in the range of 50 kΩ or lower, the base current will be an insignificant current leak from the voltage divider. The voltage in the divider (the voltage of the base) is therefore stiff against changes of the state of the circuit.

For this example we will use $R_1 = R_2 = 50\ \mathrm{k}\Omega$, which for a supply voltage $V_{CC} = 10$ V gives $V_b = 5$ V, and consequently $V_e = 4.4$ V. We can program the quiescent collector current by using $R_e = 2.2\ \mathrm{k}\Omega$ to achieve the required $I_c \simeq I_e = 2$ mA. The transconductance is, as before, $g_m = I_c/(25\ \mathrm{mA}) = 0.08\ \Omega^{-1}$.

$I_e = I_b + I_c = (\beta^{-1} + 1)I_c \simeq I_c$.

We can now calculate the small signal gain using again the equivalent small signal circuit (figure 22).

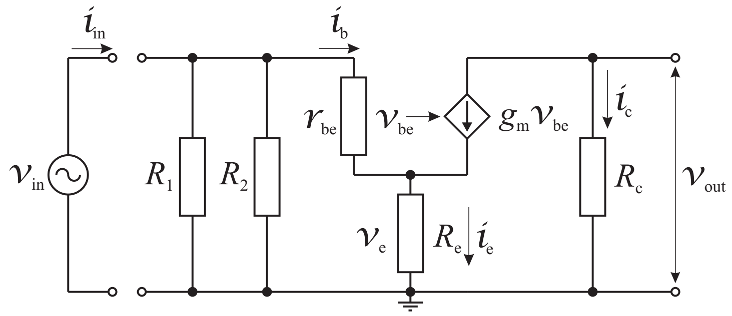


Figure 22: Small signal equivalent of the common-emitter amplifier circuit in fig. 21.

We write down the KVL for the input and the output loops, and the KCL for the node where they come together.

$$v_{in} = v_{be} + v_e = v_{be} + i_e R_e,$$
$$v_{out} = i_c R_c = -g_m v_{be} R_c,$$
$$i_e = g_m v_{be} + \frac{v_{be}}{r_{be}}.$$

Note that in these equations the resistors in the input voltage divider R_1 and R_2 are not present, they are only needed to define the working point.

Inserting the last equation into the first gives

$$v_{in} = v_{be} + g_m v_{be} R_e + \frac{v_{be} R_e}{r_{be}}.$$

We can then use the second equation to eliminate v_{be} to get

$$v_{in} = -\left(\frac{1}{g_m R_c} + \frac{R_e}{R_c} + \frac{R_e}{g_m r_{be} R_c}\right) v_{out}.$$

The small signal gain can also be written as

$$G = \frac{v_{out}}{v_{in}} = -\frac{R_c}{\frac{1}{g_m} + R_e\left(1 + \frac{1}{g_m r_{be}}\right)}.$$

In the Ebers-Moll model $r_{be} = \beta/g_m$, so that

$$G = -\frac{R_c}{\frac{1}{g_m} + R_e\left(1 + \frac{1}{\beta}\right)}.$$

Remember that we chose $R_e = 2.2\ \mathrm{k}\Omega$ to get $I_c = 2$ mA, which resulted in $g_m \simeq 0.08\ \Omega^{-1}$, and therefore to good approximation

$$G \simeq -\frac{R_c}{R_e}, \tag{10}$$

and the gain is no longer sensitive to device values like β or the bias values. But this didn't come for free, the gain is now reduced.

This is an example how negative feedback can reduce non-linearity: The input to the circuit is the voltage of the base to ground. The input voltage to the transistor as a transconductance amplifier is V_{be}, which is the former voltage reduced by $R_e I_e$, proportional to the output current.

Now that we have a circuit which has satisfactory amplification properties, we can study its input and output impedances.

Let's first look at the input impedance. It is given by

$$R_{\rm in} = \frac{v_{\rm in}}{i_{\rm in}}.$$

We start with

$$i_{\rm in} = i_1 + i_2 + i_b = v_{\rm in}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{v_{\rm be}}{r_{\rm be}},$$

and replace $v_{\rm be}$ using $v_{\rm out} = -g_{\rm m} v_{\rm be} R_{\rm c}$ and the gain $v_{\rm out}/v_{\rm in} = -R_{\rm c}/R_{\rm e}$, and $r_{\rm be}$ using $r_{\rm be} = \beta/g_{\rm m}$, so that

$$i_{\rm in} = v_{\rm in}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{\beta R_{\rm e}}\right),$$

and

$$R_{\rm in} = \frac{v_{\rm in}}{i_{\rm in}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{\beta R_{\rm e}}\right)^{-1}.$$

So, $R_{\rm in}$ is simply the parallel combination of R_1, R_2 and $\beta R_{\rm e}$. β is about 100, and the last term therefore small. The input impedance is reasonably high and dominated by the resistors in the input voltage divider.

Reminder: To effectively couple a voltage from one device to another we need to keep the output impedance of the source low and the input impedance of the receiver high.

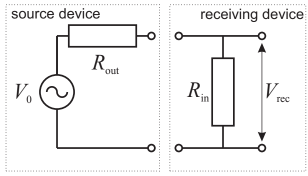


Figure 23: Input and output impedance of coupled devices.

This, and the preference to keep the load on the supply through the input voltage divider small, motivates us to make R_1 and R_2 large (but still small enough that the voltage divider is not disturbed by the current flowing into the transistor base).

To find the output impedance we look for the Thevenin and Norton equivalent of the circuit in figure 22. We already know the output (open circuit) voltage $v_{\rm out} = -(R_{\rm c}/R_{\rm e})v_{\rm in}$. We therefore need to find the short-circuit current of this circuit.

When the output is shorted, then $R_{\rm c}$ in figure 22 is shorted out, and the short-circuit current is given by the current from the current source in the small-signal transistor replacement

$$I_{\rm sc} = -g_{\rm m} v_{\rm be}.$$

Furthermore we can use KVL and KCL

$$v_{\rm in} + v_{\rm be} + v_{\rm e} = 0$$
$$i_{\rm b} + g_{\rm m} v_{\rm be} = i_{\rm e}$$

together with Ohm's law to find

$$I_{\rm sc} = \frac{g_{\rm m} v_{\rm in}}{1 + \frac{R_{\rm e}}{r_{\rm be}} + g_{\rm m} R_{\rm e}},$$

and

$$R_{\rm out} = \frac{v_{\rm out}}{i_{\rm sc}} = R_{\rm c} + R_{\rm c}\left(\frac{1}{\beta} + \frac{1}{g_{\rm m} R_{\rm e}}\right) \simeq R_{\rm c}.$$

If our amplifier should have reasonable gain this will be by all means a large resistance. It is a weakness of this circuit that it has a high output impedance and will not be able to drive low-impedance loads. Luckily there are other transistor circuits which can be used to achieve very low output impedances.

The common-collector amplifier or emitter follower

The third possible connection scheme for BJTs, the common-base amplifier is not often used in low-frequency discrete circuits, which is why we will not discuss it here.

Figure 24 shows a different transistor circuit, commonly known as emitter follower, but it is also sometimes referred to as a common-collector amplifier.

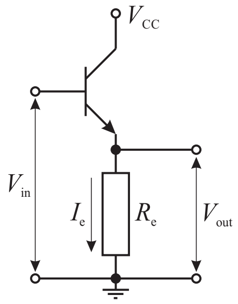


Figure 24: The emitter follower or common-collector amplifier circuit.

As long as $0.6\,\text{V} \leq V_{in} \leq 9.4\,\text{V}$ the transistor will be in the active region with

$$I_e = \frac{V_{in} - 0.6\,\text{V}}{R_e},$$

and $V_{out} = V_{in} - 0.6\,\text{V}$. The output voltage follows the input voltage, which explains the name.

If this circuit does not give us any voltage amplification, what else could it be good for? Let's study this again with the small signal equivalent (figure 25).

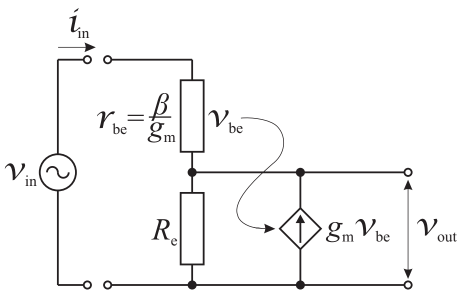


Figure 25: Small signal equivalent of the emitter follower.

First, we leave it to the students as an exercise to show that the small signal gain is

$$G = \frac{v_{out}}{v_{in}} = \frac{1}{1 + \left(\frac{\beta}{\beta+1}\right)\frac{1}{g_m R_e}} \simeq 1.$$

The small signal gain is close to 1, but slightly smaller.

Next, we can combine this with $v_{in} = v_e + v_{be}$ (KVL) and $g_m v_{be} + i_{in} = v_{out}/R_e$ (KCL) to find the input impedance, which is

$$R_{in} = \frac{\beta}{g_m} + (\beta + 1)R_e.$$

$\beta/g_m = r_{be} \simeq 1.5\,\text{k}\Omega$ but if R_e is about $2\,\text{k}\Omega$, which is a typical value, then $R_{in} \simeq 200\,\text{k}\Omega$ for this circuit, which is reasonably high.

For the output impedance we calculate again the short-circuit current

$$i_{sc} = g_m v_{be} - i_e + i_b,$$

but $i_e = 0$ because R_e is shorted out. Similarly $v_{be} = v_{in}$ because of the short, so

$$i_{sc} = v_{in}\left(g_m + \frac{g_m}{\beta}\right) = v_{in} g_m\left(\frac{\beta+1}{\beta}\right),$$

and

$$R_{out} = \frac{v_{out}}{i_{sc}} = \frac{1}{\frac{1}{R_e} + g_m\left(\frac{\beta+1}{\beta}\right)} \simeq \frac{R_e}{1 + g_m R_e} \simeq \frac{1}{g_m} = \frac{25\text{ mV}}{I_c} = r_e.$$

The latter is called the 'emitter resistance' and is rather low.

Generally, the resistance looking into the emitter of a transistor is small, whereas the resistance looking into the collector is high.

The emitter follower is not a voltage amplifier but a current amplifier, which is what makes the output impedance small. A common solution is to combine a voltage amplifier like a common emitter amplifier, which provides the voltage gain, with a current amplifier like the emitter follower, to lower the poor output impedance of the first stage in this combination.

The long-tailed pair

As we have just seen the combination of two or more transistors can be very useful. Another widely used combination of two transistors can be used to eliminate distortions (thermal or due to non-linearities) without a large reduction in gain: the long-tailed pair circuit (figure 26). Note that this circuit has two inputs (V_a and V_b), and one output V_{out}. As we will see this circuit actually amplifies the difference of the two input voltages (or signals). Many of the distorting effects will apply to both inputs the same way and will therefore not contribute to the amplified output signal.

We call such an amplifier a differential amplifier. As we have seen the operational amplifier is an example for such an amplifier. In fact, usually the input stage of an op-amp is made up of a long-tailed pair.

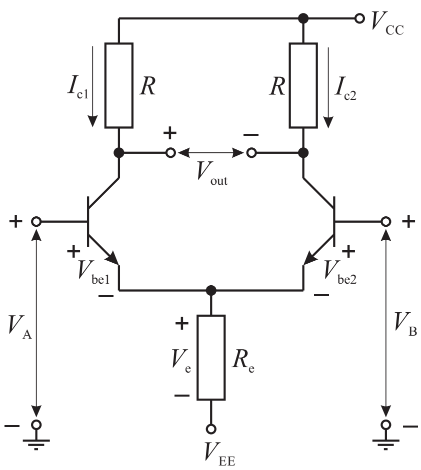


Figure 26: The long tailed pair amplifier circuit.

Often only the output from one collector is taken (the two small signal collector voltages are symmetric and opposite). This is called a 'single-ended' output. The output is then referenced to ground and can be used as input to standard circuits like a common-emitter amplifier or an emitter follower. The collector resistor at the unconnected collector is then not required (but the transistor is, it needs to steer the currents).

This circuit relies on the easy availability of matched transistors and resistors, when they are manufactured on the same wafer. This applies in particular for integrated circuits, where the whole circuit is implemented on one small piece of silicon.

'Matched' here means components with the same characteristics and key performance parameters.

To understand the response of this circuit we will not use the small-signal equivalent, but the Ebers-Moll equation, to have fewer approximations, however, the calculations become non-linear.

Reminder: The Ebers-Moll equation is

$$I_c = I_s\left(e^{\frac{eV_{be}}{k_B T}} - 1\right).$$

KCL and KVL, $P = VI$ and Ohm's law (only for resistors) still work. Superposition does not work.

We start with KVL in the top loop of the circuit

$$V_{\text{out}} = (I_{\text{c2}} - I_{\text{c1}})\,R = I_{\text{s}} R \left(e^{\frac{eV_{\text{be2}}}{k_{\text{B}}T}} - e^{\frac{eV_{\text{be1}}}{k_{\text{B}}T}} \right),$$

where we did insert the Ebers-Moll equation for the second part. We can then use KVL again to get

$$V_{\text{be1}} = V_{\text{A}} - V_{\text{e}}$$
$$V_{\text{be2}} = V_{\text{B}} - V_{\text{e}}$$

and

$$V_{\text{out}} = I_{\text{s}} R \left(e^{\frac{eV_{\text{B}}}{k_{\text{B}}T}} - e^{\frac{eV_{\text{A}}}{k_{\text{B}}T}} \right) e^{-\frac{eV_{\text{e}}}{k_{\text{B}}T}}.$$

Now we can imagine that the input voltages V_{a} and V_{b} consist of a part which changes together and a part which changes differentially

$$V_{\text{a}} = V_{\text{com}} + \frac{\Delta V_{\text{in}}}{2}$$
$$V_{\text{b}} = V_{\text{com}} - \frac{\Delta V_{\text{in}}}{2}$$

V_{com} is referred to as the common mode voltage, whereas the differential input is called the normal mode.

without loss of generality. The output then becomes

$$V_{\text{out}} = -2 I_{\text{s}} R \, e^{\frac{e}{k_{\text{B}}T}(V_{\text{com}} - V_{\text{e}})} \sinh \frac{e\,\Delta V_{\text{in}}}{2 k_{\text{B}} T}.$$

To understand the exponential in this expression we can use $V_{\text{com}} = (V_{\text{A}} + V_{\text{B}})/2 = (V_{\text{be1}} + V_{\text{be2}})/2 + V_{\text{e}} + V_{\text{EE}}$ (sum of the KVL for the two bottom loops). The supply voltage V_{EE} is constant, and the diode drops V_{be1} and V_{be2} too, to good approximation, so that $\delta V_{\text{cm}} \simeq \delta V_{\text{e}}$ and therefore the exponential $e^{\frac{e}{k_{\text{B}}T}(V_{\text{com}} - V_{\text{e}})}$ is a constant to first order, regardless of the changes in common mode voltage on terminals A and B.

The output is therefore

$$V_{\text{out}} \propto -\sinh \frac{e\,\Delta V_{\text{in}}}{2 k_{\text{B}} T},$$

which is very linear for small signals.

$\sinh x \simeq x + \frac{x^3}{3!} + \ldots$, no x^2 term.

The current mirror

By now you should be convinced that transistors are very powerful devices, so it will not come as a surprise that one can also use transistors to create excellent current sources. We will start this discussion with the circuit in figure 27.

We leave it to the students to figure out what this circuit does. Is the transistor in this circuit in the active region? What is I_{c}?

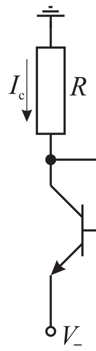


Figure 27: An interesting bias.

We can now use the base voltage defined in this circuit to bias one or more matched transistors.

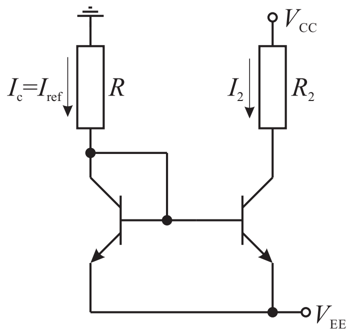


Figure 28: The current mirror.

What is I_2 in this circuit? As the base voltage corresponding to the current I_{ref}, which is programmed by the left transistor, is also the base voltage for the right transistor the current I_2 through this transistor will be the same as I_{ref}, independent of the value of R_2. It should be clear why this circuit is called a 'current mirror'.

One of the exercises in the problem sheet will address these questions and guide you through a calculation of how the current mirror works.

Appendix A
Semiconductor principles

As you will see in the quantum mechanics course this year the electrons in an atom are in states of well-defined energy. As you will also see there is a general principle in quantum mechanics which prevents that a given state is occupied by more than on electron. As a consequence, when many atoms with their electrons come in close contact within a solid, their electron energy levels will spread slightly resulting in very closely spaced energy levels, which each can be occupied by a single electron. Because of the large number of electrons in the solid the spacing is so close that it cannot be resolved, and the energy levels are observed as a band.

This is known as the Pauli exclusion principle.

The band structure of a solid defines its electrical conductivity. In an insulator there is an energy band called the 'valence' band which is full of electrons and a higher energy band called the 'conduction' band, which has no electrons. The electrons in the valence band are not free to move because of the Pauli exclusion principle: there are no free states available for the electron to move to. Therefore an insulator cannot conduct electrical current, hence its name. In a metal the conduction band is partially filled with electrons and therefore these electrons are free to move and metals have good electrical conductivity.

Imagine the M25 which is completely full of cars with no gaps. In this case no cars could move.

In a semiconductor the band gap between the valence and the conduction gap is relatively small, small enough that thermal energies are sufficient to lift a small, but non-zero fraction of the electrons into the conduction band. At low energies all the electrons will be in the valence band and as the temperature increases more and more electrons will be lifted into the conduction band. As you will hear this year in the statistical mechanics course the probability for an electron within the solid to be lifted by an energy ΔE at a temperature T is given by the Boltzmann factor $e^{-\Delta E/(k_B T)}$, where k_B is the Boltzmann constant.

For example for silicon the band gap is 1.14 eV (1.83×10^{-19} J).

$k_B = 1.38064852 \times 10^{-23}$ J/K

Commonly used semiconductor devices used in electronics are made of silicon or germanium. These are elements from the carbon group of the periodic table and as such have four electrons in the outermost shell.

Other semiconductors are used for more specialized functions, e.g. GaAs is used for semiconductor lasers and photodiodes.

A pure semiconductor of this type will arrange in a diamond cubic crystal structure with each of the outer shell electrons combining with an electron from a neighbouring atom to form a covalent bond. The four outer shell electrons of such an atom are all used up in four such bonds.

Because all its electrons are well integrated in this bond structure a pure semiconductor is a poor conductor. In terms of energies all electrons are used to completely fill the valence band. However, the conductivity can conveniently be altered by adding small amounts of elements from adjacent groups (‘doping’). Doping with elements from the boron group results in p-type material (prevalence of holes) and doping with elements from the nitrogen group results in n-type material (prevalence of electrons). These prevalent charge carriers are weakly attached to their atoms and can easily make the jump into the valence band.

If a semiconductor has two regions of different doping a semiconductor junction is created. A diode contains one junction, whereas a simple transistor contains two junctions (*npn* or *pnp*).

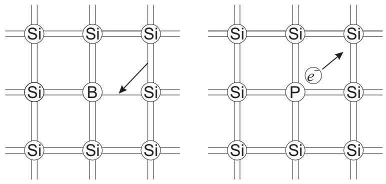


Figure A.1: The effect of placing a dopant into the silicon lattice. p-type doping (left) and n-type doping (right). For the figure the diamond cubic crystal structure has been flattened to 2D. Arrows indicate movement of electrons.

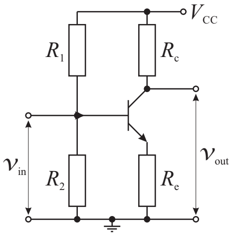
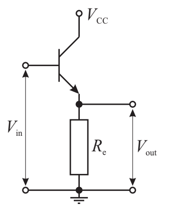
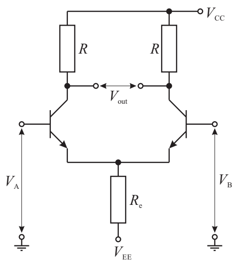
In the case of a hole it is actually the adjacent strongly bound electrons which are trying to fill the hole, but in doing so create a vacancy at their original location. Effectively this allows the hole to travel over large distances.

Appendix B

Summary of approximate transistor circuit properties

Table B.1 gives a summary of (approximate) key performance parameters for common transistor circuits for quick reference. For a derivation see the main text. These approximations should help you to quickly estimate certain performance parameters but for a serious circuit design more detailed models will be needed.

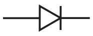
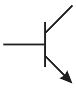
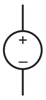
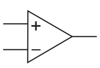
Table B.1: Key performance parameters for common transistor circuits. Approximate expressions for realistic transistors and resistor values are given where applicable.

Designation	circuit diagram	small signal gain	R_{in}	R_{out}	CMRR
Common-emitter amplifier		$-R_{\mathrm{c}}/R_{\mathrm{e}}$	$R_1\|\|R_2$	R_{c}	
Emitter follower		1	βR_{e}	r_{e}	
Long-tailed pair		$-\frac{g_{\mathrm{m}}R}{2}$			$g_{\mathrm{m}}R_{\mathrm{e}}$

Appendix C
Circuit diagram symbols

This appendix lists for reference the circuit diagram symbols used in this document.

Table C.1: Circuit diagram symbols.

	Resistor
	Capacitor
	Diode
	Transistor (*npn*)
	DC voltage source
	AC voltage source
	Controlled voltage source
	Current source
	Controlled current source
	Ground (0V)
	Differential amplifier (here: op-amp)

Appendix D
Open issues

Todd:

- Appendix A – I'm not sure if it will be useful or not, how much work is it to fill the table out the rest of the way? My inclination is to leave it in.

Tony

- The section on semiconductors is very terse. IMHO it needs to either be expanded (maybe better) reduced to a very short statement saying that in this course we will use a very simple model for a transistor and the underlying physics of semiconductors will be covered in the third year course. The electronics course manual gives a short introduction.

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