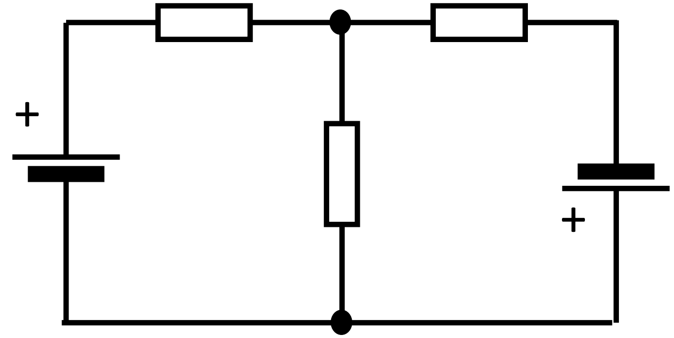
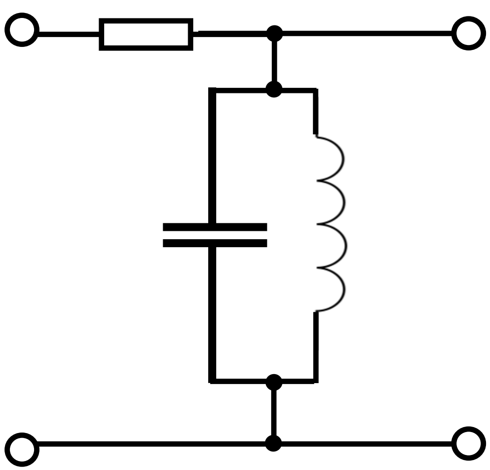
Circuit Theory

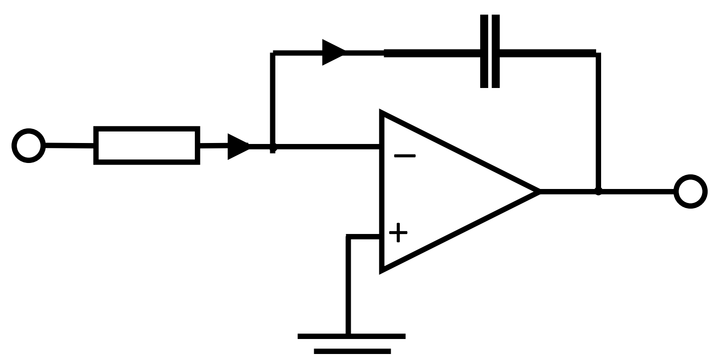


Prof. Todd Huffman

What is circuit theory?

Analysing (electrical) circuits, applying basic rules (Ohm's law, Kirchoff's law) to networks of components, to calculate the current and voltage at any point





To do this:

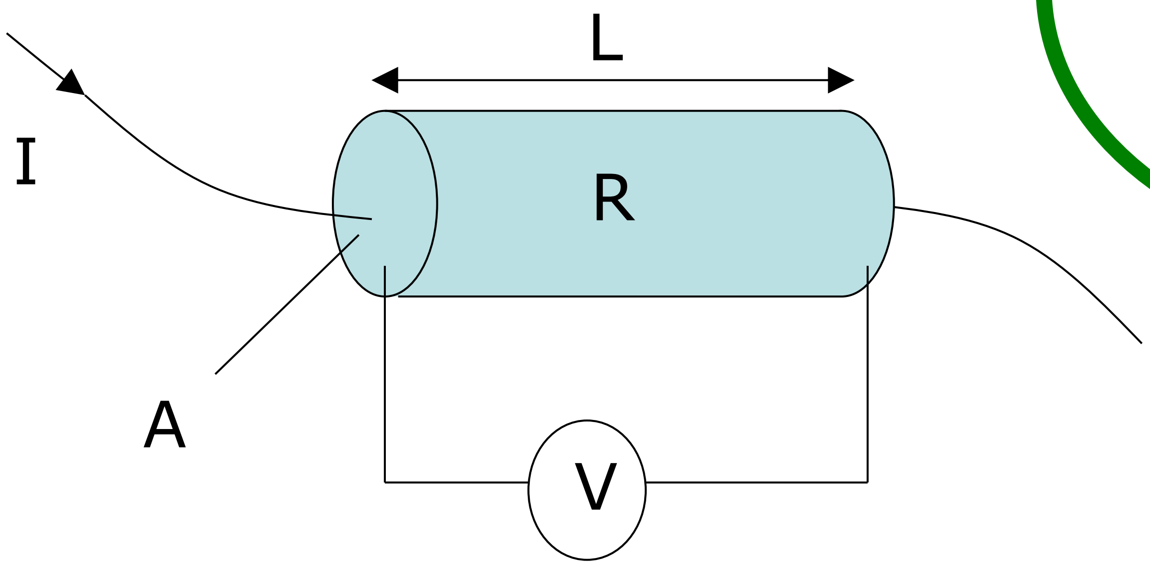
- You need to know the basic rules.
  - Ohm’s Law
  - Kirkoff’s Current law (Conservation of Charge)
  - Kirkoff’s Voltage law (Conservation of Energy)
- And you need to know the techniques
  - Mesh analysis
  - Node analysis
- And I need to show you some tricks
- And You need to practice, as with any skill.

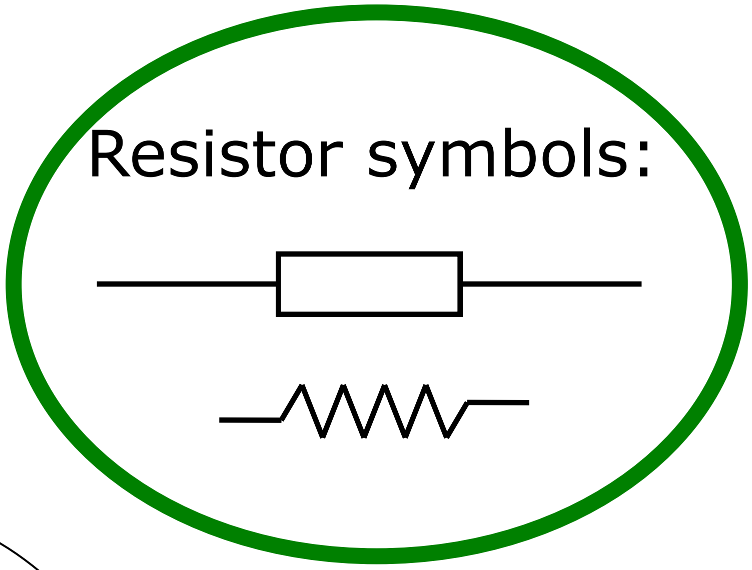
Reading List

- Electronics: Circuits, Amplifiers and Gates, *D V Bugg, Taylor and Francis*
  Chapters 1-7
- Basic Electronics for Scientists and Engineers, *D L Eggleston, CUP*
  Chapters 1,2,6
- Electromagnetism Principles and Applications, *Lorrain and Corson, Freeman*
  Chapters 5,16,17,18
- Practical Course Electronics Manual
  http://www-teaching.physics.ox.ac.uk/practical_course/ElManToc.html
  Chapters 1-3 (Very Good one to read!)
- Elementary Linear Circuit Analysis, *L S Bobrow, HRW*
  Chapters 1-6
- The Art of Electronics, *Horowitz and Hill, CUP*

Ohm's law

Voltage difference ∝ current



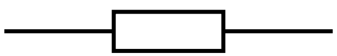


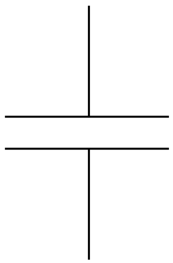
$$V = IR$$ R=Resistance Ω[ohms]

Other Circuit Symbols

Passive Devices

or

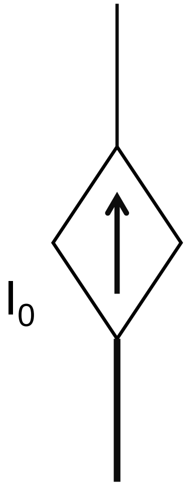


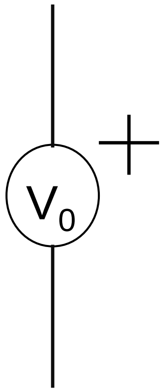


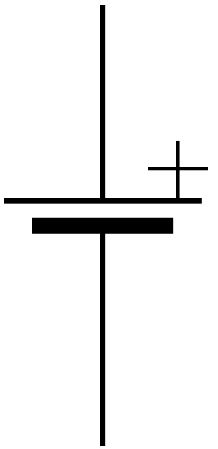
Capacitor

Inductor

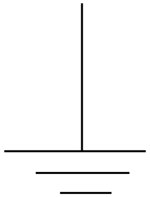
Sources of Electrical Power







Direction of Current or
+/- Terminals must be shown.



Ground – Reference Potential
V ≡ Zero volts (by definition)

Passive Sign Convention

Passive devices ONLY - **Learn it**; **Live it**; **Love it**!

R=Resistance Ω[ohms]

$$V = IR$$

Three deceptively Simple questions:

Which way does the current flow, left or right?

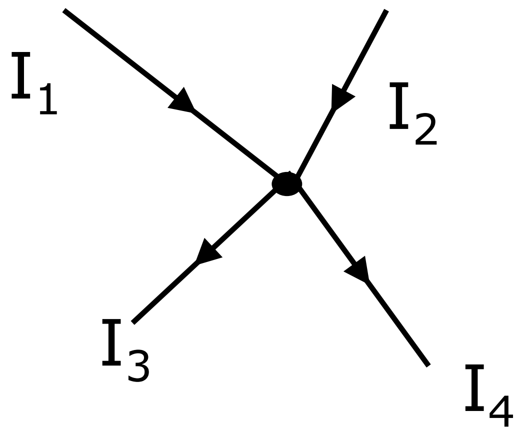
Voltage has a ‘+’ side and a ‘-’ side (you can see it on a battery) on which side should we put the ‘+’? On the left or the right?

Given V=IR, does it matter which sides for V or which direction for I?

Explain on white board – Trick

Kirchoff's Laws

I Kirchoff's current law: Sum of all currents at a node is zero



$$I_1+I_2-I_3-I_4=0$$

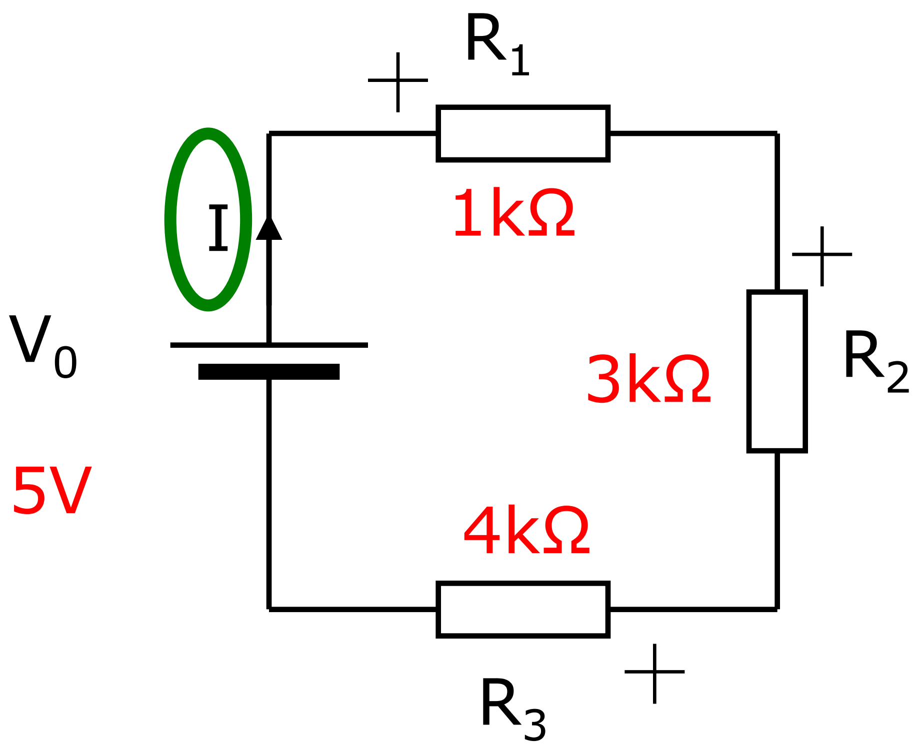
$$\sum I_n = 0$$

(conservation of charge)

Prof. Huffman – before you get carried away, show them what a 'node' is

Passive Sign Convention:
If you follow it, you can arbitrarily choose whether "incoming" or "outgoing" currents are Positive at each node independently from all other nodes!

II Kirchoff's voltage law: Around a closed loop the net change of potential is zero



$$-V_0+IR_1+IR_2+IR_3=0$$

$$\sum V_n = 0$$

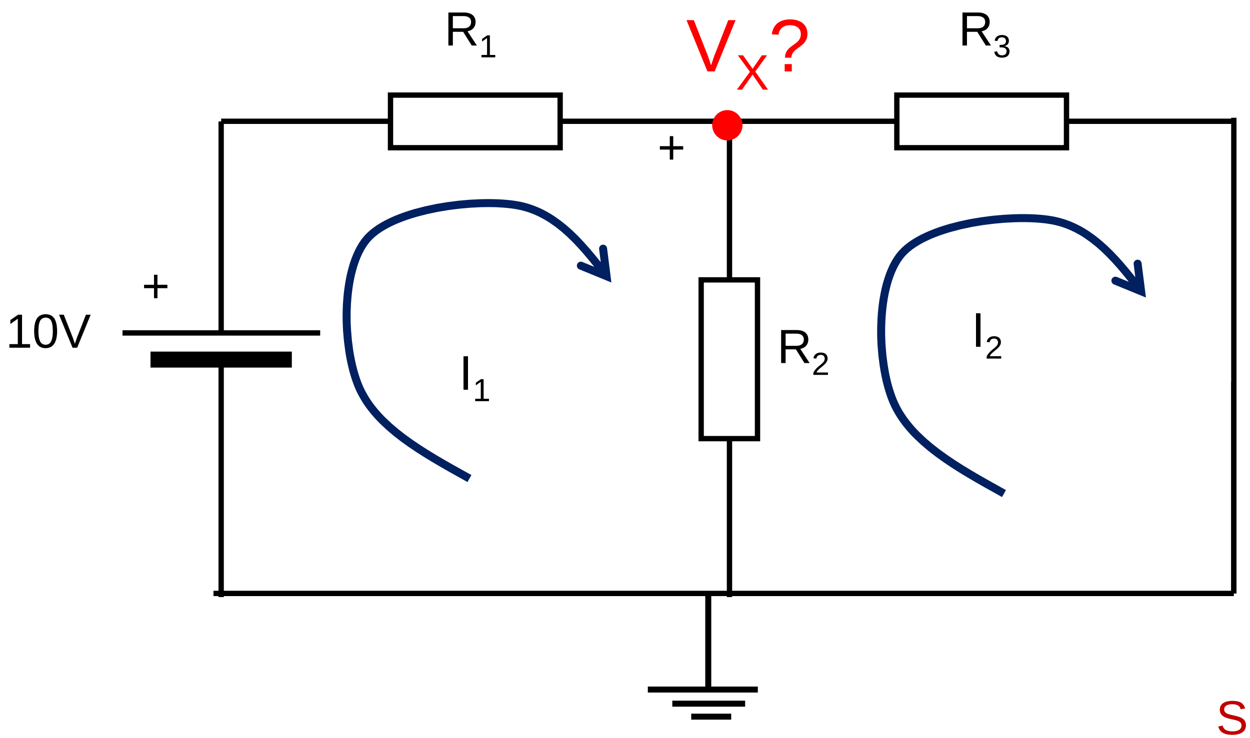
Calculate the voltage across R_2

$$5V = I(1+3+4)k\Omega$$

$$I = \frac{5V}{8000\Omega} = 62.5mA$$

$$V_{R2} = 62.5mA \times 3k\Omega = 1.9V$$

Passive Sign Convention really helps!!! Notice "I"



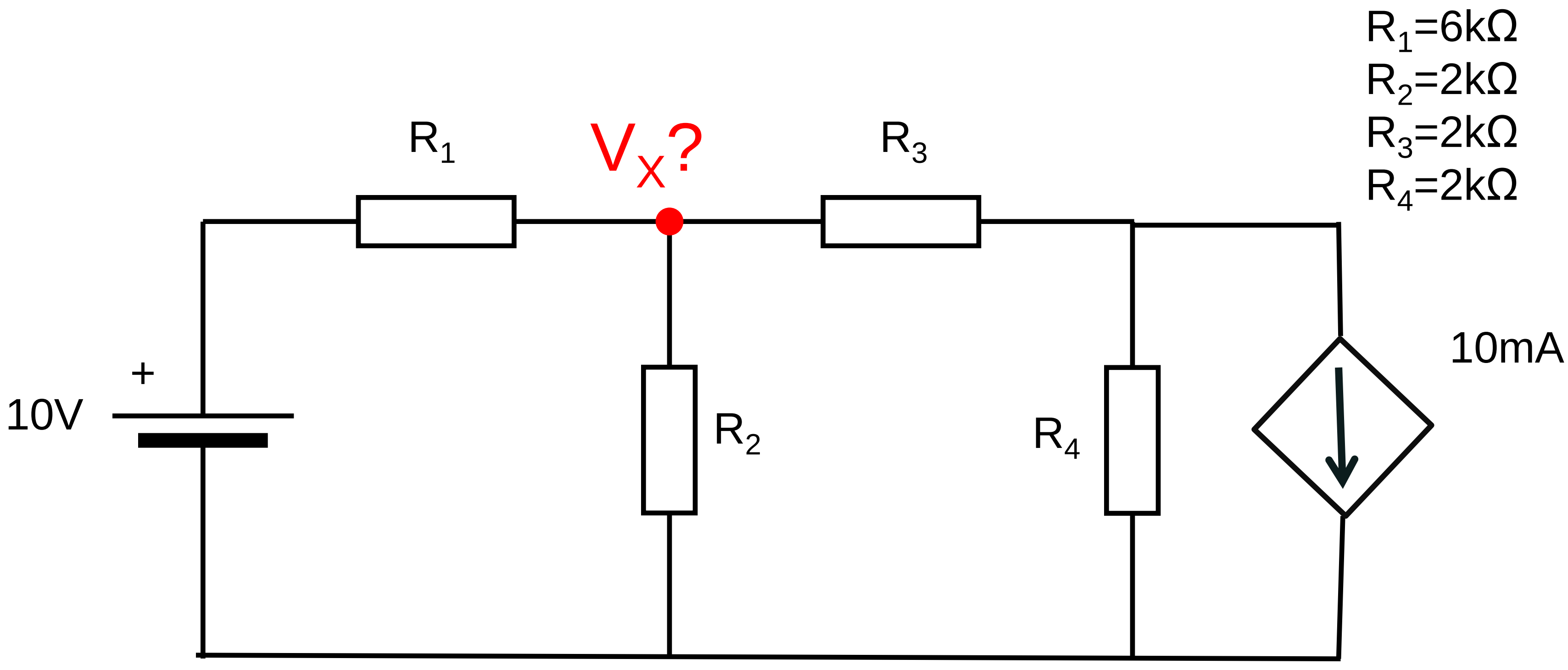
R_1=1kΩ
R_2=2kΩ
R_3=4kΩ

Solve example on board

Let us use these rules to find V_x.

This technique is called “mesh analysis”.
Start by labelling currents and using KVL. Then apply KCL at nodes.

If you then use Passive Sign Convention, the direction you chose for the currents DOES NOT MATTER!



You might think, up to now, that Passive Sign convention is a bit silly.

OK…So which is the “right” direction for V_x now?

What if we had a circuit with 5 loops and an additional current source?

Let’s solve the above circuit and find out what V_x is.

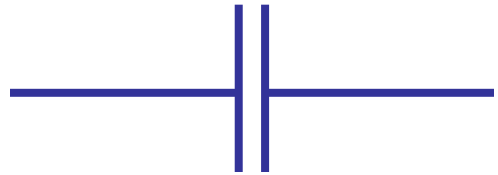
Node Analysis

- Instead of currents around loops – voltages at nodes

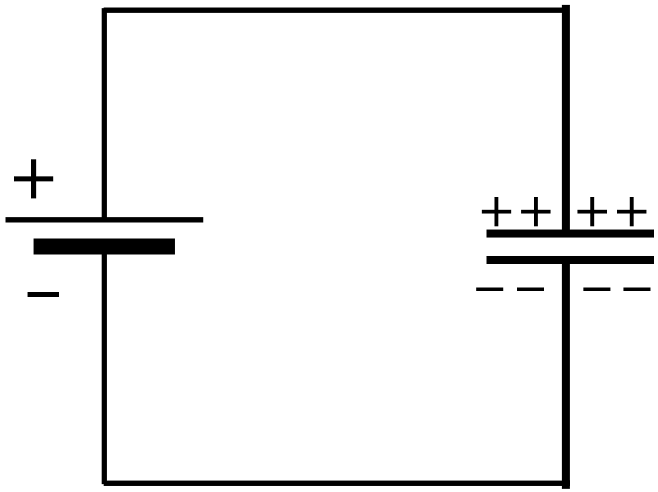
1. Choose one node as a “ground”. The reference
2. Now label all nodes with a voltage. This is positive wrt ground.
3. Now at each node (that isn’t ground) use KCL

There are Tricks you can use IF you use ***passive sign convention*** at each node!

Capacitors



Unit – “Farad”
C = εA/d



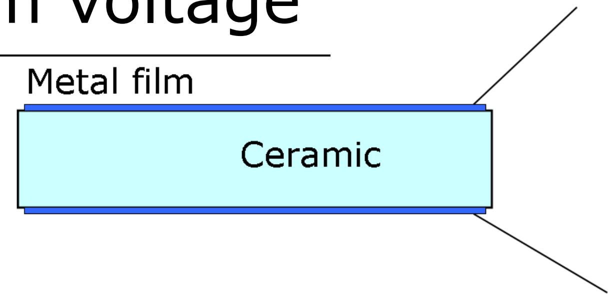
Capacitors are also PASSIVE –
They too have a kind of “ohm’s law” that relates voltage and current.

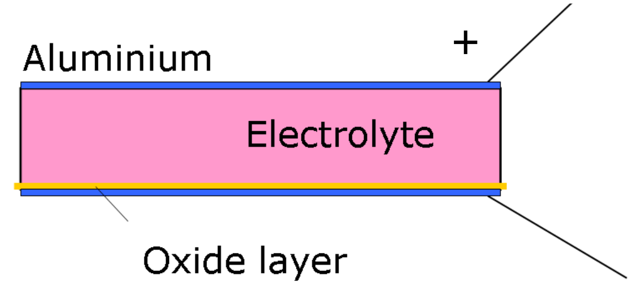
$$Q = CV$$

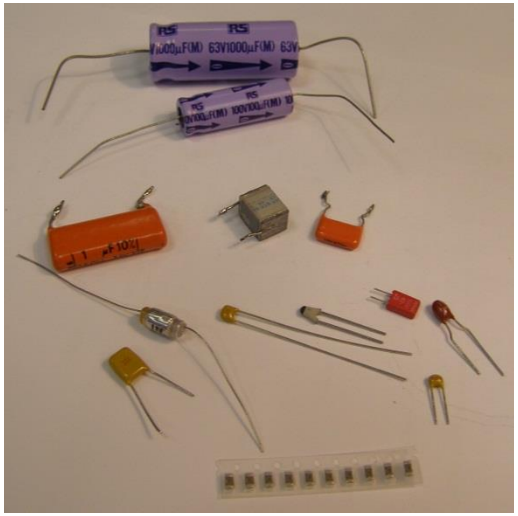
$$I = \frac{dQ}{dt} \longrightarrow I = C\frac{dV}{dt}$$

Capacitor types

	Range	Maximum voltage
Ceramic	1pF–1μF	30kV
Mica	1pF–10nF	500V (very stable)
Plastic	100pF–10μF	1kV
Electrolytic	0.1μF–0.1F	500V
Parasitic	~pF	



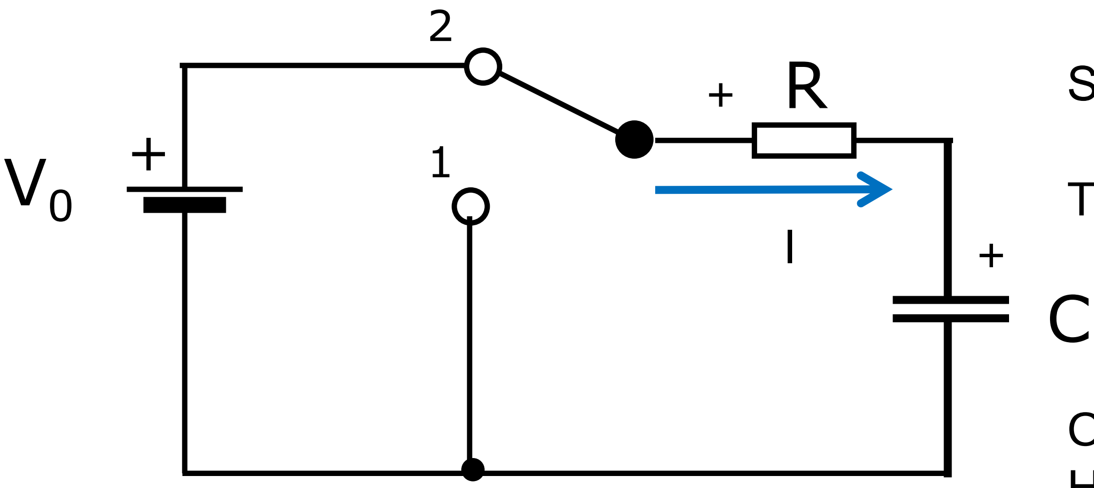




Polarised

RC circuits

1 Initially $V_R=0$ $V_C=0$



Switch in Position "1" for a long time.

Then instantly flips at time t = 0.

Capacitor has a derivative!
How do we analyze this?

There are some tricks…

ALWAYS start by asking these three questions:

1.) What does the Circuit do up until the switch flips?
(Switch has been at pos. 1 for a VERY long time. easy)

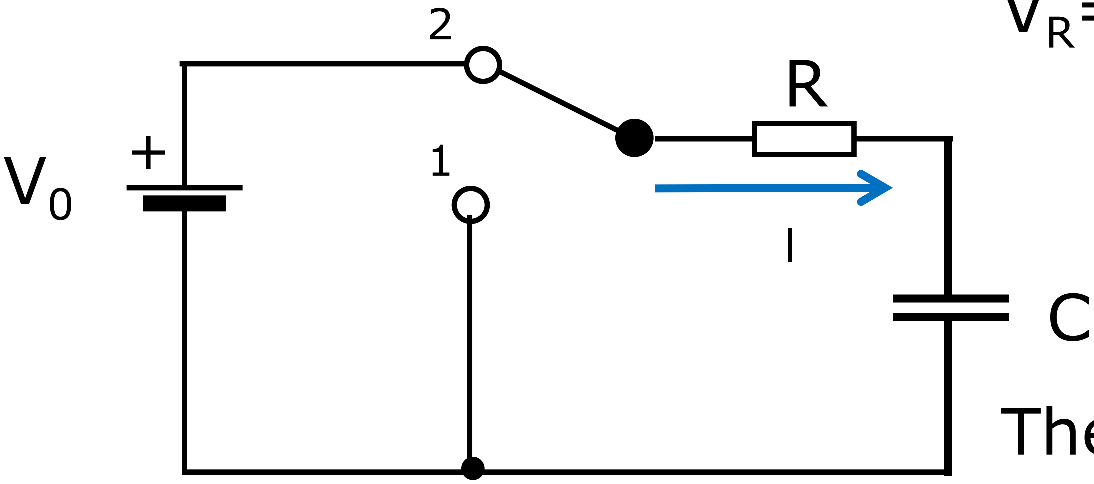
2). What does the circuit do a VERY long time AFTER the switch flips? (easy)

3). What can we say about the INSTANT after switch flips? (easy if you know trick)

The Trick!!

- Remember → $I = C\frac{dV}{dt}$
- Suppose the voltage on a 1 farad Capacitor changes by 1 volt in 1 second.
  - What is the current?
  - What if the same change in V happens in 1 microsecond?
  - So…What if the same change in V happens instantly?
- Rule: It is impossible to change the voltage on a capacitor instantly!
  - Another way to say it:
    The voltage at $t = 0-\varepsilon$ is the same as at $t = 0+\varepsilon$.

RC circuits



1 Initially at t = 0^-

$V_R=0 \quad V_C=0 \quad I=0$

Then at t = 0^+

Must be that $V_C=0$

If $V_C=0$ then KVL says $V_R=V_0$ and $I=V_0/R$.

Capacitor is acting like a short-circuit.

Finally at t = $+\infty$

$V_R=0 \quad V_C=V_0 \quad I=0$

2

$$-V_0 + V_R + V_C = 0$$

$$V_0 = V_R + V_C$$

$$= IR + \frac{Q}{C}$$

differentiate wrt t

$$0 = \mathrm{R}\frac{\mathrm{dI}}{\mathrm{dt}} + \frac{\mathrm{I}}{\mathrm{C}}$$

$$0 = R\frac{dI}{dt} + \frac{I}{C}$$

$$\frac{-I}{RC} = \frac{dI}{dt}$$

$$\int \frac{-1}{RC}\,dt = \int \frac{1}{I}\,dI$$

$$\frac{-t}{RC} = \ln I + a$$

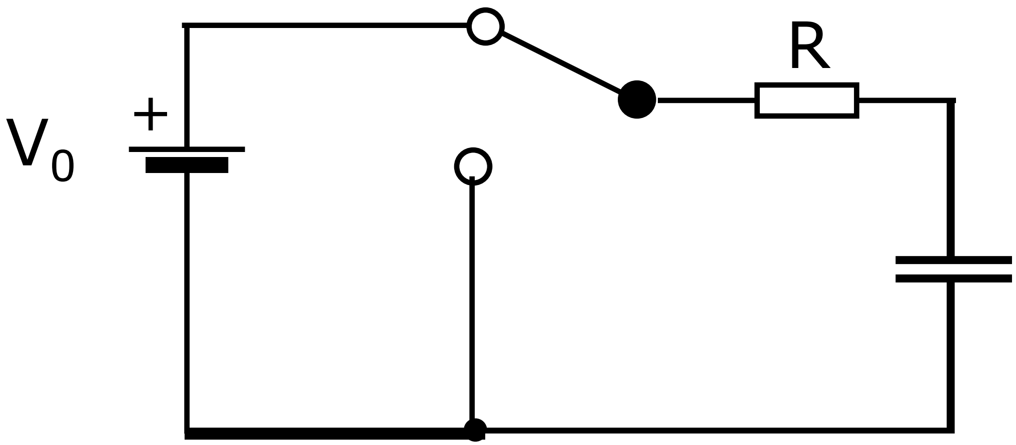
$$= \ln I + \ln b$$

$$= \ln(bI)$$

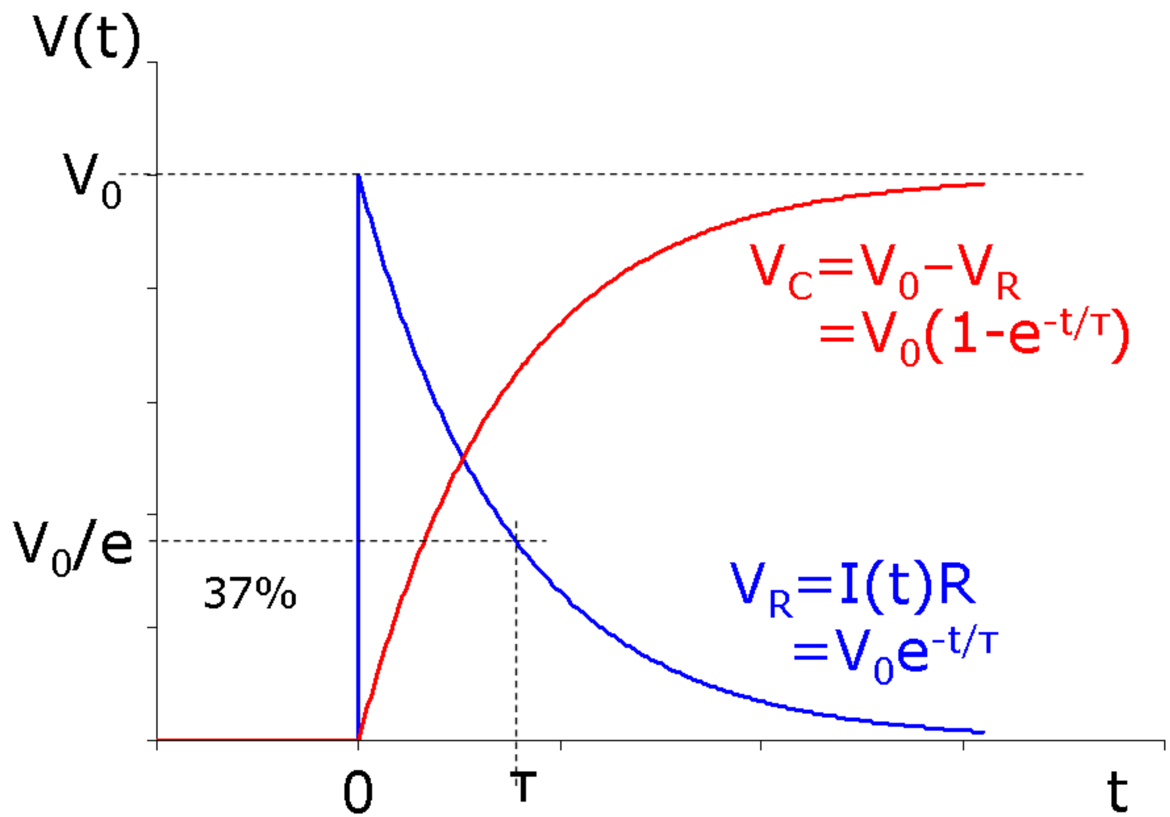
$$e^{\left(\frac{-t}{RC}\right)} = bI$$

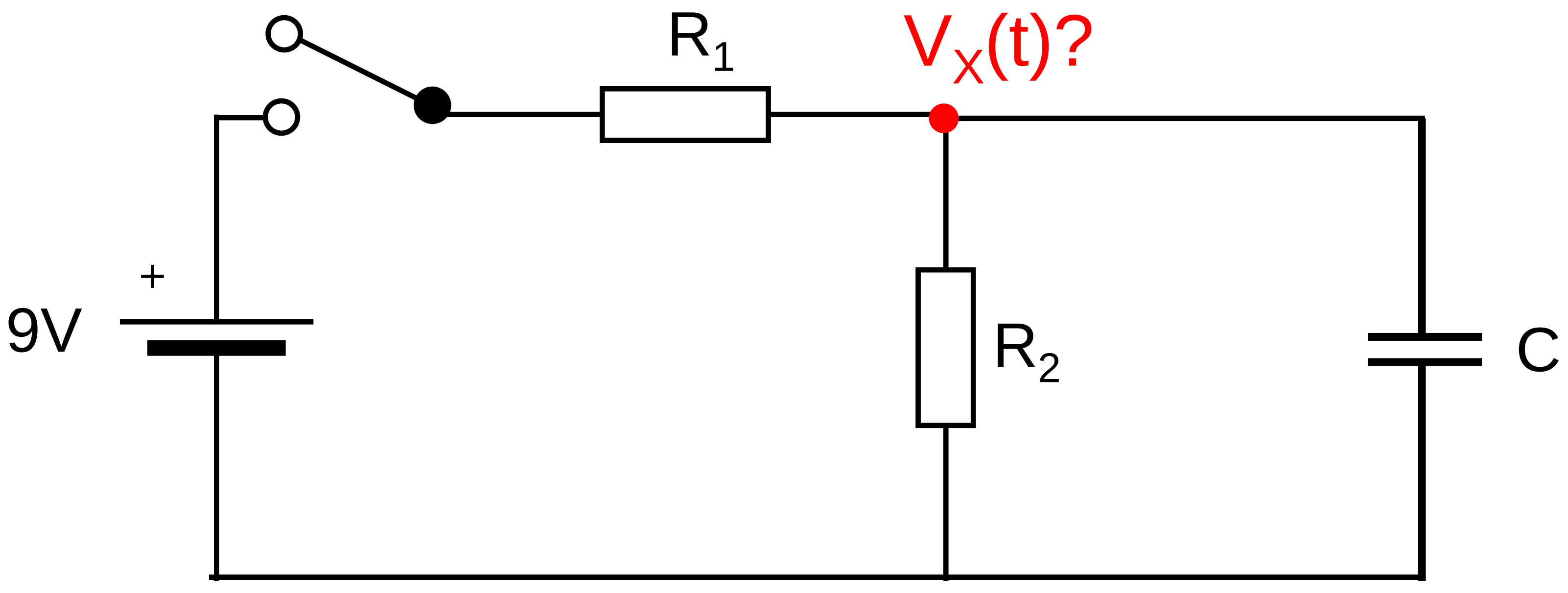
$$I = \left(\frac{1}{b}\right)e^{\left(\frac{-t}{RC}\right)}$$

$$= I_0 e^{\left(\frac{-t}{RC}\right)}$$



$$I(t) = \frac{V_0}{R} e^{-t/\tau}$$



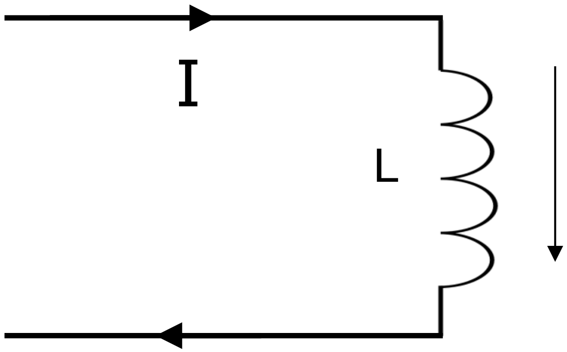


Switch Closes at time t=0.
What is the voltage V_X as a function of time?

a) $R_1=R_2$
b) $R_1=2R_2$

Work it out with student's help.

Inductance



$$L = \mu_0 A \frac{N^2}{\ell}$$

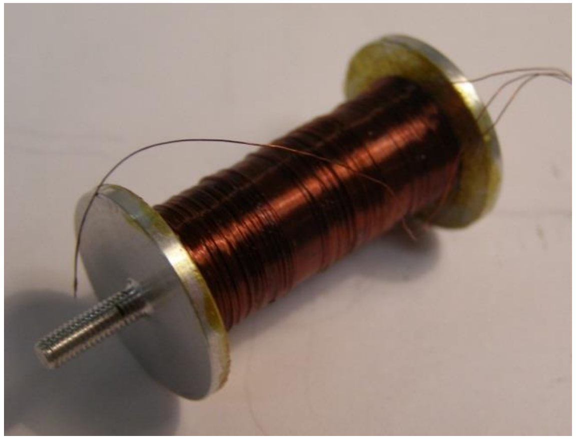
for solenoid
Unit – "Henry"

Inductors are also PASSIVE –
They too have a kind of "ohm's law" that relates voltage and current.

$$V = L \frac{dI}{dt}$$

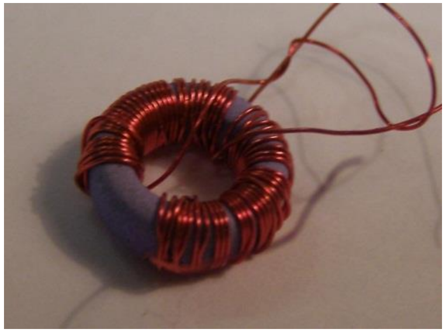
How should the inductor's voltage be labelled in the above diagram?

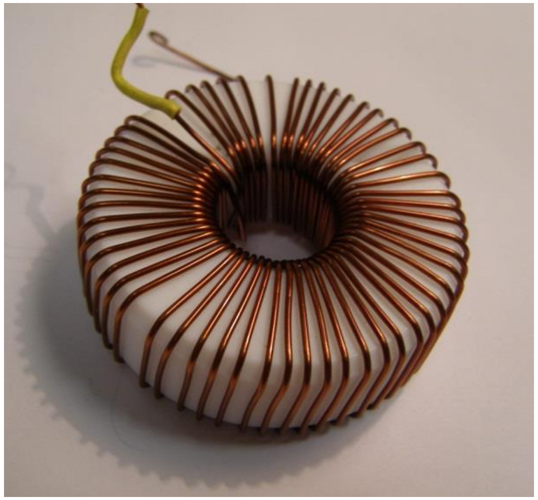
Inductors

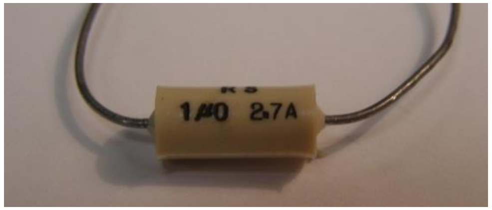


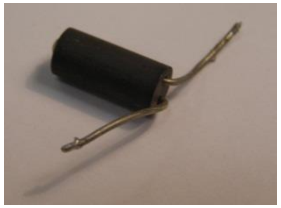
Wire wound coils - air core

- ferrite core



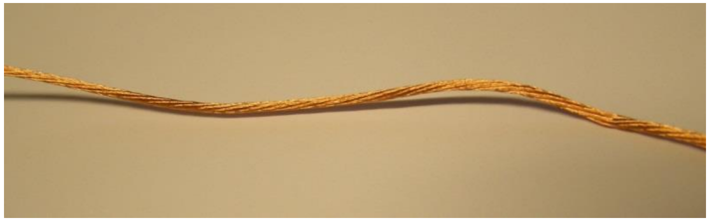


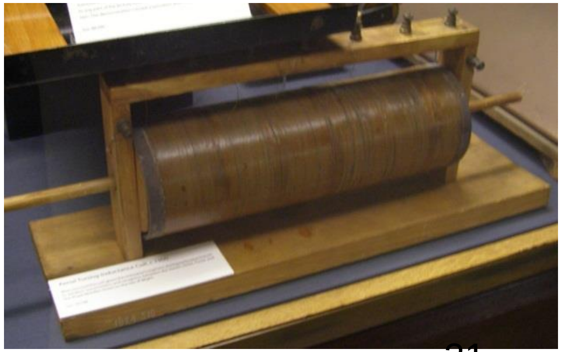




Wire loops

Straight wire

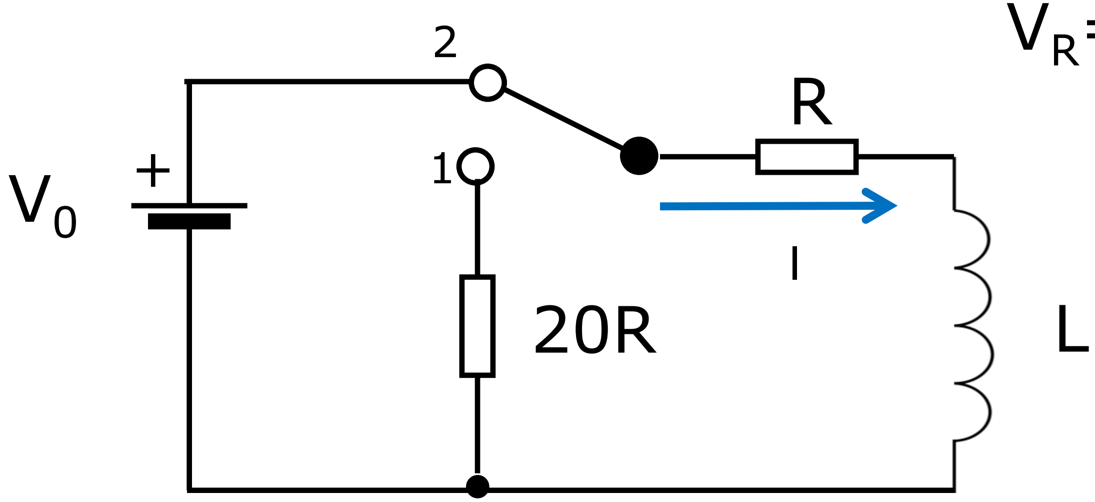




RL circuits

Initially $t=0^-$

$V_R=V_0 \quad V_L=0 \quad I=V_0/R$



This one is going to be fun, I promise!

1.) What does the Circuit do up until the switch flips?
(Switch has been at pos. 2 for a VERY long time. easy)

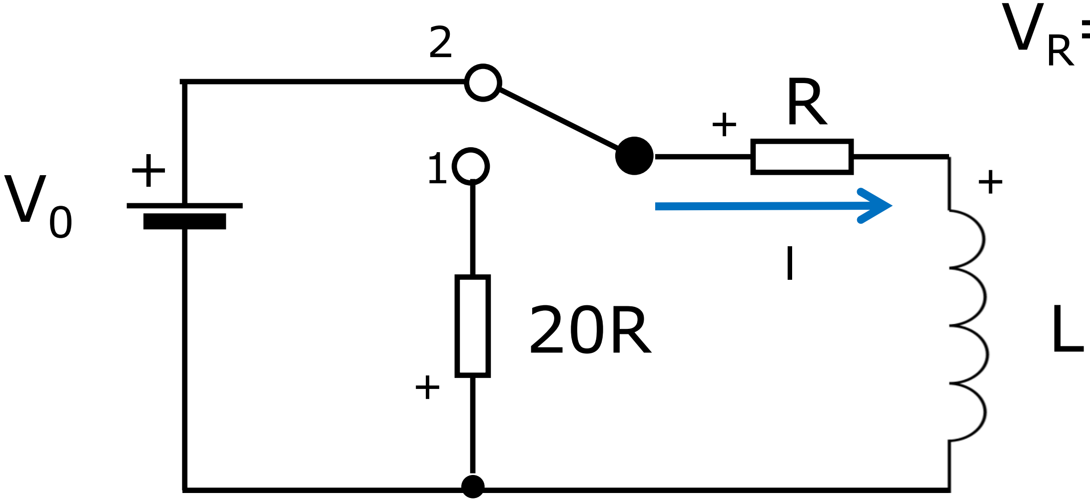
2). What does the circuit do a VERY long time AFTER the switch flips? (easy)

3). What can we say about the INSTANT the switch flips? (easy if you know trick)

The Next Trick!!

- Remember → $V = L\frac{dI}{dt}$
- Suppose the current on a 1 henry Capacitor changes by 1 amp in 1 second.
  - What is the voltage?
  - What if the same change in I happens in 1 microsecond?
  - So…What if the same change in I happens instantly?
- Rule: It is impossible to change the current on an inductor instantly!
  - Another way to say it:
    The current at t = 0-ε is the same as at t = 0+ε.
  - Does all this seem familiar? It is the concept of "duality".

RL circuits



Initially $t=0^-$

$V_R=V_0$ $V_L=0$ $I=V_0/R$

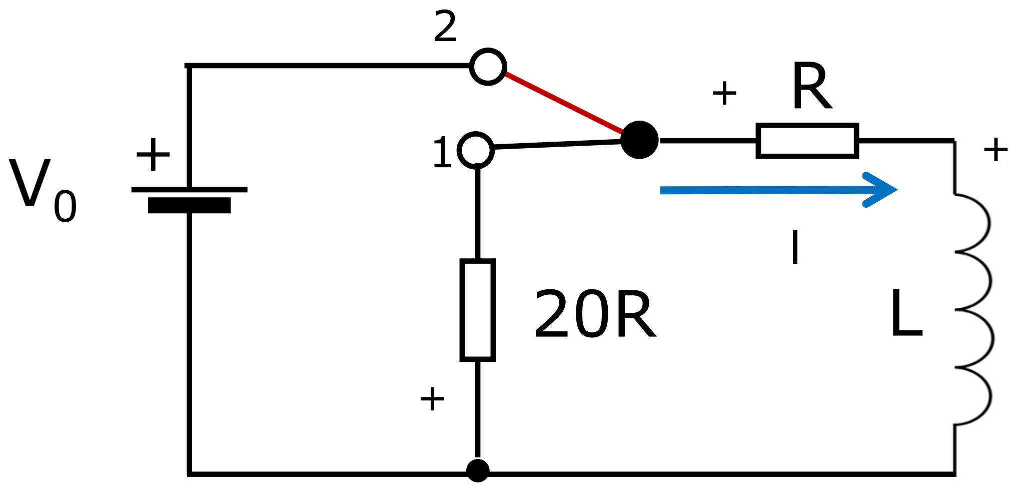
And at $t=\infty$ (Pos. "1")

$V_R=0$ $V_L=0$ $I=0$

What about at $t=0^+$?

USE A TRICK!

RL circuits



Initially $t=0^-$

$V_R=V_0$ $V_L=0$ $I=V_0/R$

At $t=0^+$ → Current in L MUST be the same

$I=V_0/R$ So…

$V_R=V_0$ $V_{R20}=20V_0$

and $V_L = -21V_0$!!!

For times t > 0

$$0 = V_{R20} + V_R + V_L$$

$$0 = 20IR + IR + L\frac{dI}{dt}$$

$$0 = 21IR + L\frac{dI}{dt}$$

And at $t=\infty$ (Pos. “1”)

$V_R=0$ $V_L=0$ $I=0$

$$\frac{dI}{dt} + \frac{21R}{L} I = 0$$

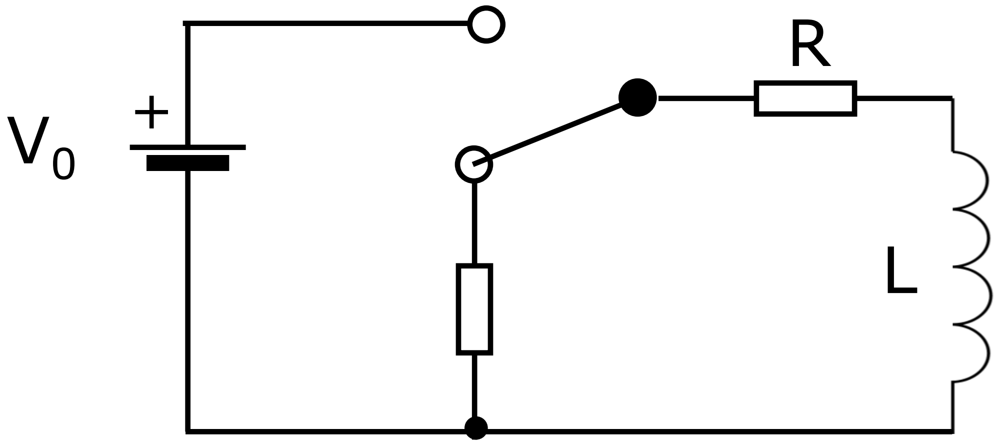
$$\int_{I(0)}^{I(t)} \frac{dI}{I} = -\int_0^t \frac{dt}{\tau} \qquad \tau = \frac{L}{21R}$$

$$\ln\left(\frac{\mathrm{I}(\mathrm{t})}{\mathrm{I}(0)}\right) = -\frac{\mathrm{t}}{\tau}$$

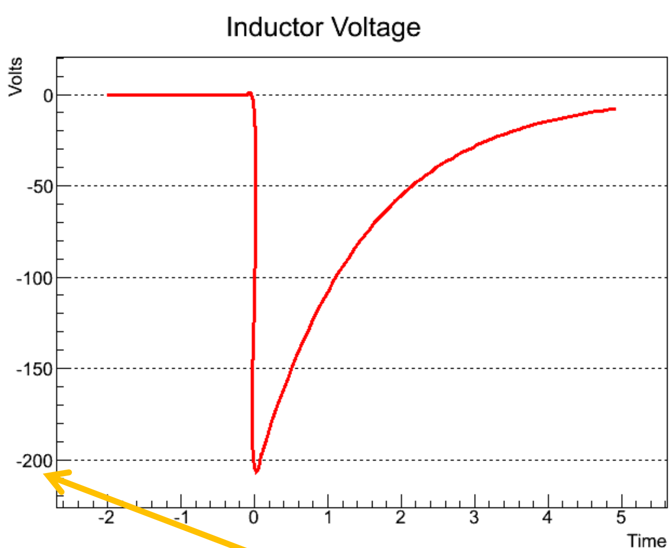
$$\mathrm{I}(\mathrm{t}) = \mathrm{I}(0)\exp\left(-\frac{\mathrm{t}}{\tau}\right)$$

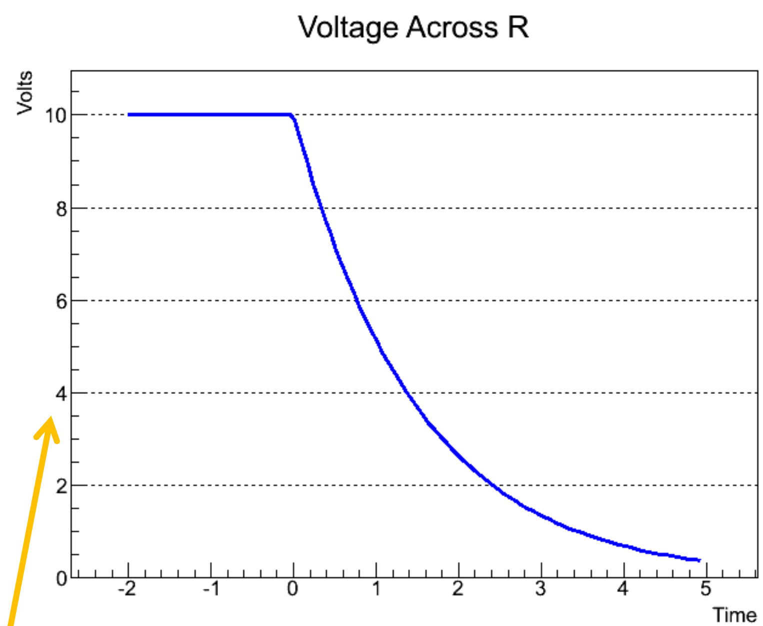
$$\mathrm{I}(\mathrm{t} = 0) = 0 \longrightarrow I(0) = \frac{V_0}{R}$$

$$I(t > 0) = \frac{V_0}{R}\exp\left(-\frac{t}{\tau}\right) \qquad I(t \le 0) = \frac{V_0}{R}$$

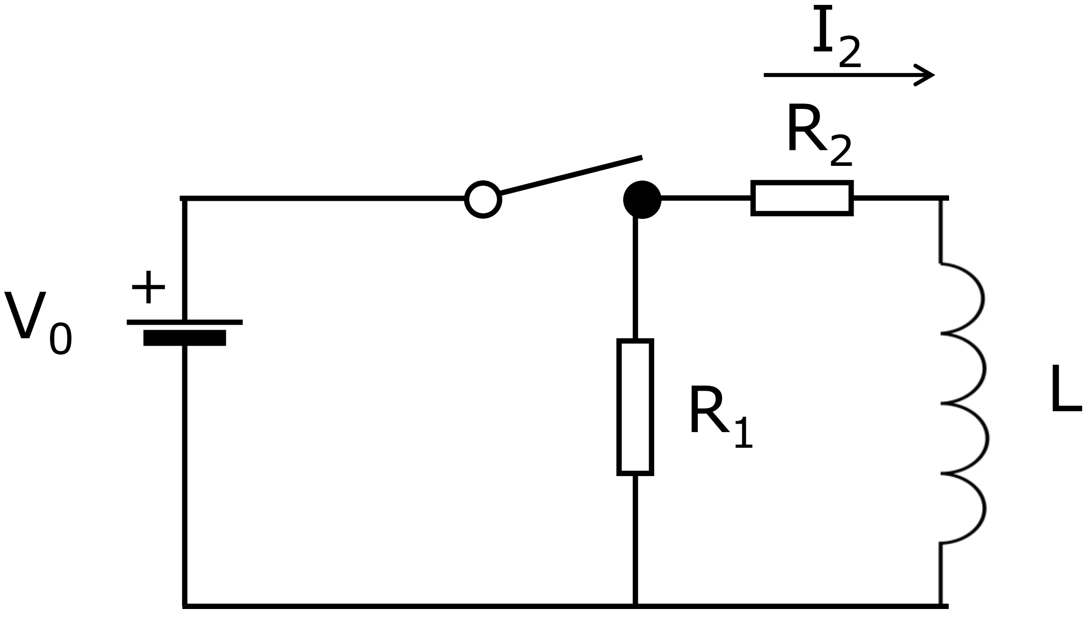


$$I(t) = \frac{V_0}{R}\left(\exp\left(-\frac{t}{\tau}\right)\right)$$





Notice Difference in Scale!



t<0 switch closed I_2=?

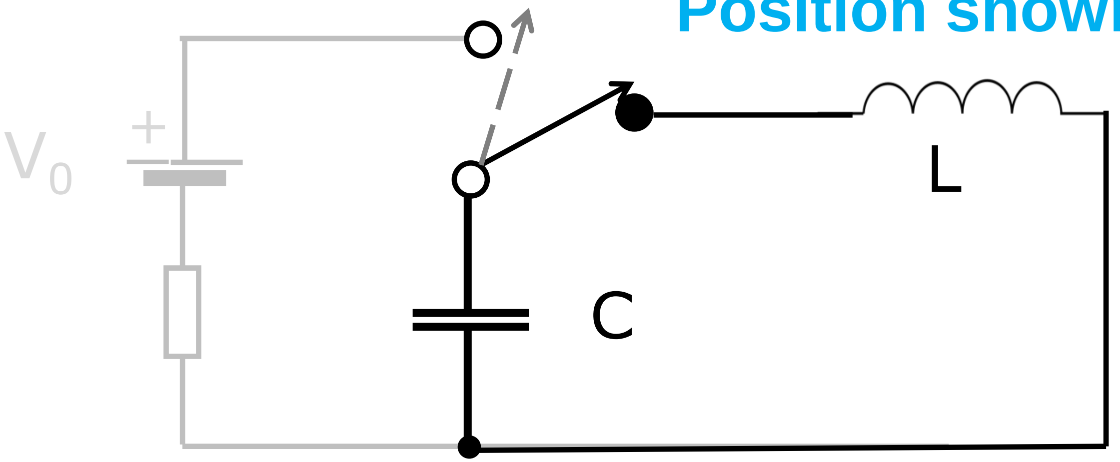
t=0 switch opened

t>0 $I_2(t)$=?

voltage across R_1=?

Write this problem down and DO attempt it at home

LC circuit



$I = I_0$ for $t = 0$

Return to white board

$$t \geq 0$$

$$V_L + V_C = 0$$

$$L\frac{dI}{dt} + \frac{Q}{C} = 0$$

$$L\frac{d^2I}{dt^2} + \frac{1}{C}I = 0$$

$$\frac{d^2 I}{dt^2} = -\frac{1}{LC} I$$

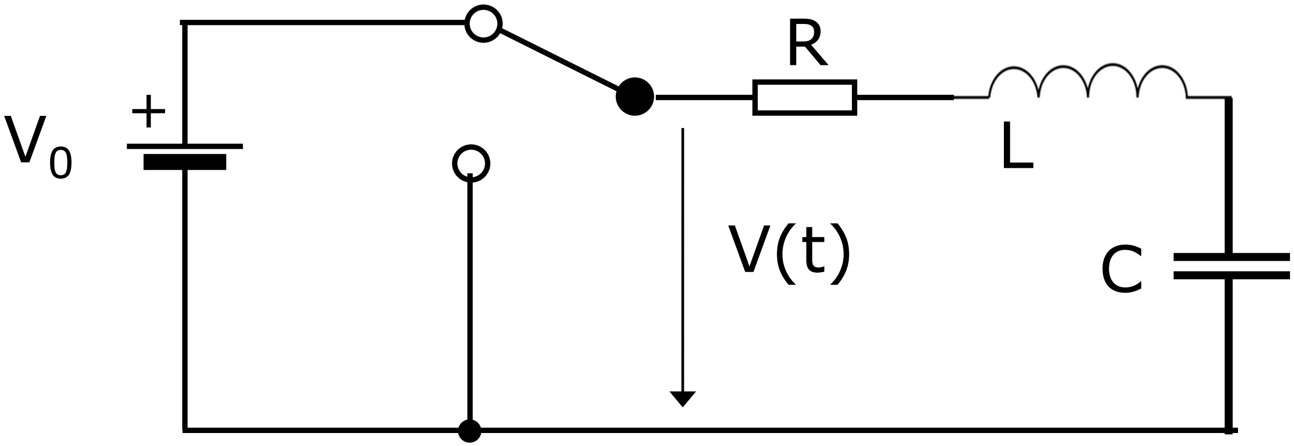
Initial conditions?

$I(0)=I_0$

$$\frac{dI}{dt}(0) = ?$$

LCR circuit

$$V(t)=\begin{cases}0 & \text{for } t<0\\ V_0 & \phantom{\text{for }} t\geq 0\end{cases}$$



$I(t)=0$ for $t<0$

$$t \geq 0$$

$$V_R + V_L + V_C = V_0$$

$$IR + L\frac{dI}{dt} + \frac{Q}{C} = V_0$$

$$\frac{d^2I}{dt^2} + \frac{R}{L}\frac{dI}{dt} + \frac{1}{LC}I = 0$$

$$\frac{d^2I}{dt^2} + \frac{R}{L}\frac{dI}{dt} + \frac{1}{LC}I = 0$$

R=6Ω
L=1H
C=0.2F

Initial conditions?

$I(0)$

$$\frac{dI}{dt}(0)$$

LCR circuit

$$I = k_1 e^{\lambda_1 t} + k_2 e^{\lambda_2 t}$$

$$\lambda = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$

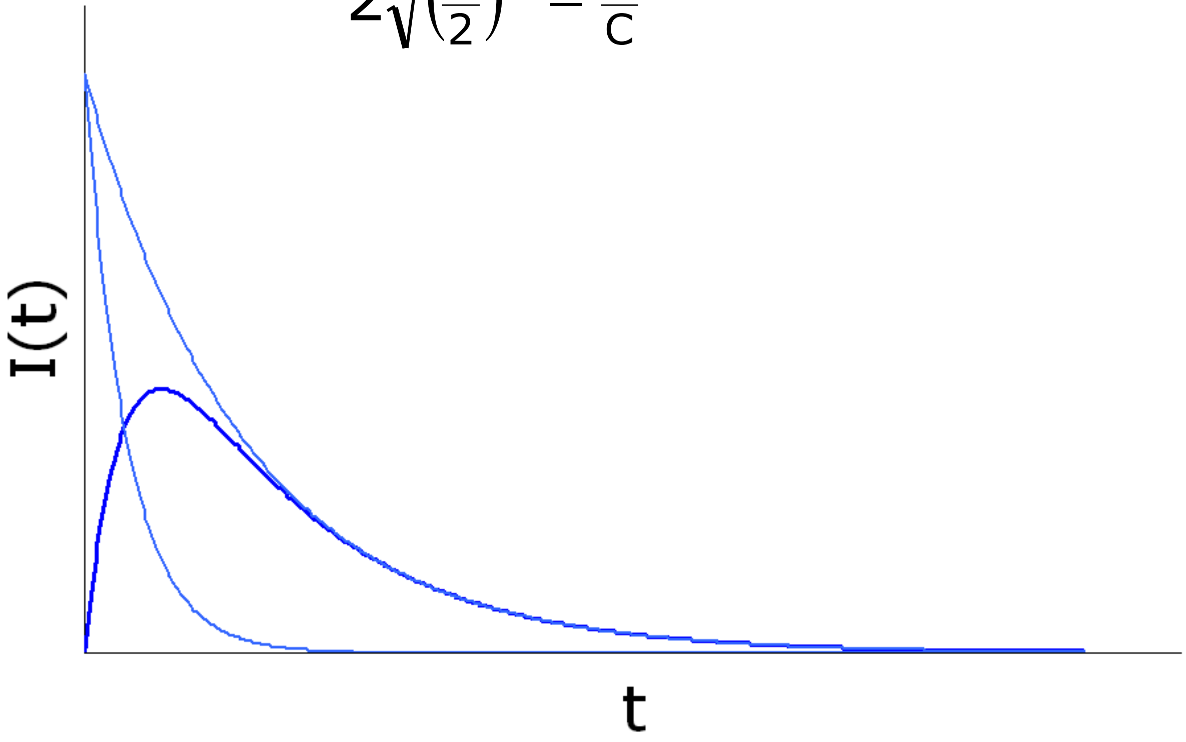
$$\lambda_{1,2} = -\gamma \pm \beta$$

$$\beta = \sqrt{\gamma^2 - \omega_0^2}$$

$$\gamma = \frac{R}{2L}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$I(t) = \frac{V_0}{2\sqrt{\left(\frac{R}{2}\right)^2 - \frac{L}{C}}}\left(e^{-(\gamma-\beta)t} - e^{-(\gamma+\beta)t}\right)$$



LCR circuit

$$\gamma^2 < \omega_0^2$$

$$I(t \geq 0) = \frac{V_0}{\sqrt{\frac{L}{C} - \left(\frac{R}{2}\right)^2}} e^{-\gamma t} \cdot \sin(\omega_R t)$$