AC Circuit Theory

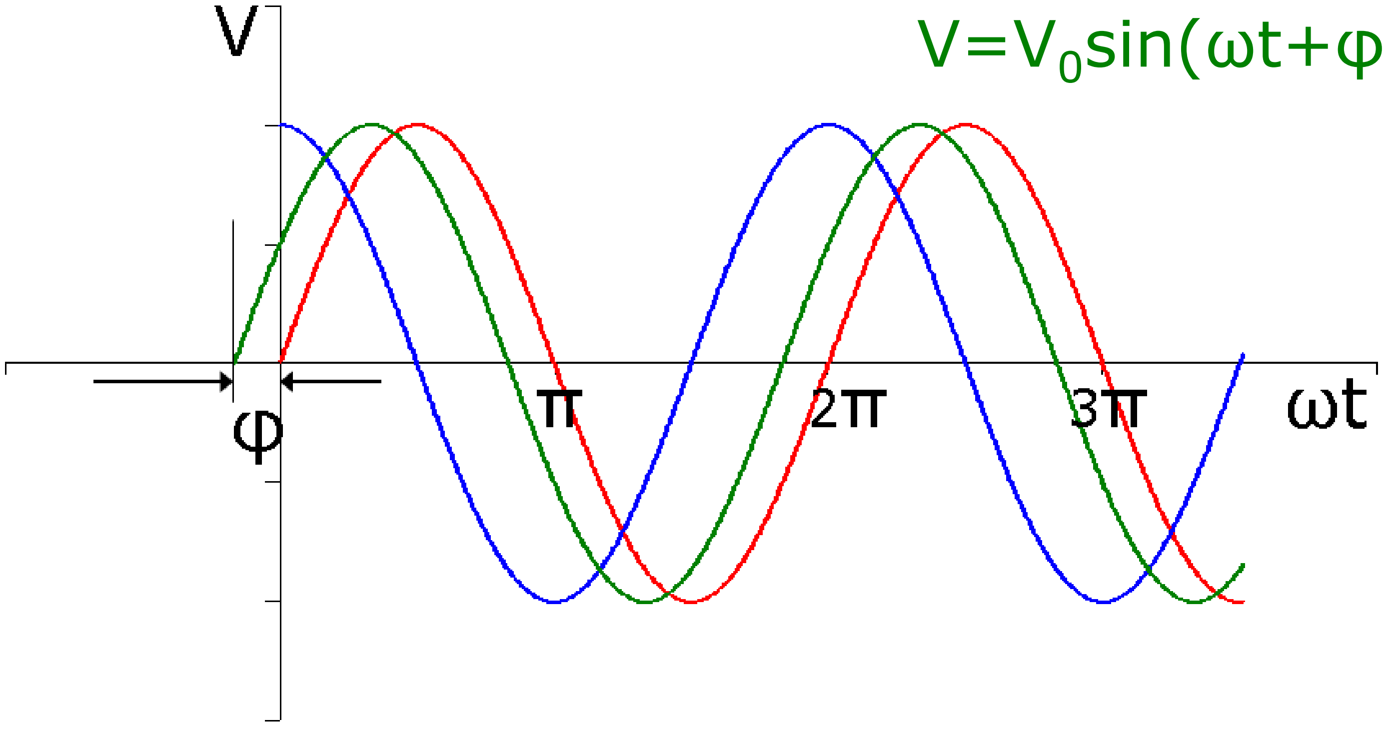
Amplitude

Phase

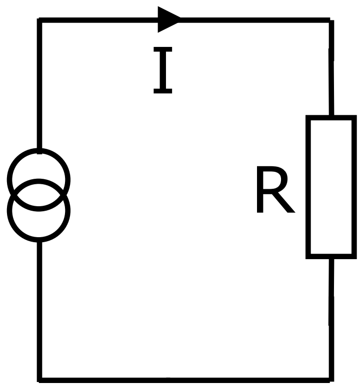
$V=V_0\sin(\omega t)$

$V=V_0\cos(\omega t)$

$V=V_0\sin(\omega t+\varphi)$



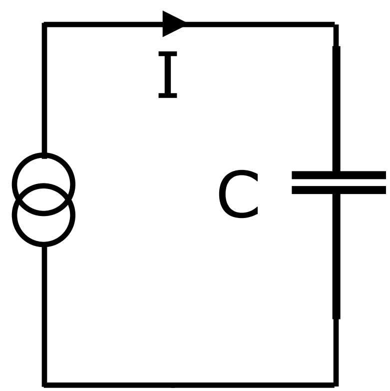
Apply AC current to a resistor…



$I = I_0 \sin(\omega t)$

$V_R = IR = I_0 R \sin(\omega t)$

…capacitor…



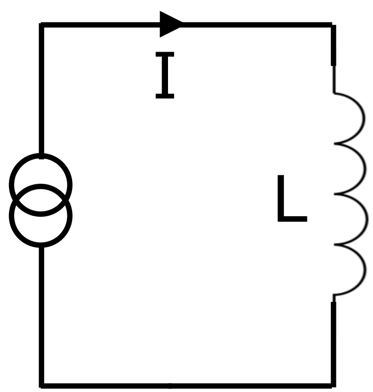
$$V_C = \frac{Q}{C} = \frac{1}{C}\int I_0 \sin(\omega t)\,dt$$

$$= -\frac{1}{\omega C} I_0 \cos(\omega t)$$

$$= \frac{1}{\omega C} I_0 \sin\left(\omega t - \tfrac{\pi}{2}\right)$$

Voltage lags current by $\pi/2$

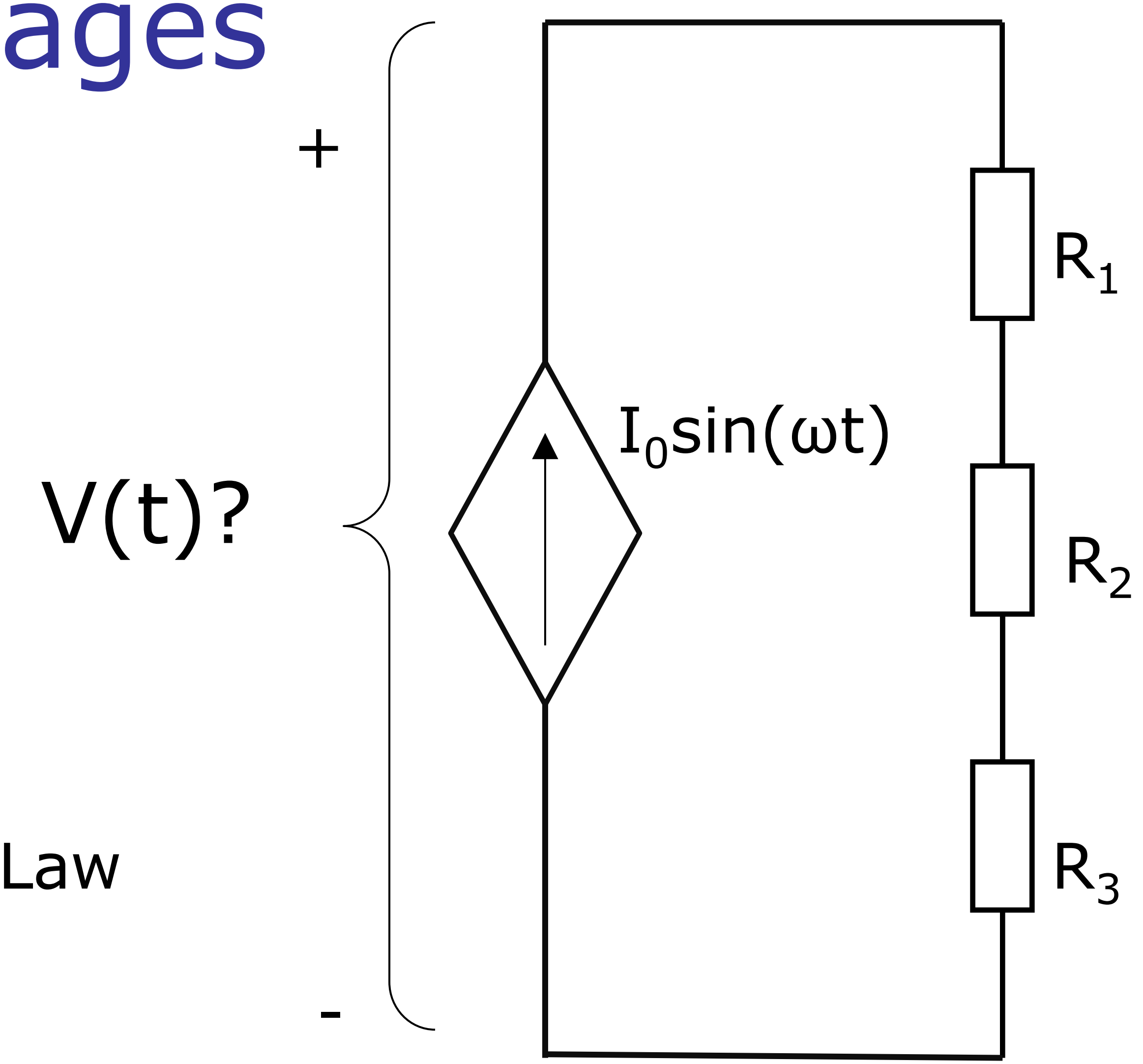
…and inductor



$$V_L = L\frac{dI}{dt} = \omega L I_0 \cos(\omega t) = \omega L I_0 \sin\left(\omega t + \tfrac{\pi}{2}\right)$$

Voltage leads current by $\pi/2$

Adding voltages



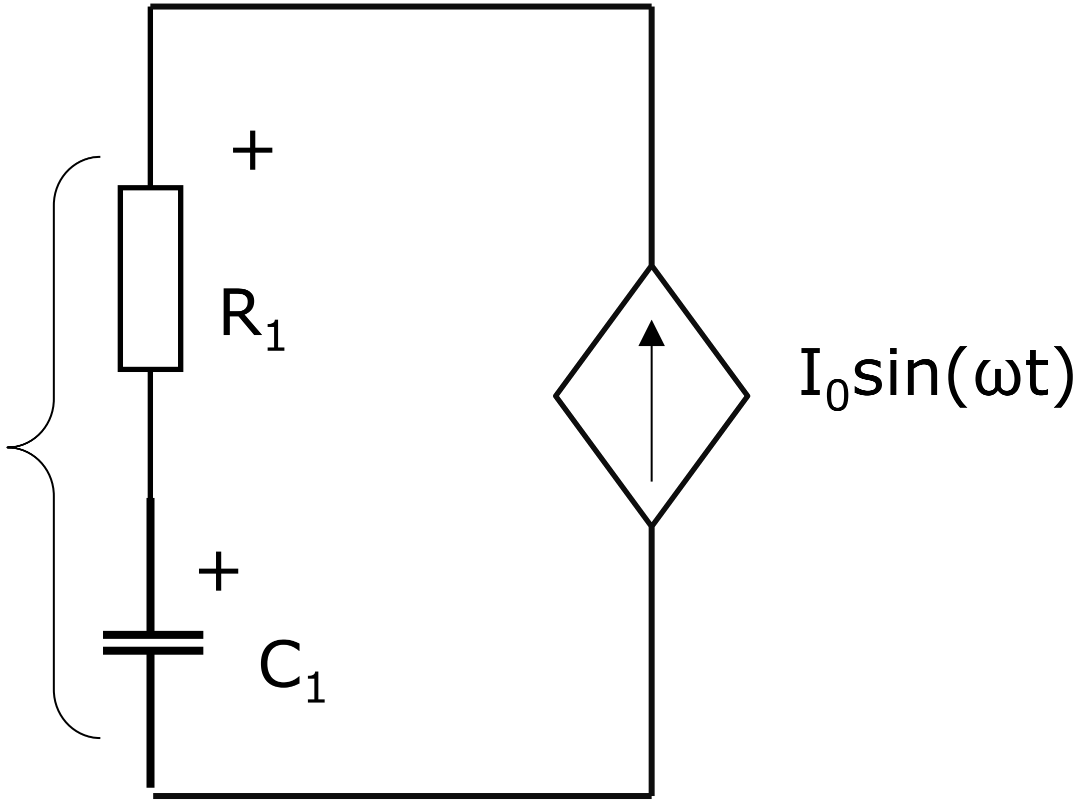
Kirchoff's Voltage Law
and
Ohm's law

$$V = I_0 R_1 \sin(\omega t) + I_0 R_2 \sin(\omega t) + I_0 R_3 \sin(\omega t)$$

$$= I_0 (R_1 + R_2 + R_3) \sin(\omega t)$$

Adding voltages

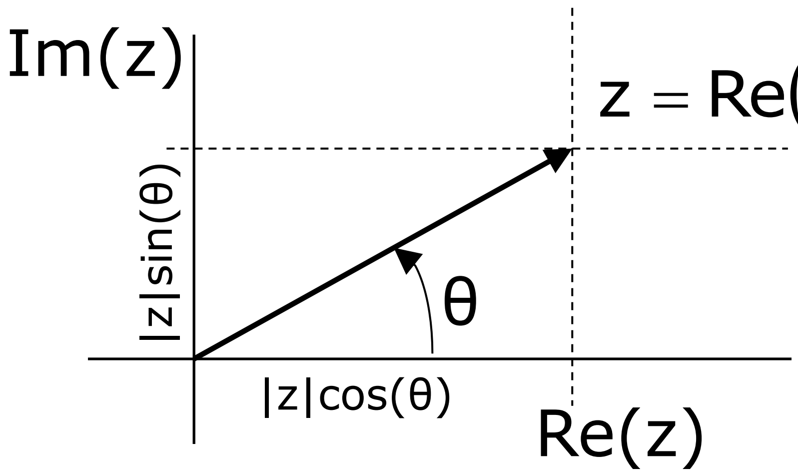
$V(t) = V_R + V_C$



$$V = I_0 R_1 \sin(\omega t) - \frac{I_0}{\omega C}\cos(\omega t)$$

voltages out of phase…

Complex numbers in AC circuit theory



$$z = \mathrm{Re}(z) + j\,\mathrm{Im}(z) = |z|\exp(j\theta)$$

$$j^2 = -1$$

Reason:

'i' is already taken

$$V(t) = V_0 \cos(\omega t) = \mathcal{R}e\left(V_0 e^{j\omega t}\right)$$

$$V(t) = V_0 \sin(\omega t) = \mathcal{I}m\left(V_0 e^{j\omega t}\right)$$

Complex Impedance

The ratio of the voltage across a component to the current through it when both are expressed in complex notation

$$I=I_0\sin(\omega t) \;\rightarrow\; I=I_0 e^{j\omega t}$$

$$V_R = IR = I_0 R\, e^{j\omega t} \;\rightarrow\; Z_R = R$$

$$V_C = \frac{1}{C}\int I\,dt = \frac{I_0}{j\omega C} e^{j\omega t} \;\rightarrow\; Z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

$$V_L = L\frac{dI}{dt} = j\omega I_0 L e^{j\omega t} \;\rightarrow\; Z_L = j\omega L$$

Complex Impedance

Real part: resistance (R)

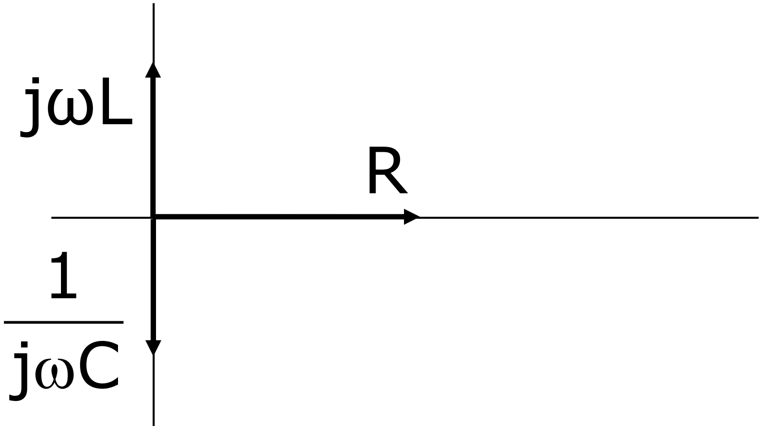
Imaginary part: reactance $\left(\omega L - \frac{1}{\omega C}\right)$

Ohm's law

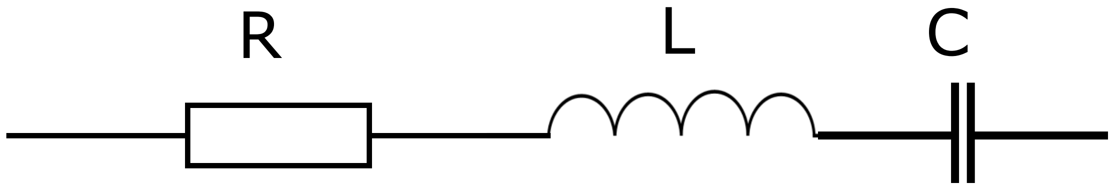
$$V = Z \cdot I$$

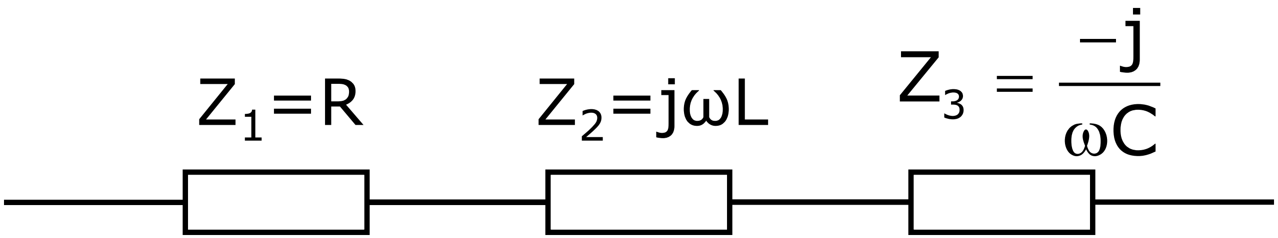
$$|V| = |Z| \cdot |I|$$

$$\text{phase}(V) = \text{phase}(Z) + \text{phase}(I)$$



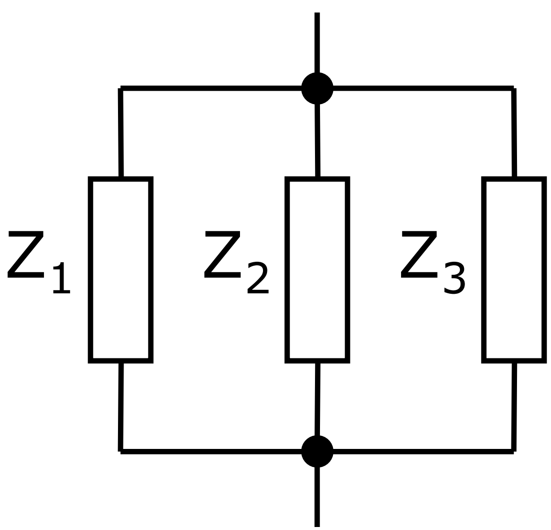
Series / parallel impedances





$$Z_T = \sum_n Z_n$$

Impedances in series: $Z_{Total}=Z_1+Z_2+Z_3\ldots$

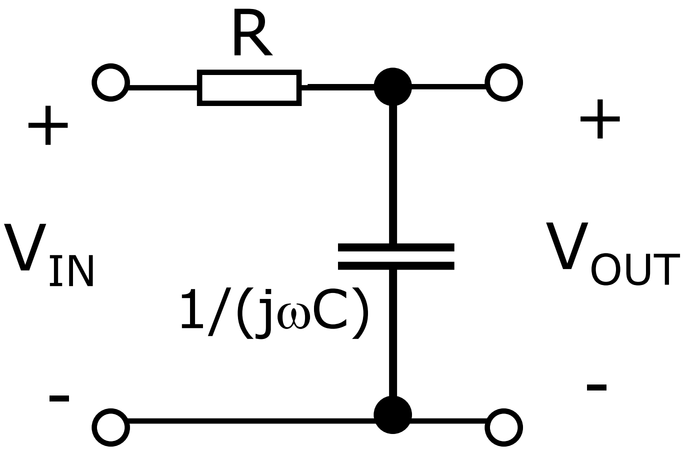


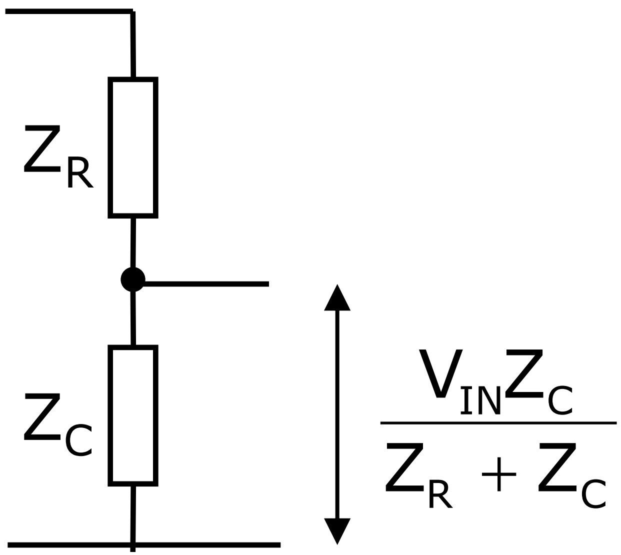
Impedances in parallel

$$\frac{1}{Z_T} = \sum_n \frac{1}{Z_n}$$

$$= \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\ldots$$

RC low pass filter





$$V_{OUT} = \frac{V_{IN}Z_C}{Z_R + Z_C} = V_{IN}\frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

$$= V_{IN}\frac{1}{1 + j\omega RC}$$

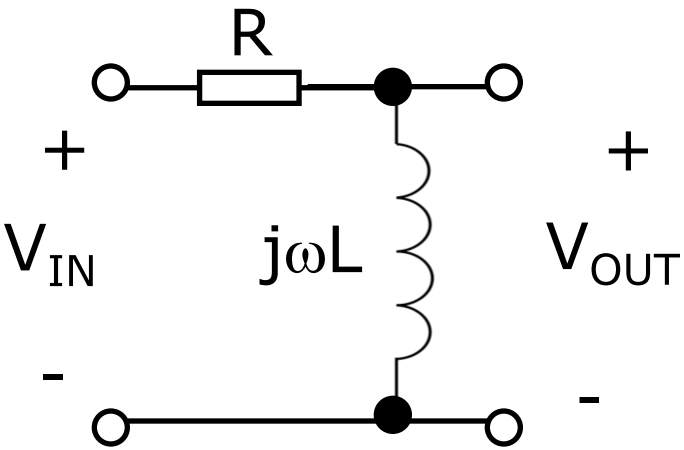
$$= V_{IN}\frac{1}{1 + j\omega RC}\frac{(1 - j\omega RC)}{(1 - j\omega RC)}$$

$$= V_{IN}\frac{1 - j\omega RC}{1 + (\omega RC)^2}$$

$$= V_{IN}\frac{1}{\sqrt{1 + (\omega RC)^2}}e^{j\varphi}$$

$$\varphi = \tan^{-1}(-\omega RC)$$

RL high pass filter



$$V_{OUT} = V_{IN}\frac{j\omega L}{R + j\omega L}$$

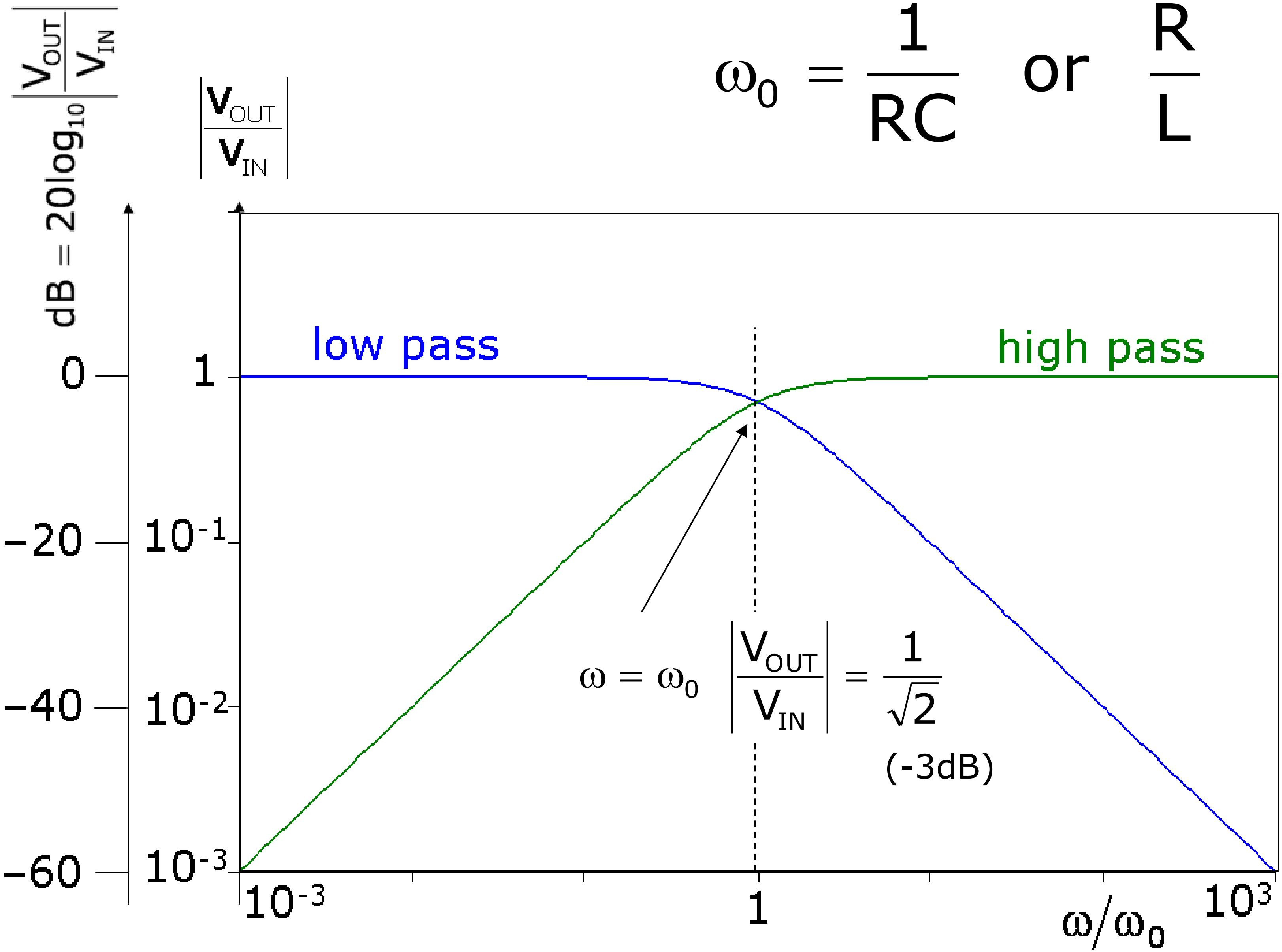
$$V_{OUT} = V_{IN}\frac{\omega \frac{L}{R}}{\sqrt{1 + \left(\omega \frac{L}{R}\right)^2}} e^{j\varphi}$$

$$\varphi = \tan^{-1}\left(\frac{R}{\omega L}\right)$$

Here is a slight trick: Get comfortable with the **complex result.**

Bode plot

$$\omega_0 = \frac{1}{RC} \quad \text{or} \quad \frac{R}{L}$$



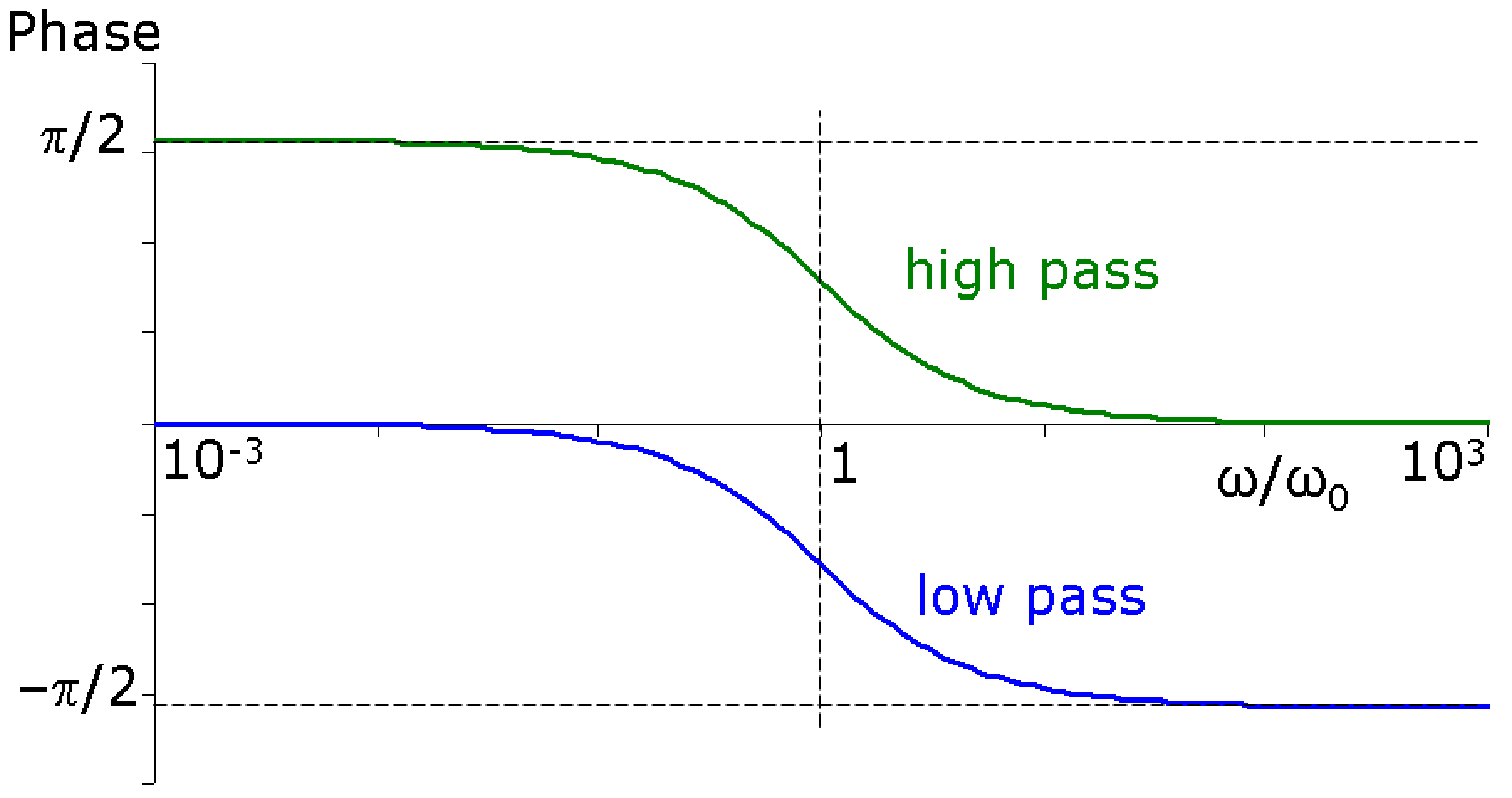
Decibels

Logarithm of power ratio

$$dB = 10\log_{10}\left|\frac{V_{OUT}^2}{V_{IN}^2}\right| = 20\log\left|\frac{V_{OUT}}{V_{IN}}\right|$$

V_{OUT}/V_{IN}	dB
10	20
1	0
0.1	−20
0.01	−40
0.001	−60

Bode plot



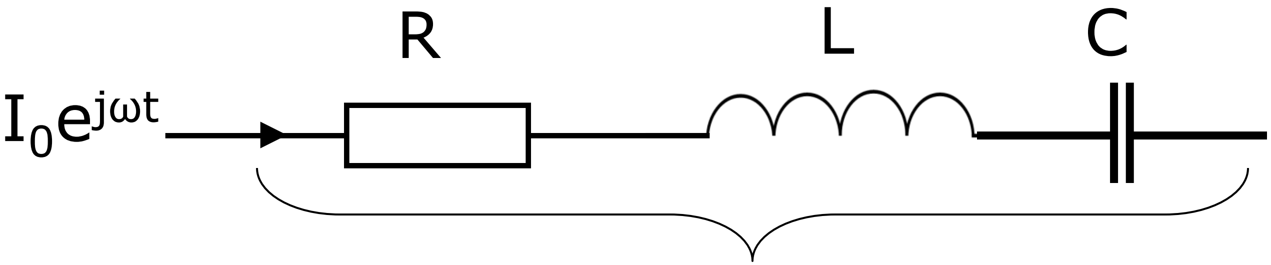
Something Interesting

- **Capacitor**
  - At low frequencies (like DC) → **Open circuit**
    - Not a surprise, it's got a gap!
  - At high frequencies ("fast") → **Short circuit!**
- **Inductor**
  - At low frequencies → **Short circuit**
    - Not a surprise, it's just a wire really
  - At high frequencies → **Open circuit!**
- **Sometimes this can help you with your intuition on the circuit's behaviour.**

Go to black Board and

EXPLAIN PHASORS

LRC series circuit



$$V = V_R + V_L + V_C$$
$$= (Z_R + Z_L + Z_C)I$$
$$= \left(R + j\left(\omega L - \frac{1}{\omega C}\right)\right)I$$

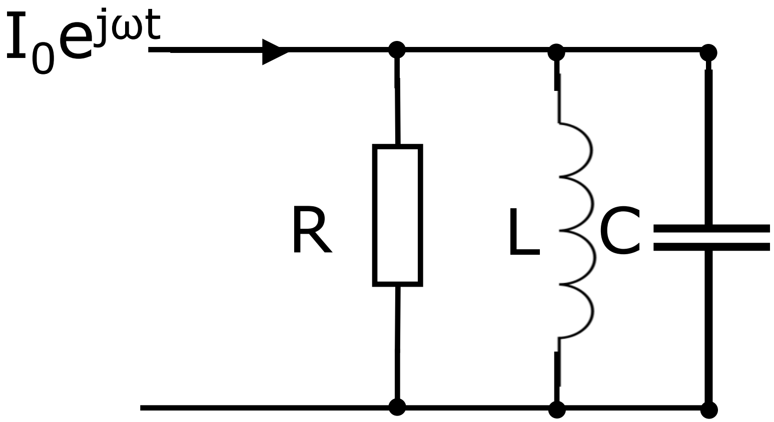
$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\varphi = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

Purely resistive at $\omega_R = \frac{1}{\sqrt{LC}}$ $\varphi=0$ $Z=R$

$$V = I_0 \cdot \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \cdot e^{j\left(\omega t + \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)\right)}$$

LRC parallel circuit



$$I = I_R + I_L + I_C = \frac{V}{Z}$$

$$\frac{1}{Z} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

$$Z = \frac{1}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)}$$

$$= \frac{\frac{1}{R} + j\left(\frac{1}{\omega L} - \omega C\right)}{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L} - \omega C\right)^2}$$

Purely resistive at $\omega_R = \frac{1}{\sqrt{LC}}$ $\varphi=0$ $Z=R$

$$|Z| = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{\omega L} - \omega C\right)^2}}$$

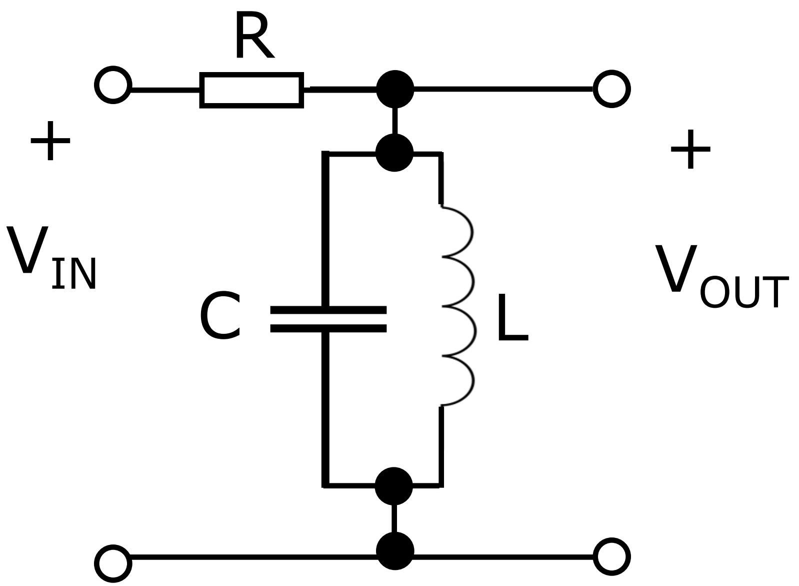
$$\varphi = \tan^{-1}\left(R\left(\frac{1}{\omega L} - \omega C\right)\right)$$

Bandpass filter

$$V_{OUT} = \frac{Z_{LC}}{R + Z_{LC}} V_{IN}$$

$$\frac{1}{Z_{LC}} = \frac{1}{j\omega L} + j\omega C$$

$$Z_{LC} = j\frac{1}{\left(\frac{1}{\omega L} - \omega C\right)}$$



$$V_{OUT} = \frac{j\omega L}{j\omega L + R\left(1 - \omega^2 LC\right)} V_{IN}$$

$$\left|\frac{V_{OUT}}{V_{IN}}\right| = \frac{1}{\sqrt{1 + R^2\left(\frac{1}{\omega L} - \omega C\right)^2}}$$

$$\omega \to \infty \quad \left|\frac{V_{OUT}}{V_{IN}}\right| \to 0 \qquad \omega \to 0 \quad \left|\frac{V_{OUT}}{V_{IN}}\right| \to 0$$

ON BLACK BOARD TOO!

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}} \qquad \left|\frac{V_{OUT}}{V_{IN}}\right| = 1$$

Bandpass filter

build a radio filter

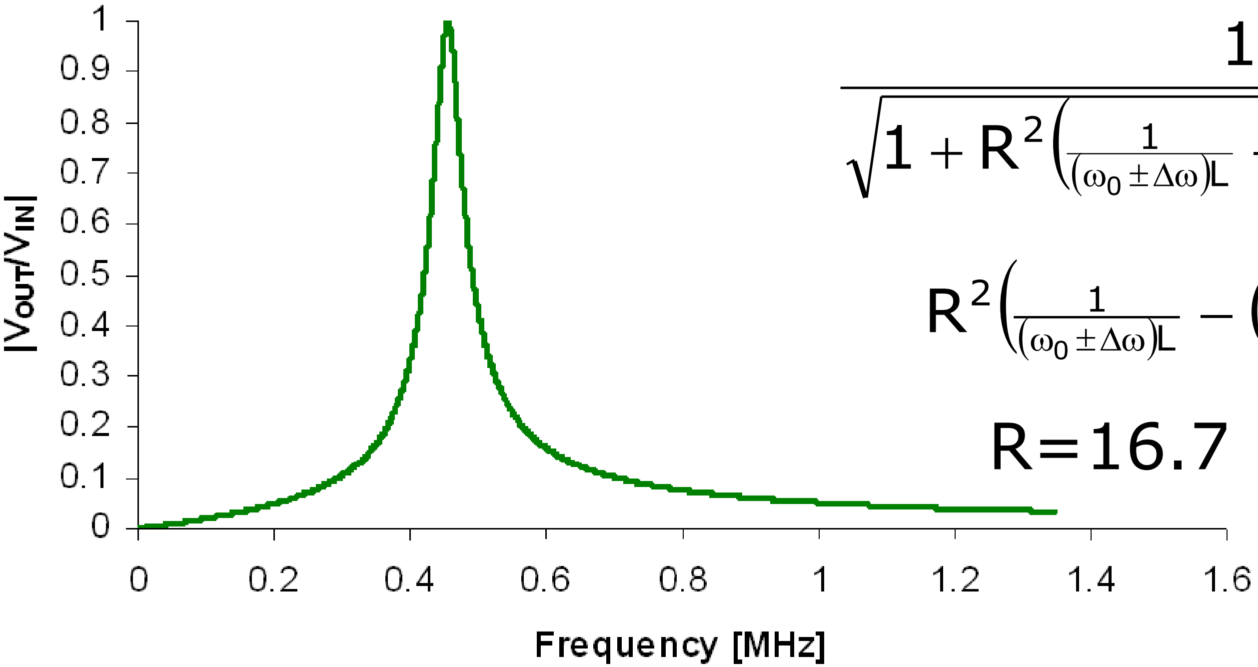
$$\left|\frac{V_{OUT}}{V_{IN}}\right| = \frac{1}{\sqrt{1 + R^2\left(\frac{1}{\omega L} - \omega C\right)^2}}$$

$f_0 = 455\text{kHz}$

$\Delta f = 20\text{kHz}$

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}} \longrightarrow \begin{array}{l} C = 10\text{nF} \\ L = 12.2\mu\text{H} \end{array}$$

$$\Delta\omega = 2\pi\Delta f \qquad \omega = \omega_0 \pm \Delta\omega$$



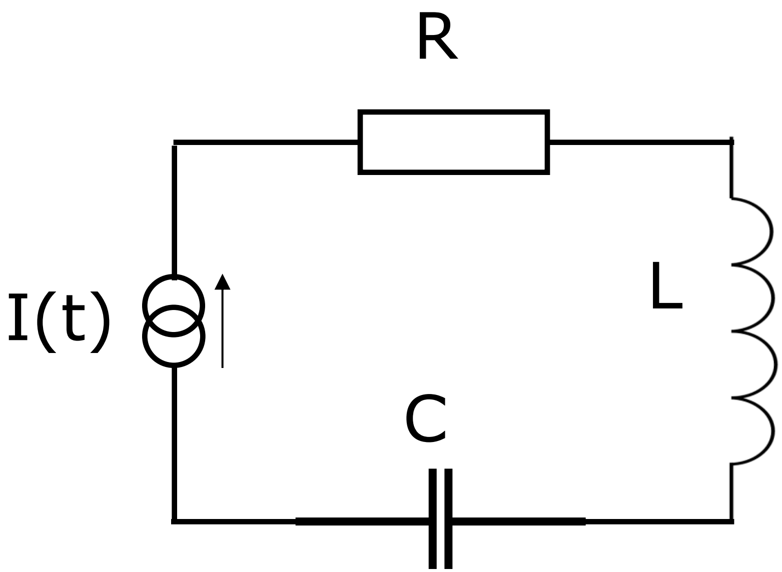
$$\frac{1}{\sqrt{1 + R^2\left(\frac{1}{(\omega_0 \pm \Delta\omega)L} - (\omega_0 \pm \Delta\omega)C\right)^2}} = \frac{1}{\sqrt{2}}$$

$$R^2\left(\frac{1}{(\omega_0 \pm \Delta\omega)L} - (\omega_0 \pm \Delta\omega)C\right)^2 = 1$$

$R = 16.7\ \text{k}\Omega$

LCR series circuit – current driven

$$I(t) = I_0 e^{j\omega t}$$



$$Z = R + j\omega L + \frac{1}{j\omega C}$$

$$Z = R + j\left[\omega L - \frac{1}{\omega C}\right]$$

$$V = IZ = I_0 \cdot |Z| \cdot e^{j\varphi} \cdot e^{j\omega t}$$

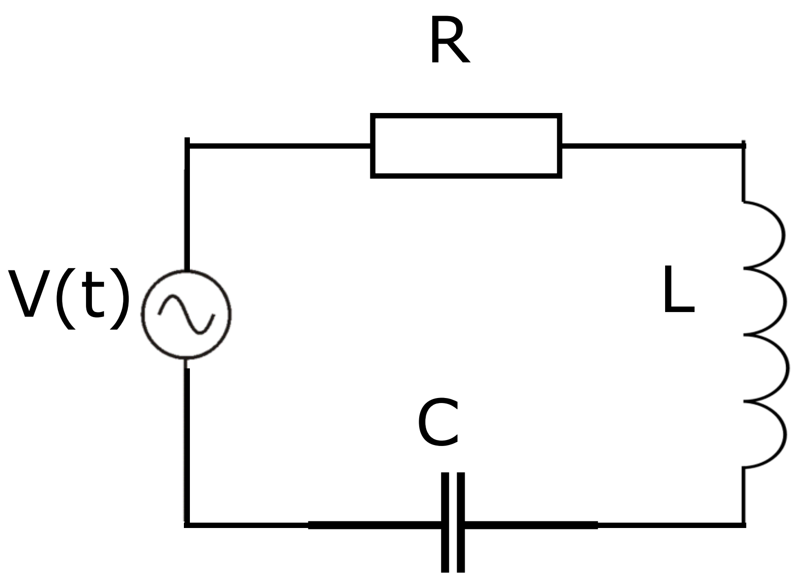
Low Freq.→Wild Stuff!

NOT a Very Good Idea

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\varphi = \tan^{-1}\left(\frac{\mathrm{Im}}{\mathrm{Re}}\right) = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

LCR series circuit – voltage driven



$$V(t)=V_0e^{j\omega t}$$

$$I=\frac{V}{Z}=\frac{V_0}{|Z|}e^{-j\varphi}$$

$$|Z|=\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$$

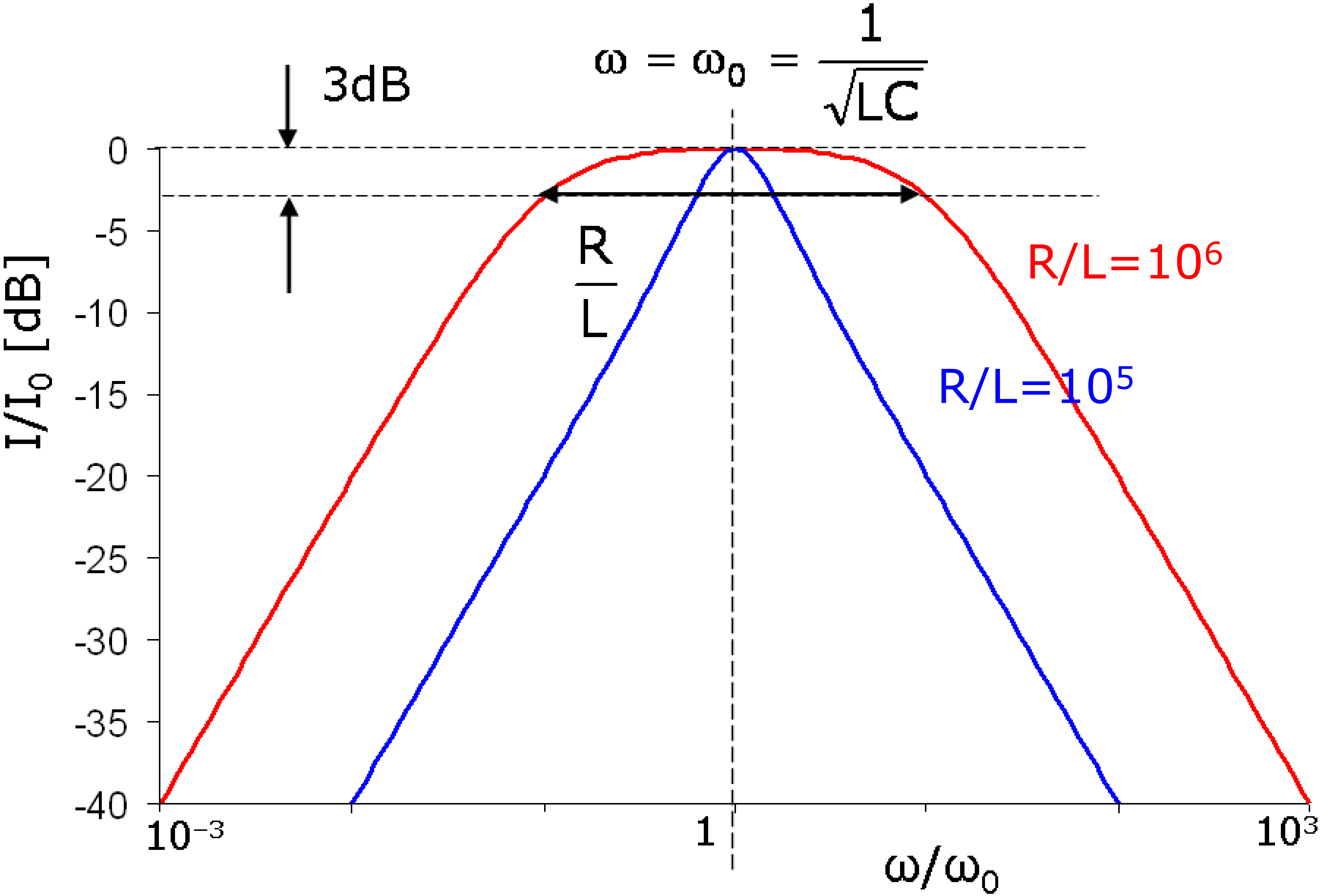
$$\varphi=\tan^{-1}\left(\frac{\omega L-\frac{1}{\omega C}}{R}\right)$$

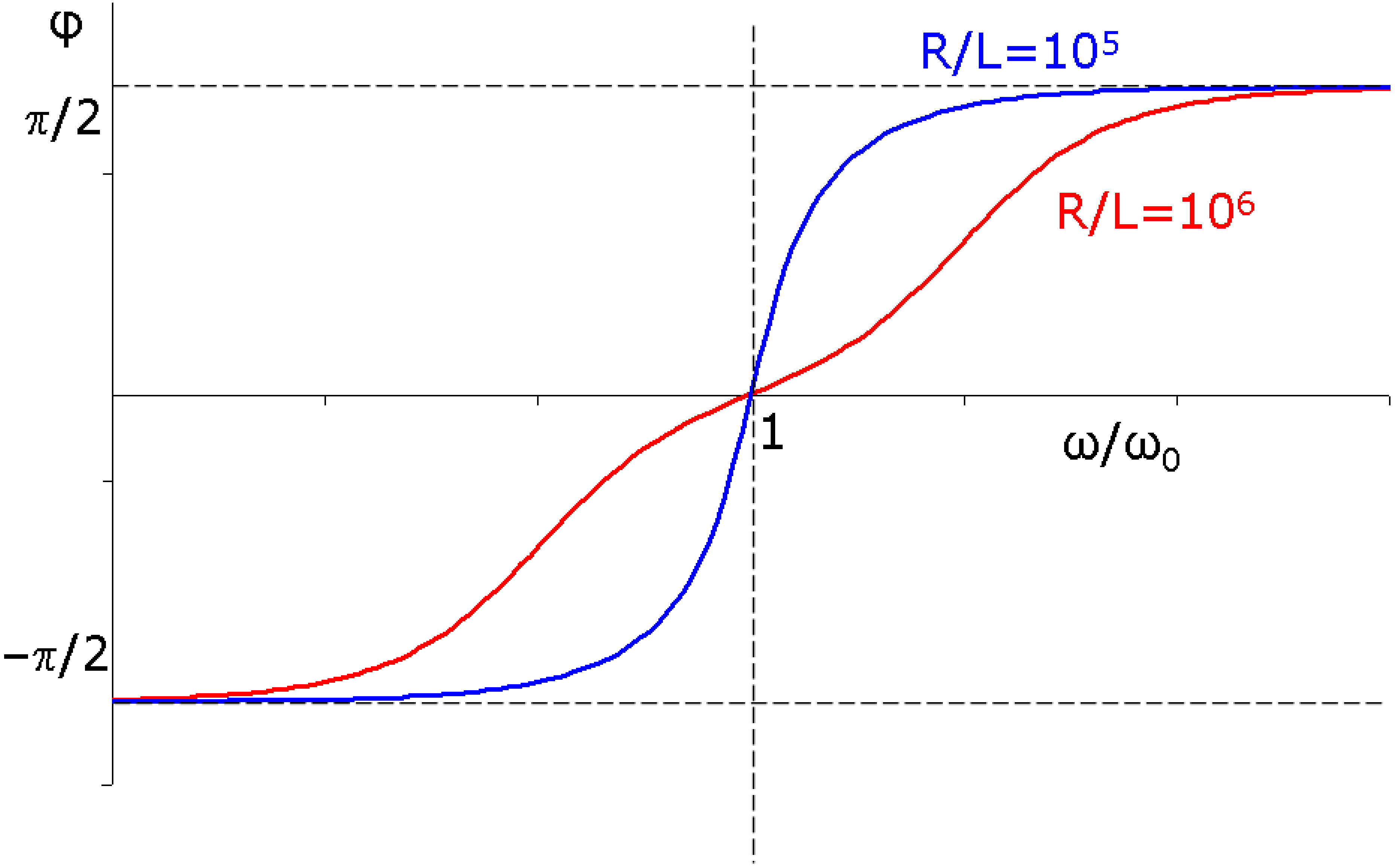
$$|I|=\frac{V_0}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}$$

$\omega\to\infty \qquad |I|\to 0$

$\omega\to 0 \qquad |I|\to 0$

$\omega\to\omega_0 \qquad |I|=\frac{V_0}{R}$ maximum


3dB
$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$
$\frac{R}{L}$
R/L=10^6
R/L=10^5
I/I_0 [dB]
0
-5
-10
-15
-20
-25
-30
-35
-40
10^{-3}
1
10^3
ω/ω_0


φ
π/2
−π/2
R/L=10⁵
R/L=10⁶
1
ω/ω₀

Q value

No longer on CP2 syllabus (moved to CP3)

$$Q = 2\pi \frac{\text{peak energy stored}}{\text{energy dissipated per cycle}}$$

NOT ALWAYS HELPFUL

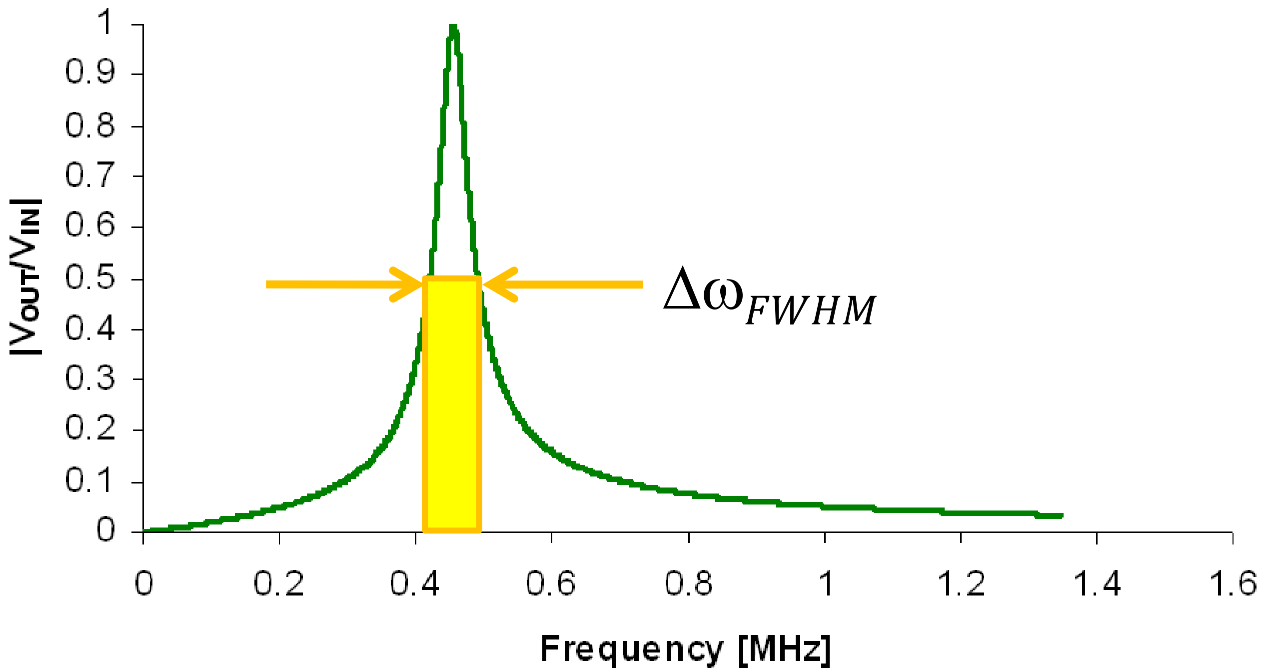
$$Q = \frac{\omega_0}{\Delta\omega_{FWHM}} = \frac{\text{Frequency at resonance}}{\text{Diff. in Freq. to FWHM}}$$

Power
not voltage

High Q value – narrow resonance

Q value

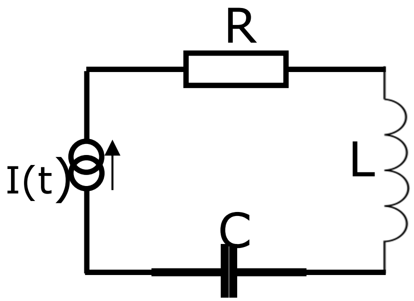
For what we did last time.



$$Q = \frac{\omega_0}{\Delta\omega_{FWHM}} = \frac{\omega_0}{2\Delta\omega} = \frac{1}{2} \cdot 2R\sqrt{\frac{C}{L}} = R\sqrt{\frac{C}{L}}$$

High Q value – narrow resonance

LC circuit – power dissipation



Power is only dissipated in the resistor

$$\langle P \rangle = \frac{1}{T}\int_0^T I(t)V(t)dt \qquad T = \frac{2\pi}{\omega}$$

proof – power dissipated inductor and capacitor – none

$$I(t) = I_0 \sin(\omega t)$$

$$V_C = -\frac{1}{\omega C} I_0 \cos(\omega t)$$

$$V_L = \omega L I_0 \cos(\omega t)$$

$$V_X = X I_0 \cos(\omega t)$$

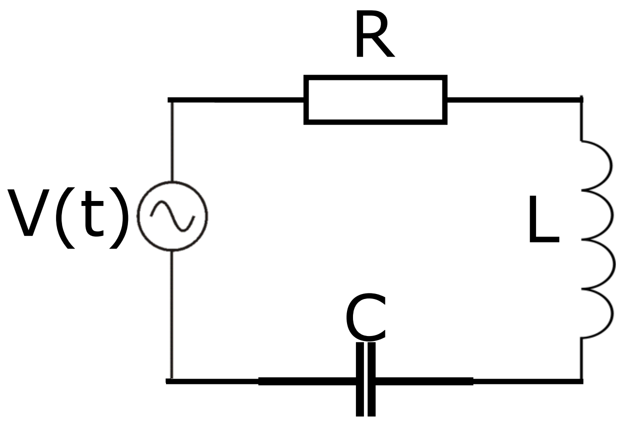
reactance

$$X = -\frac{1}{\omega C} \text{ or } \omega L$$

$$\langle P \rangle_{L,C} = \frac{1}{T}\int_0^T X I_0 \sin(\omega t)\cos(\omega t)dt = \left[\frac{1}{2\omega T} X I_0^2 \sin^2(\omega t)\right]_0^{T=2\pi/\omega} = 0$$

LRC Power dissipation

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$



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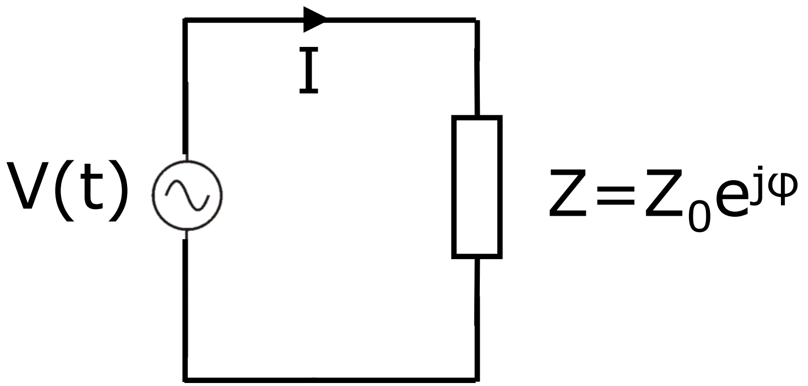


$$V(t) = V_0 \sin(\omega t)$$

$$I(t) = \frac{V_0}{Z_0}\sin(\omega t - \varphi)$$

$$Z_0 = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\tan\varphi = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$$

$$P(t) = V(t)I(t) = \frac{V_0^2}{Z_0}\sin(\omega t)\sin(\omega t - \varphi)$$

Instantaneous power

Power dissipation

$$P(t) = V(t)I(t) = \frac{V_0^2}{Z_0}\sin(\omega t)\sin(\omega t - \varphi)$$

$$= V_0 I_0 \sin(\omega t)\left[\sin(\omega t)\cos(\varphi) - \cos(\omega t)\sin(\varphi)\right]$$

$$\langle P \rangle = \frac{1}{T}\int_0^T V_0 I_0 \left[\sin^2(\omega t)\cos(\varphi) - \sin(\omega t)\cos(\omega t)\sin(\varphi)\right] dt$$

$$\langle P \rangle = \frac{V_0 I_0}{T}\int_0^T \tfrac{1}{2}\left(1 - \cos(2\omega t)\right)\cos(\varphi)\, dt \qquad - \qquad 0$$

$$= \frac{V_0 I_0}{T}\left[\frac{t}{2}\cos\varphi\right]_0^T$$

$$= \frac{V_0 I_0}{2}\cos\varphi \qquad = V_{RMS} I_{RMS}\cos\varphi$$

$\cos\varphi$ = power factor

On black board

But then go ahead and flip to next slide now anyway.

POWER IN COMPLEX CIRCUIT ANALYSIS

Power factor

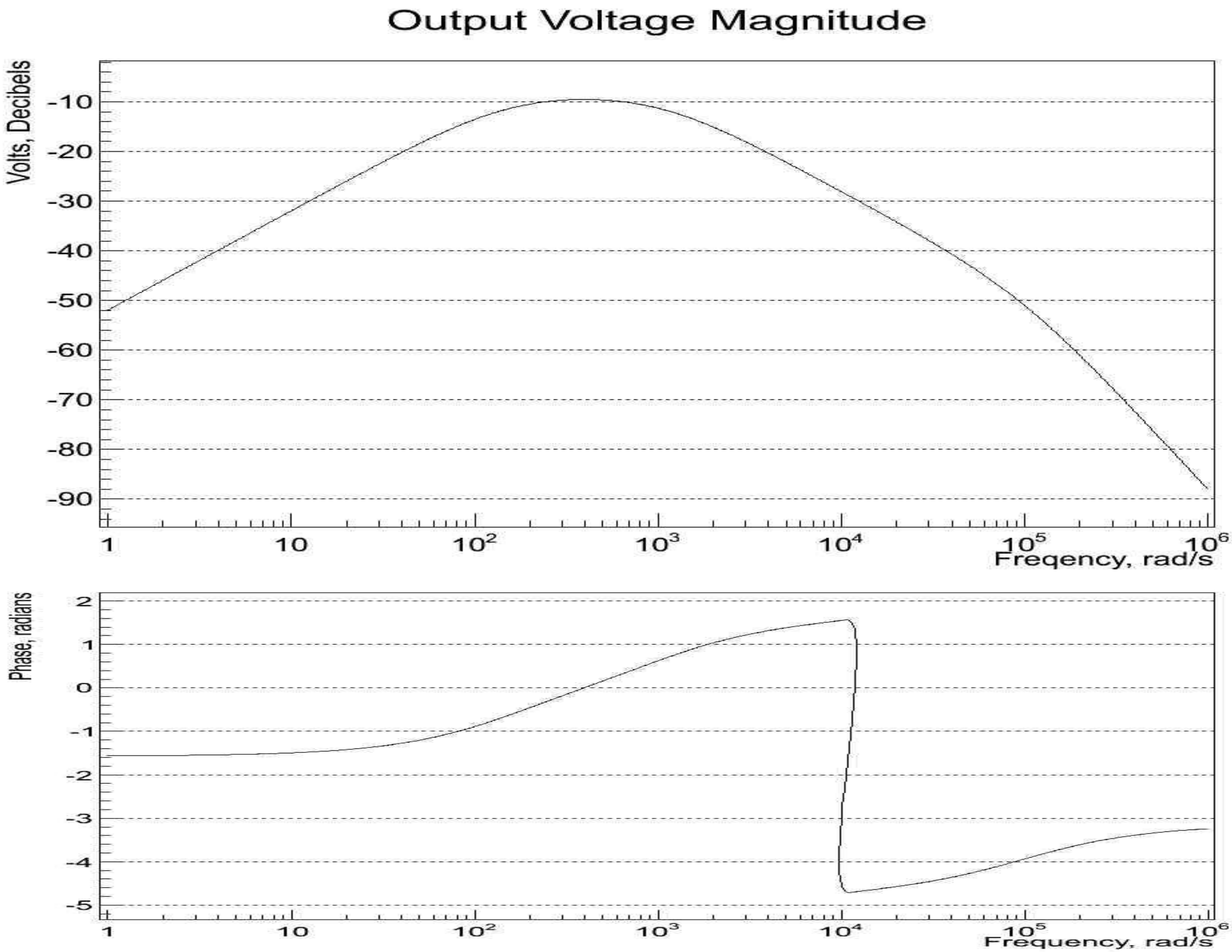
$$\cos\varphi = \frac{\text{average power}}{\text{apparent power}} = \frac{P}{V_{RMS} I_{RMS}}$$

Resistive load $Z=R \quad \cos\varphi=1$

Reactive load $Z=X \quad \cos\varphi=0$

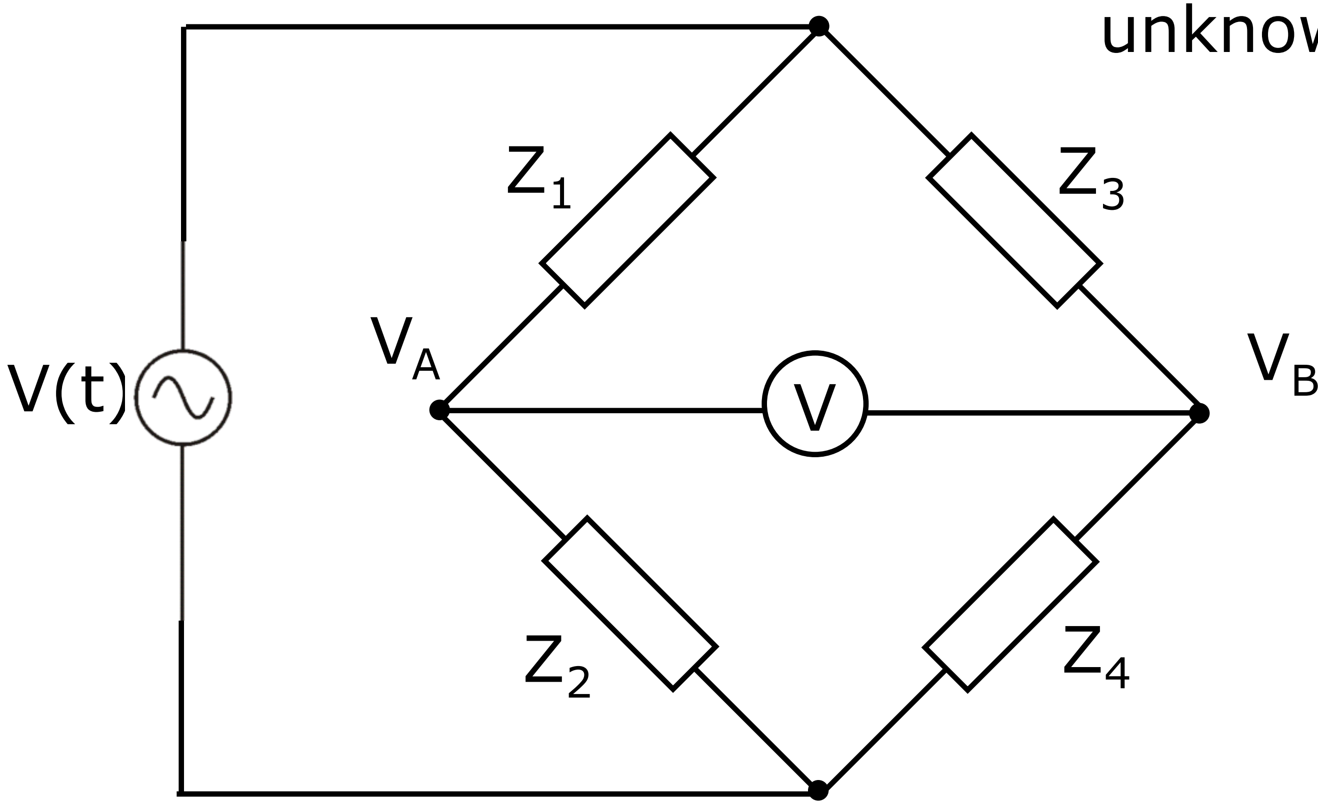
RECALL
THE PREVIOUS CIRCUIT:

R=1 kΩ; C=2.5 μF; L = 10 mH



Bridge circuits

To determine an unknown impedance



Bridge balanced when $V_A - V_B = 0$

$$\frac{Z_2}{Z_1 + Z_2} V(t) - \frac{Z_4}{Z_3 + Z_4} V(t) = 0$$

$$\frac{Z_2}{Z_1 + Z_2} = \frac{Z_4}{Z_3 + Z_4}$$

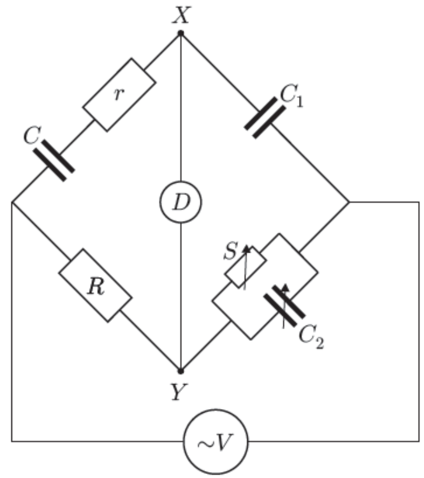
$$Z_2 (Z_3 + Z_4) = Z_4 (Z_1 + Z_2)$$

$$Z_2 Z_3 = Z_4 Z_1$$

Phasors and stuff

BACKUP SLIDES

9. (i) A Schering bridge is used to determine the capacitance C and resistance r of a lossy capacitor. In the diagram below the instrument D detects when X and Y are at the same potential, i.e. the bridge is balanced.



Show that the conditions for balancing the bridge are

$$C = C_1 S/R \quad \text{and} \quad r = C_2 R/C_1 \,. \qquad [10]$$

(ii) A source of alternating voltage $V_0 \exp(j\omega t)$ with internal complex impedance $Z_1 = R_1 + jX_1$ is connected to a load of complex impedance $Z_2 = R_2 + jX_2$.

Show that the current through the load is given by

$$I = I_0 \exp[j(\omega t - \phi)],$$

where

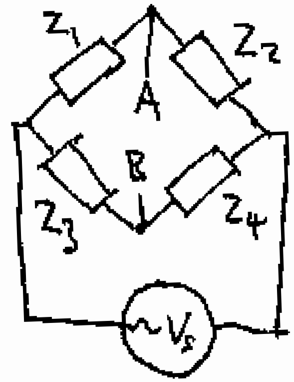
$$I_0 = V_0 \left[(R_1 + R_2)^2 + (X_1 + X_2)^2\right]^{-1/2},$$

and find an expression for ϕ. [4]

Hence show that the mean power $\langle P \rangle$ consumed by the load over one cycle is

$$\langle P \rangle = \tfrac{1}{2} I_0^2 R_2 . \qquad [6]$$

i) Bridge relations



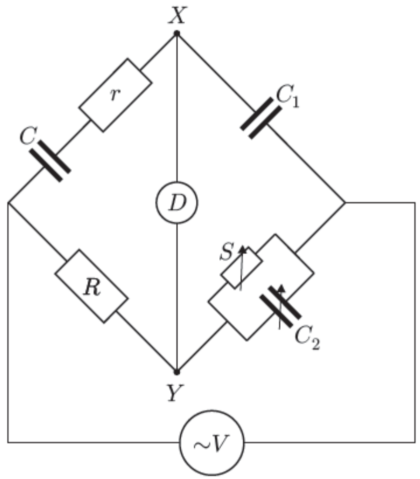
balanced when $V_A = V_B$

$$V_A = \frac{Z_1}{Z_1+Z_2} V_s \qquad V_B = \frac{Z_3}{Z_3+Z_4} V_s$$

$$Z_1(Z_3+Z_4) = Z_3(Z_1+Z_2)$$

$$Z_1 Z_4 = Z_3 Z_2$$

Must be satisfied for both real and imaginary parts



$$Z_1 = Z_C + Z_r = \frac{-j}{\omega C} + r$$

$$Z_2 = \frac{-j}{\omega C_1}$$

$$Z_3 = R$$

$$\frac{1}{Z_4} = \frac{1}{Z_S} + \frac{1}{Z_{C_2}} = \frac{1}{S} + j\omega C_2 = \frac{1 + j\omega S C_2}{S}$$

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$$

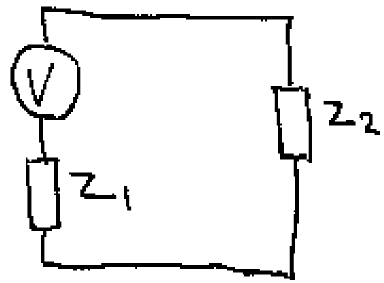
$$\left(r - \frac{j}{\omega C}\right) j\omega C_1 = \frac{R}{S}\left(1 + j\omega S C_2\right)$$

$$j\omega C_1 r + \frac{C_1}{C} = \frac{R}{S} + j\omega R C_2$$

equate real and imaginary parts

$$C = \frac{C_1 S}{R} \qquad r = \frac{C_2 R}{C_1}$$

ii)



$Z_1 = R_1 + jX_1$

$Z_2 = R_2 + jX_2$

$V = V_0 e^{j\omega t}$

$$V = (Z_1 + Z_2) I$$

$$= [(R_1 + R_2) + j(X_1 + X_2)] I$$

$$I = V_0 [(R_1+R_2)^2 + (X_1+X_2)^2]^{-\frac{1}{2}} e^{j(\omega t - \phi)} \qquad \tan\phi = \frac{X_1+X_2}{R_1+R_2}$$

Power dissipated in load $P = I^2 R_2$

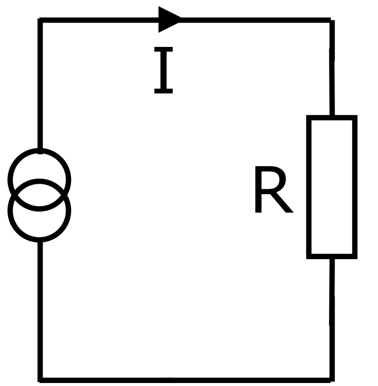
only resistive part of impedance contributes

$$P = I_0^2 R_2 \cos^2(\omega t - \phi)$$

$\cos^2$ or $\sin^2$ averaged over 1 cycle $= \frac{1}{2}$

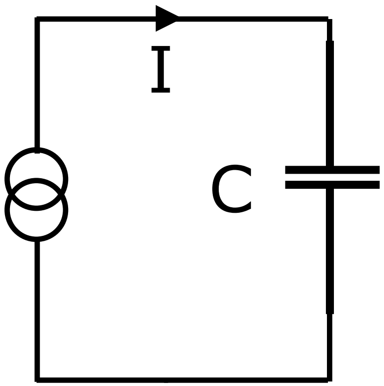
$$\langle P \rangle = \frac{1}{2} I_0^2 R$$

Phasors



$$I = I_0 \sin(\omega t)$$

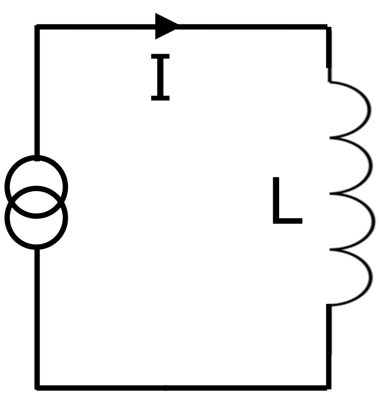
$$V_R = IR = I_0 R \sin(\omega t)$$



$$V_C = \frac{Q}{C} = \frac{1}{C}\int I_0 \sin(\omega t)dt$$

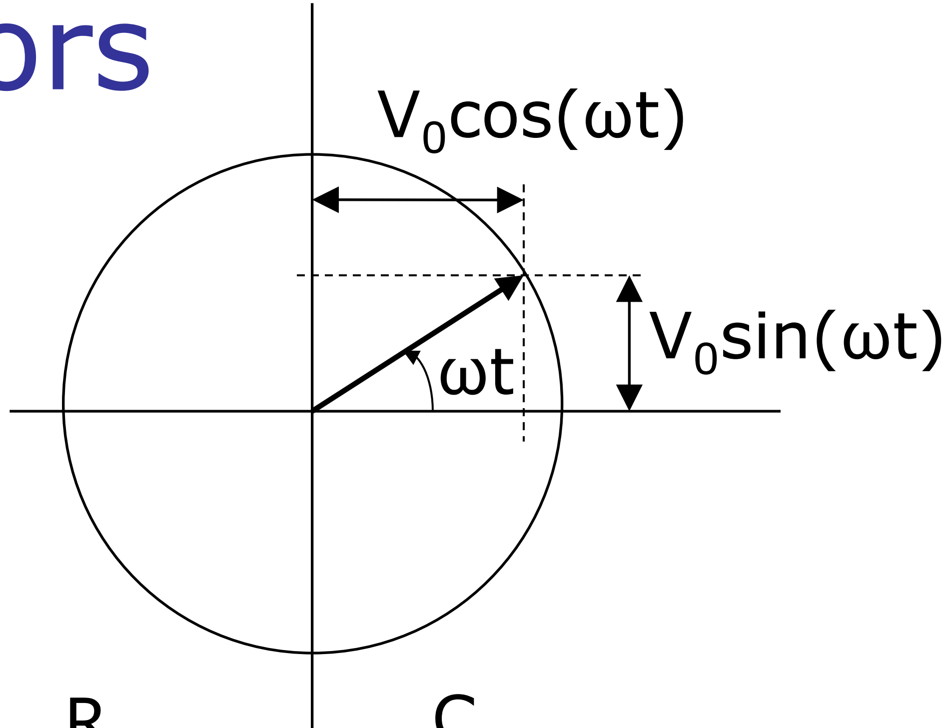
$$= -\frac{1}{\omega C} I_0 \cos(\omega t)$$

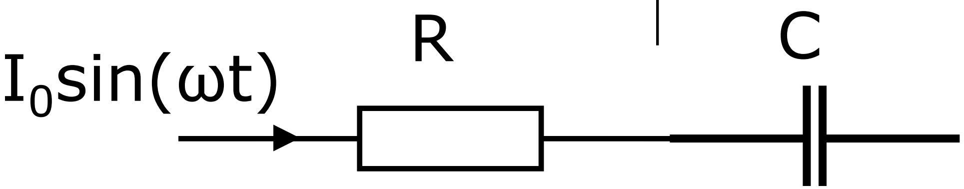
$$= \frac{1}{\omega C} I_0 \sin\left(\omega t - \tfrac{\pi}{2}\right)$$



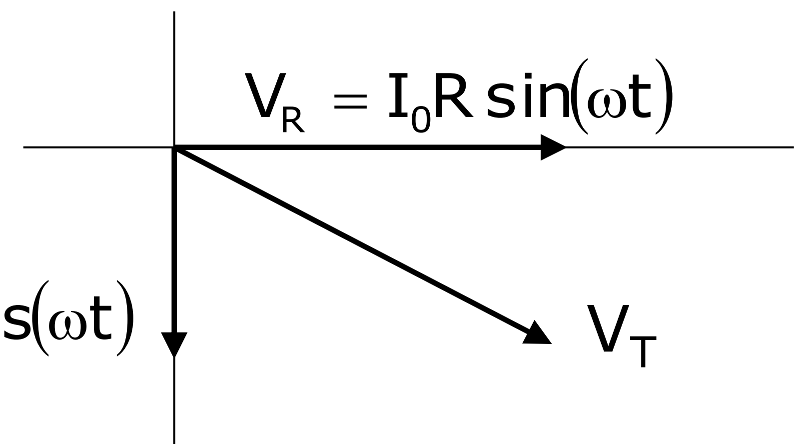
$$V_L = L\frac{dI}{dt} = \omega L I_0 \cos(\omega t) = \omega L I_0 \sin\left(\omega t + \tfrac{\pi}{2}\right)$$

Phasors



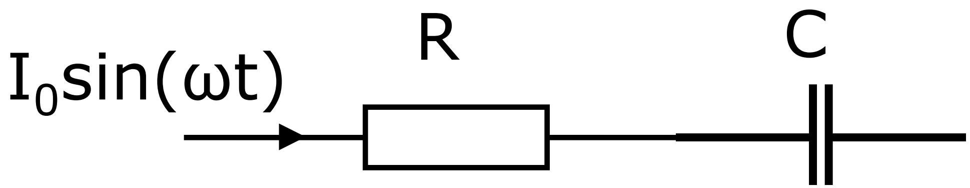


$$V_T = I_0 R \sin(\omega t) - \frac{I_0}{\omega C}\cos(\omega t)$$

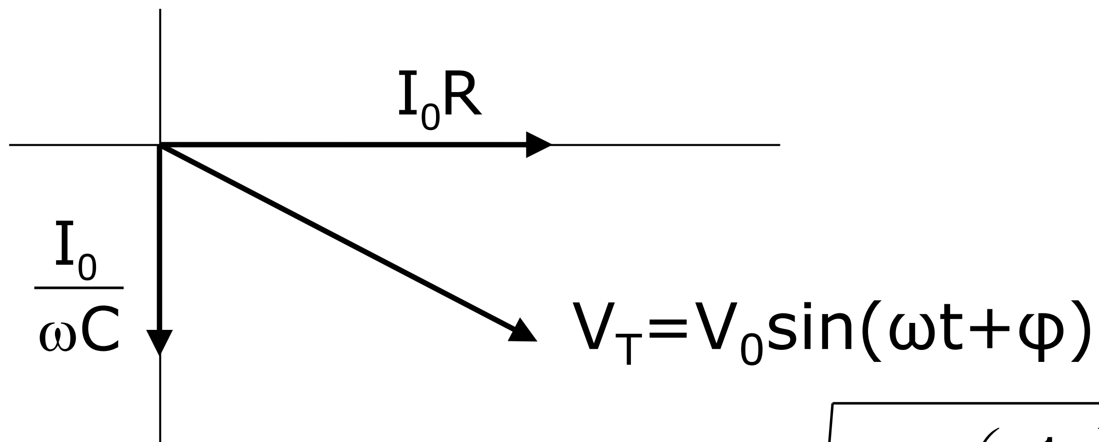


$$V_C = -\frac{I_0}{\omega C}\cos(\omega t)$$

RC phasors

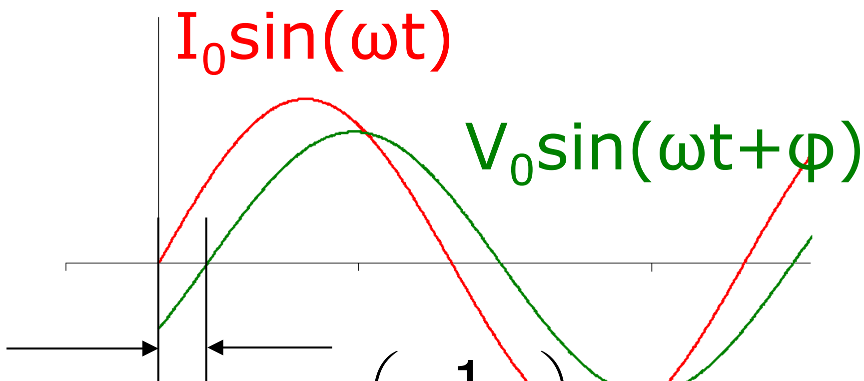


$$V_T = I_0 R \sin(\omega t) - \frac{I_0}{\omega C}\cos(\omega t)$$



$$V_0 = I_0\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\tan\varphi = \frac{1}{-\omega CR}$$



$$-\varphi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

Phasors – mathematics

$$V_T = I_0 R \sin(\omega t) - \frac{I_0}{\omega C}\cos(\omega t)$$

$$V_T = V_0 \sin(\omega t + \varphi)$$

$$V_0 = I_0 \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\tan\varphi = \frac{1}{-\omega C R}$$

$A\sin(\omega t) + B\cos(\omega t) = R\sin(\omega t + \varphi)$

$R\sin(\omega t + \varphi) = R\sin(\omega t)\cos(\varphi) + R\cos(\omega t)\sin(\varphi)$

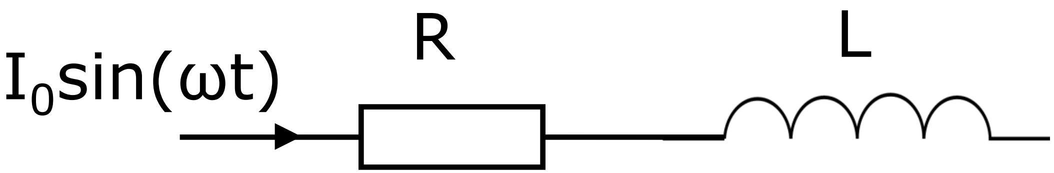
$A = R\cos(\varphi)$ $B = R\sin(\varphi)$

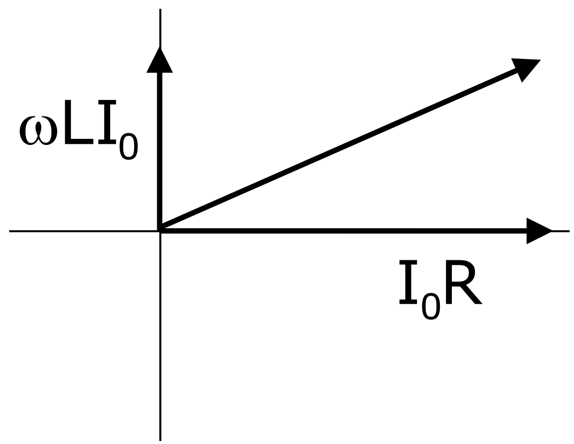
$A^2 + B^2 = R^2$

$B/A = \tan(\varphi)$

RL phasors

$$V_T = I_0 R\sin(\omega t) + \omega L I_0\cos(\omega t)$$

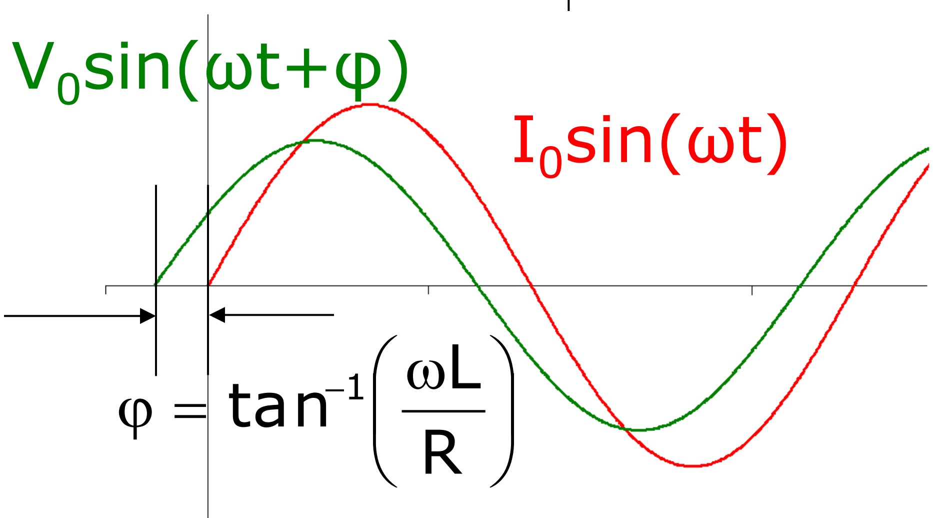




$$V_T = V_0\sin(\omega t + \varphi)$$

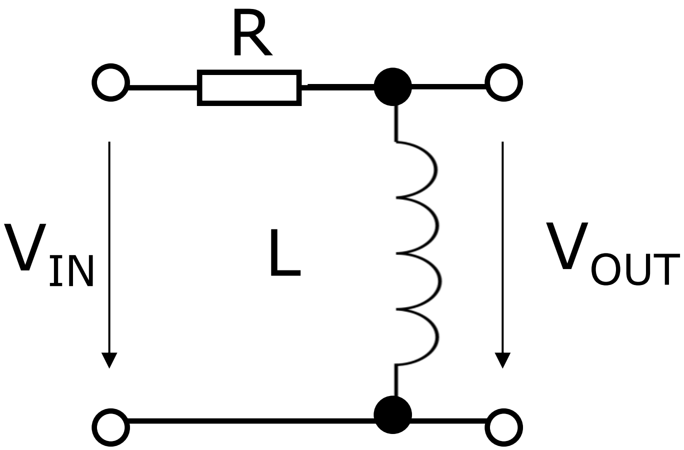
$$V_0 = I_0\sqrt{R^2 + (\omega L)^2}$$

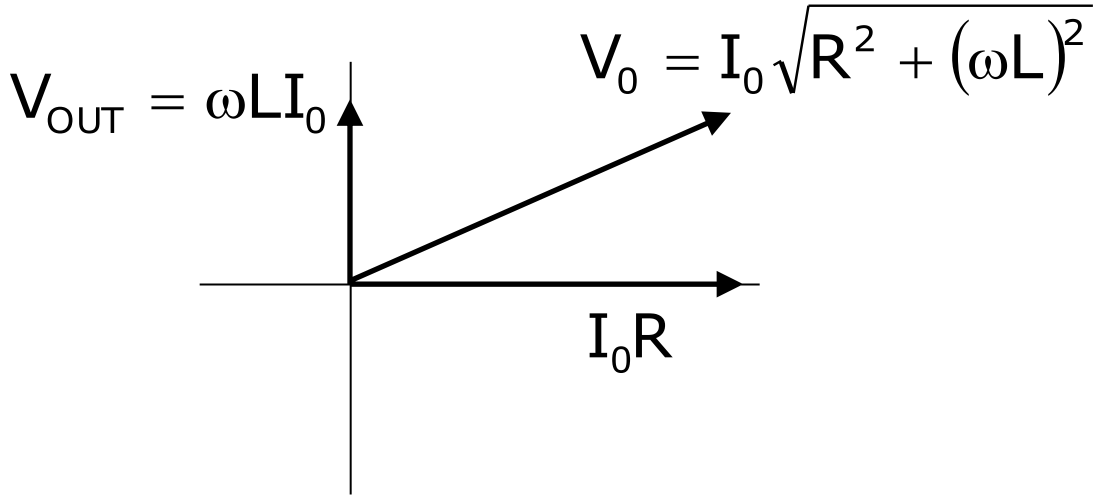
$$\tan\varphi = \frac{\omega L}{R}$$



$$\varphi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

RL filter





$$\left|\frac{V_{OUT}}{V_{IN}}\right| = \omega L \frac{1}{\sqrt{R^2 + (\omega L)^2}}$$

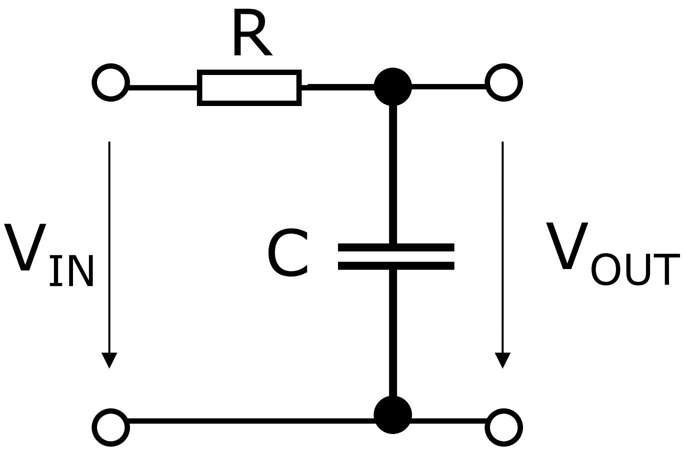
$$= \frac{\omega L/R}{\sqrt{1 + (\omega L/R)^2}}$$

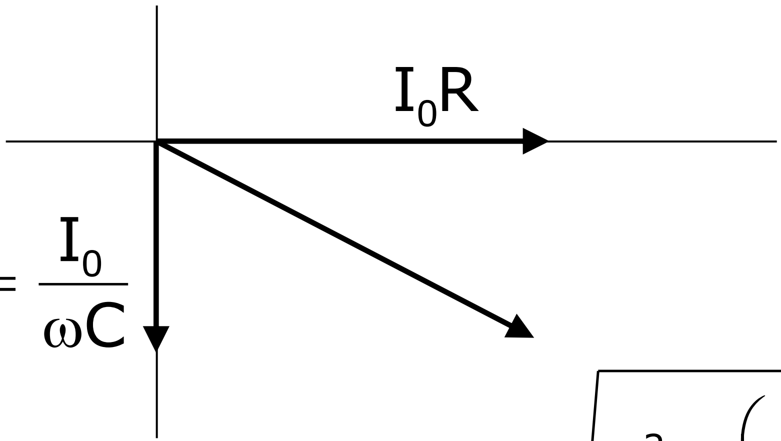
$$\omega \to \infty \qquad \left|\frac{V_{OUT}}{V_{IN}}\right| \to 1$$

$$\omega \to 0 \qquad \left|\frac{V_{OUT}}{V_{IN}}\right| \to 0$$

high pass filter

Phasors: RC filter





$$V_{OUT} = \frac{I_0}{\omega C}$$

$$V_{IN} = V_0 = I_0\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\left|\frac{V_{OUT}}{V_{IN}}\right| = \frac{1}{\omega C}\frac{1}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}}$$

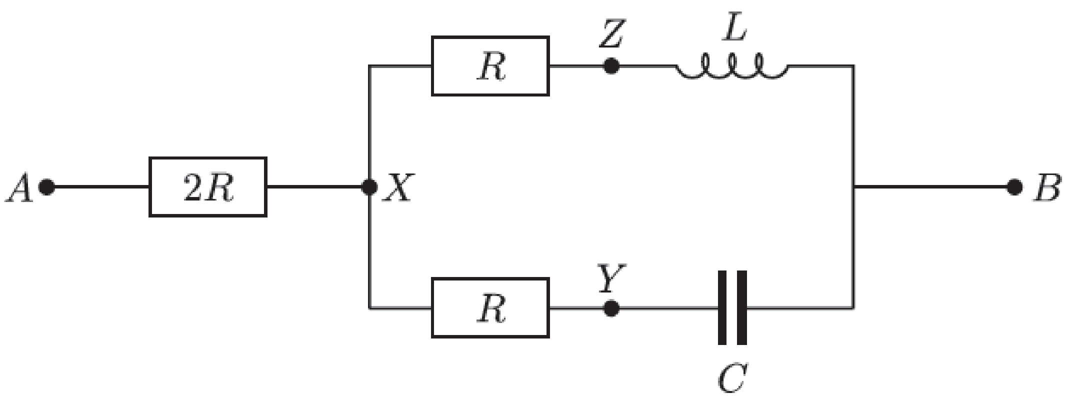
$$= \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$\omega \to \infty \qquad \left|\frac{V_{OUT}}{V_{IN}}\right| \to 0$$

$$\omega \to 0 \qquad \left|\frac{V_{OUT}}{V_{IN}}\right| \to 1$$

low pass filter

7. Define what is meant by a complex impedance. What are the complex impedances of a resistor, a capacitor and an inductor? [6]



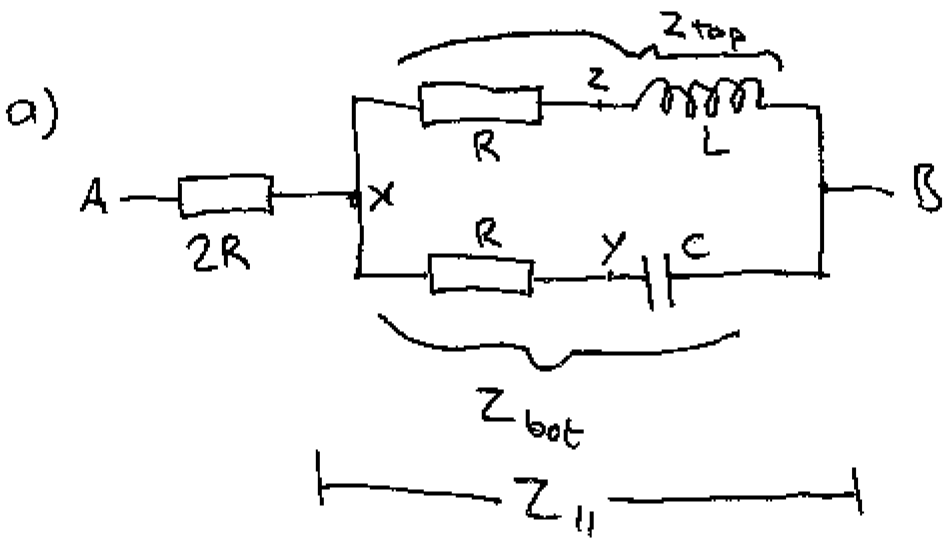
A voltage $V_{AB} = V_0 \cos \omega t$, where V_0 is a real amplitude, is applied between the points A and B in the network shown. Given that $\omega C = 1/(R\sqrt{15})$ and $\omega L = R\sqrt{15}$,

(a) calculate the total impedance between A and B, [4]

(b) verify that voltages of equal amplitude are developed between the point X and the points A, Y and Z, and [6]

(c) determine the phase of V_{XZ} relative to V_{AB}. [4]

a)



$\omega C = \dfrac{1}{R\sqrt{15}}$

$\omega L = R\sqrt{15}$

$Z_{AB} = 2R + Z_{\parallel}$ $\qquad \dfrac{1}{Z_{\parallel}} = \dfrac{1}{Z_{top}} + \dfrac{1}{Z_{bot}}$ impedances in parallel

$Z_{top} = R + j\omega L$

$= R + jR\sqrt{15}$

impedances in series

$Z_{bot} = R + \dfrac{1}{j\omega C}$

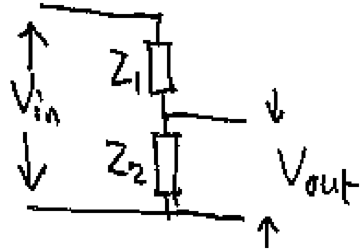
$= R(1 - j\sqrt{15})$

$\dfrac{1}{Z_{\parallel}} = \dfrac{1}{R(1+j\sqrt{15})} + \dfrac{1}{R(1-j\sqrt{15})}$

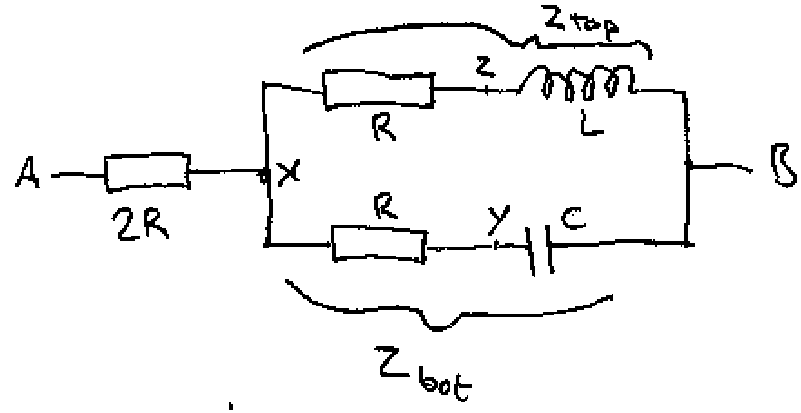
$Z_{\parallel} = 8R$ $\qquad Z_{AB} = 10R$ real

no net phase shift

b) To calculate voltage drops use general potential divider



$$V_{out} = \frac{Z_2}{Z_1 + Z_2} V_{in}$$



$$V_{XA} = \frac{2R}{Z_{AB}} V_{AB} = \frac{1}{5} V_{AB}$$

$$V_{XB} = \frac{4}{5} V_{AB}$$

$$V_{XY} = \frac{R}{Z_{bot}} V_{XB} = \frac{R}{R(1 - j\sqrt{15})} \frac{4}{5} V_{AB} = \frac{1}{4} e^{j\phi_{XY}} \cdot \frac{4}{5} V_{AB} = \frac{1}{5} V_{AB}\, e^{j\phi_{XY}}$$

$$V_{XZ} = \frac{R}{Z_{top}} V_{XB} = \frac{R}{R(1 + j\sqrt{15})} \frac{4}{5} V_{AB} = \frac{1}{5} V_{AB}\, e^{j\phi_{XZ}}$$

So $|V_{XA}| = |V_{XY}| = |V_{XZ}| = \frac{1}{5} V_0$

c) V_{XZ} lags behind V_{AB} by $\phi_{XY} = \tan^{-1}\sqrt{15}$