CP2 Circuit Theory

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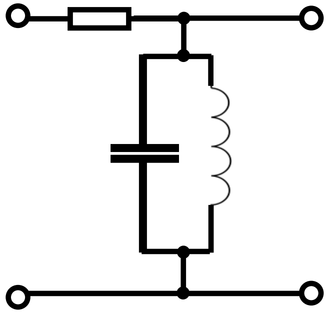
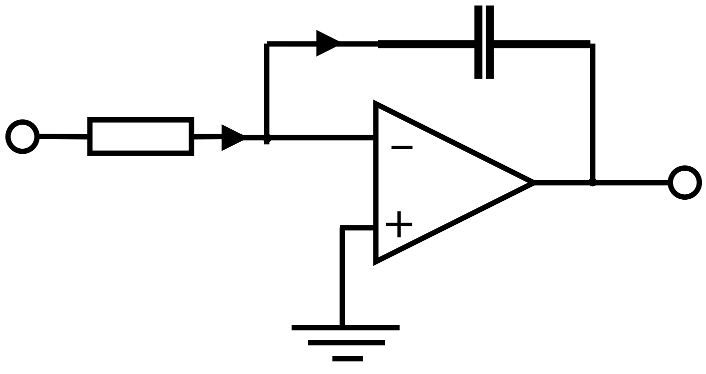
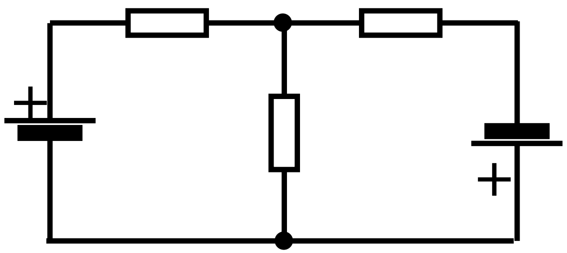
Aims of this course:

Understand basic circuit components (resistors, capacitors, inductors, voltage and current sources, op-amps)

Analyse and design simple linear circuits







Circuit Theory: Synopsis

Basics: voltage, current, Ohm's law…

Kirchoff's laws: mesh currents, node voltages…

Thevenin and Norton's theorem: ideal voltage and current sources…

Capacitors:
Inductors: } Stored energy, RC and RL transient circuits

AC theory: complex notation, phasor diagrams, RC, RL, LCR circuits, resonance, bridges…

Op amps: ideal operational amplifier circuits…

Op-amps are on the exam syllabus

Reading List

- Electronics: Circuits, Amplifiers and Gates, *D V Bugg, Taylor and Francis*
  Chapters 1-7
- Basic Electronics for Scientists and Engineers, *D L Eggleston, CUP*
  Chapters 1,2,6
- Electromagnetism Principles and Applications, *Lorrain and Corson, Freeman*
  Chapters 5,16,17,18
- Practical Course Electronics Manual
  http://www-teaching.physics.ox.ac.uk/practical_course/ElManToc.html
  Chapters 1-3
- Elementary Linear Circuit Analysis, *L S Bobrow, HRW*
  Chapters 1-6
- The Art of Electronics, *Horowitz and Hill, CUP*

Why study circuit theory?

- Foundations of electronics: analogue circuits, digital circuits, computing, communications…
- Scientific instruments: readout, measurement, data acquisition…
- Physics of electrical circuits, electromagnetism, transmission lines, particle accelerators, thunderstorms…
- Not just electrical systems, also thermal, pneumatic, hydraulic circuits, control theory

Mathematics required

- Differential equations $$\frac{d^2I}{dt^2} + \frac{R}{L}\frac{dI}{dt} + \frac{1}{LC}I = 0$$
- Complex numbers $$V(t)=V_0e^{j\omega t}$$ $$I = \frac{V}{Z} \qquad Z = R + jX$$
- Linear equations $$V_0 - I_1R_1 - (I_1 - I_2)R_3 = 0$$ $$(I_1 - I_2)R_3 - I_2R_2 + 2 = 0$$

Covered by Complex Nos & ODEs /
Vectors & Matrices lectures (but with fewer dimensions)

Charge, voltage, current

Charge: determines strength of electromagnetic force
quantised: $e = 1.62 \times 10^{-19} C$ [coulombs]

Potential difference: $V = V_A - V_B$ [volts]

Energy to move unit charge from A to B in electric field

$$V = -\int_A^B \mathbf{E} \cdot \mathbf{ds} \qquad \mathbf{E} = -\nabla V$$

$$W = -\int_A^B Q\mathbf{E} \cdot \mathbf{ds}$$

Current: rate of flow of charge

$$I = \frac{dQ}{dt} = nAve$$

Charge Q=e

No. electrons/unit vol

Cross-section area of conductor

Drift velocity

[amps]

Power: rate of change of work

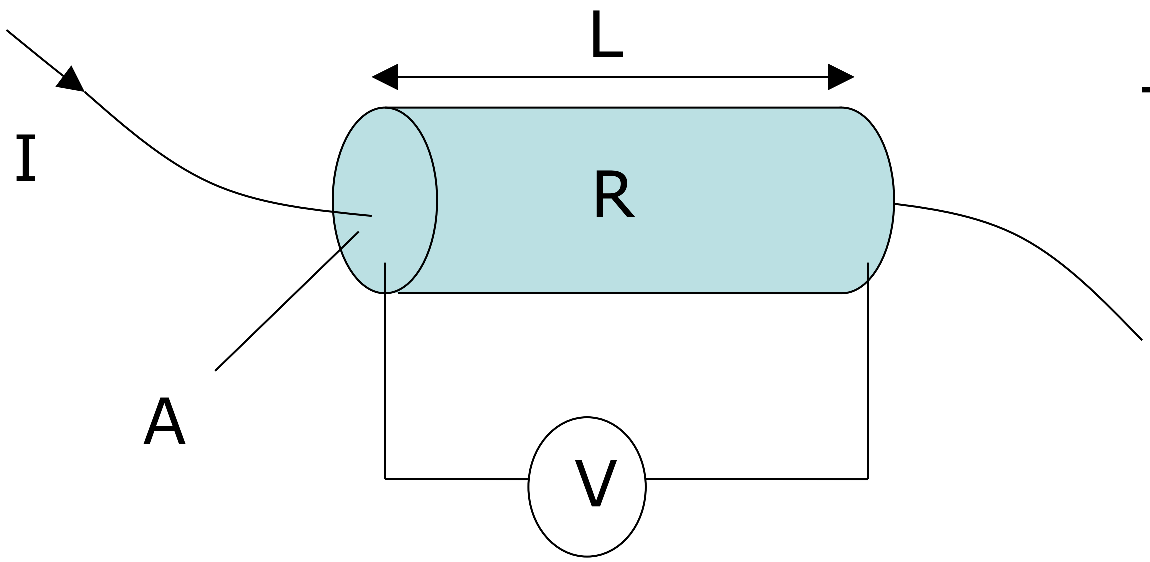
$$P = \frac{dW}{dt} = \frac{d}{dt}(QV) = Q\frac{dV}{dt} + V\frac{dQ}{dt}$$

$$= IV$$

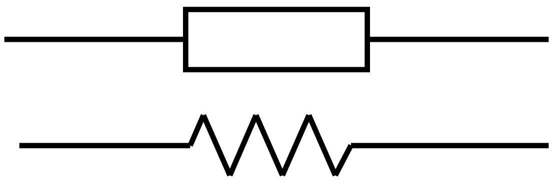
[watts]

Ohm's law

Voltage difference $\propto$ current



Resistor symbols:



$$V = IR$$

R=Resistance Ω[ohms]

$$R = \frac{\rho L}{A}$$

ρ=Resistivity Ωm

Resistivities

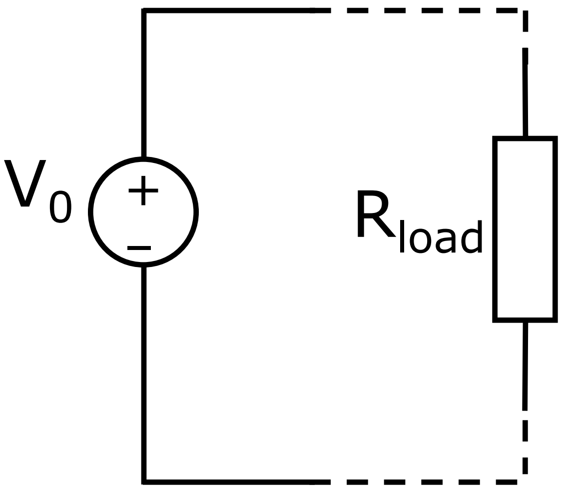
Silver	$1.6\times10^{-8}\,\Omega\mathrm{m}$
Copper	$1.7\times10^{-8}\,\Omega\mathrm{m}$
Manganin	$42\times10^{-8}\,\Omega\mathrm{m}$
Distilled water	$5.0\times10^{3}\,\Omega\mathrm{m}$
PTFE (Teflon)	$\sim10^{19}\,\Omega\mathrm{m}$

Conductance [seimens] $$g = \frac{1}{R}$$

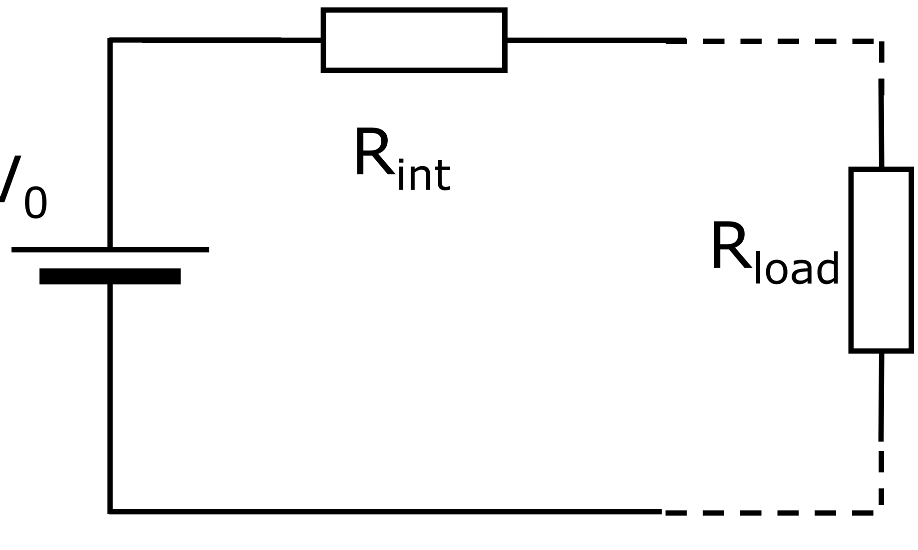
conductivity [seimens/m] $$\sigma = \frac{1}{\rho}$$

Power dissipation by resistor: $$P = IV = I^2R = \frac{V^2}{R}$$

Voltage source



Ideal voltage source: supplies V_0 independent of current

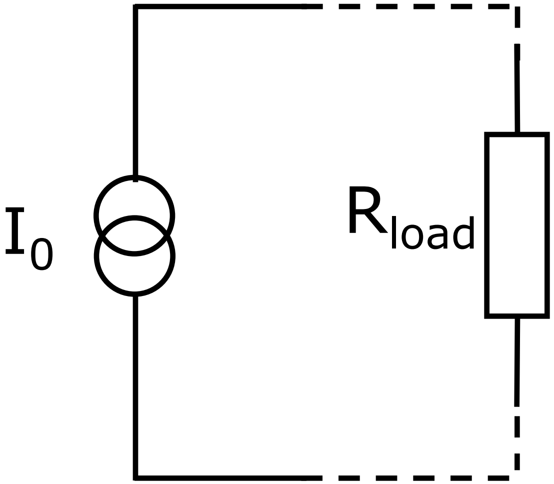


battery cell

Real voltage source:

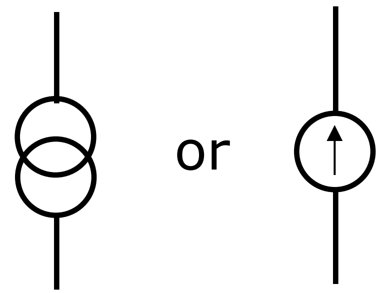
$V_{load} = V_0 - IR_{int}$

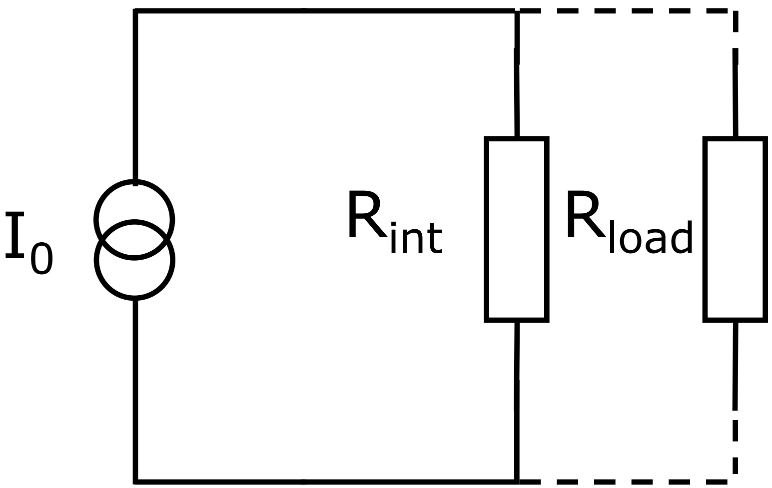
Constant current source



Ideal current source: supplies I_0 amps independent of voltage

Symbol:



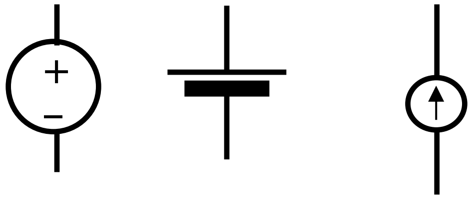


Real current source:

$$I_{load} = I_0 - \frac{V}{R_{\text{int}}}$$

AC and DC

DC (Direct Current):

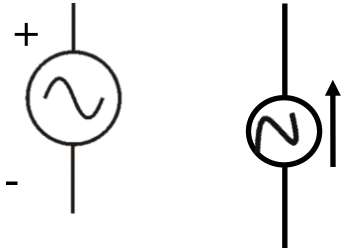


Constant voltage or current

Time independent

$V=V_0$

AC (Alternating Current):

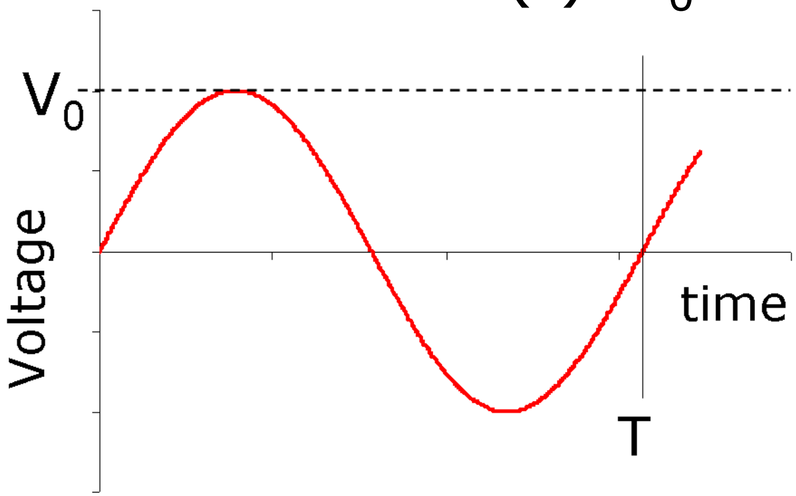


Time dependent

Periodic

$I(t)=I_0\sin(\omega t)$

$$\omega = 2\pi f = \frac{2\pi}{T}$$



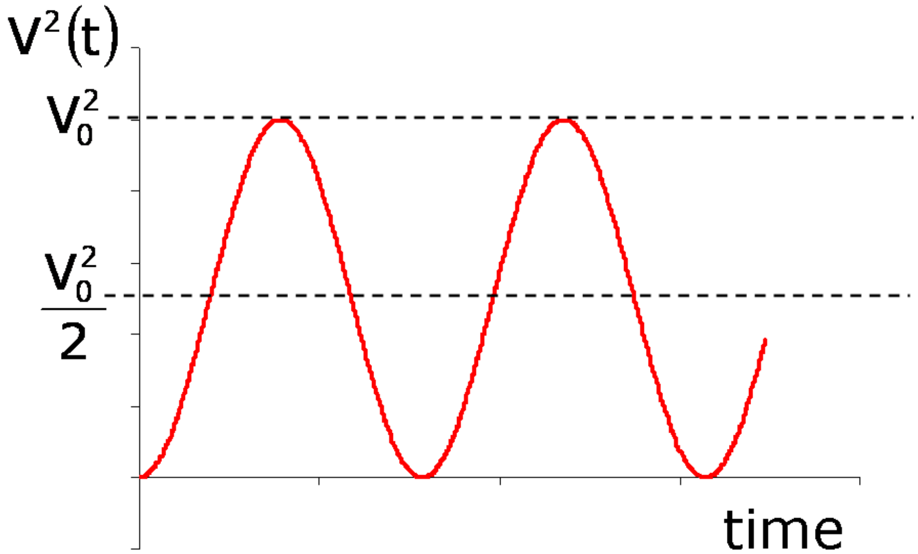
50Hz power, audio, radio...

RMS values

AC Power dissipation $\overline{P} = I_{RMS}V_{RMS} = \frac{V_{RMS}^2}{R}$

$$V_{RMS} = \frac{V_0}{\sqrt{2}}$$

Why $\sqrt{2}$? Square root of mean of $V(t)^2$



$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T V^2(t)dt}$$

Passive Sign Convention

Passive devices ONLY - **Learn it**; **Live it**; **Love it**!

R=Resistance Ω[ohms]

$$V = IR$$

Two seemingly Simple questions:

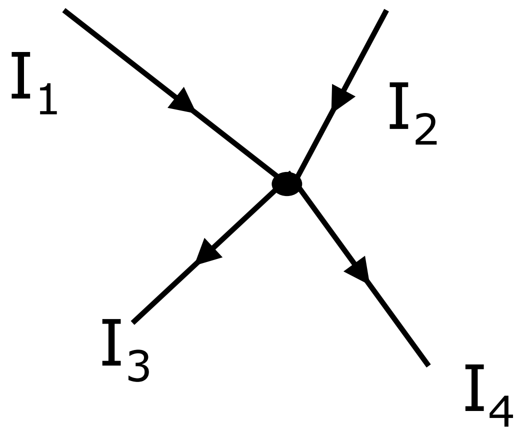
Which way does the current flow, left or right?

Voltage has a '+' side and a '-' side (you can see it on a battery) on which side should we put the '+'? On the left or the right?

Given V=IR, does it matter which sides for V or which direction for I?

Kirchoff's Laws

I Kirchoff's current law: Sum of all currents at a node is zero



$$I_1+I_2-I_3-I_4=0$$

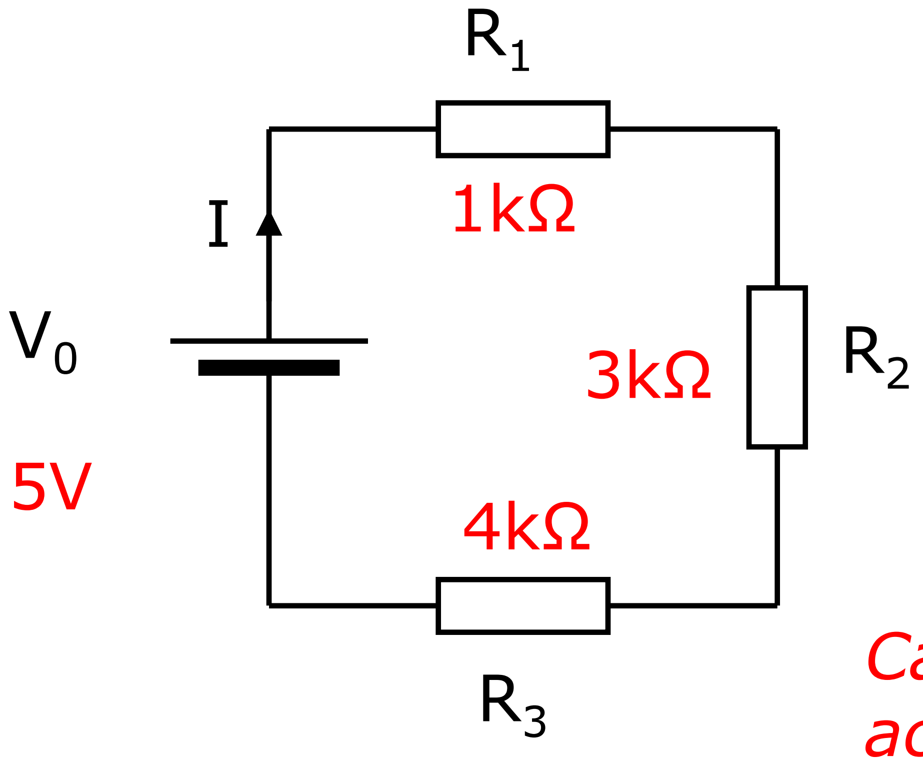
$$\sum I_n = 0$$

(conservation of charge)

Here is a cute trick:
It does not matter whether you pick "entering" or "leaving" currents as positive.

BUT keep the same convention for all currents on one node!

II Kirchoff's voltage law: Around a closed loop the net change of potential is zero (Conservation of Energy)



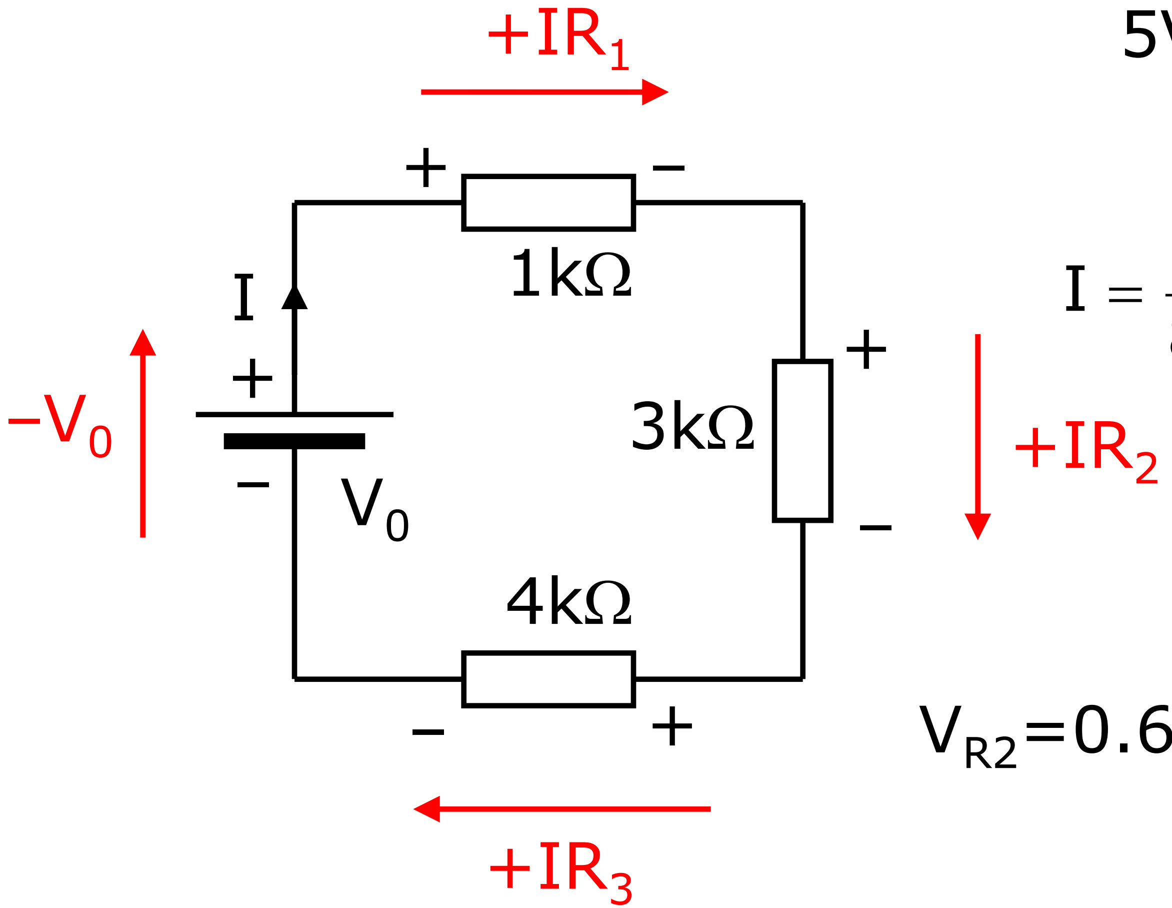
$$\sum V_n = 0$$

Calculate the voltage across R_2

Kirchoff's voltage law:

$$\sum V_n = 0$$

$$-V_0 + IR_1 + IR_2 + IR_3 = 0$$

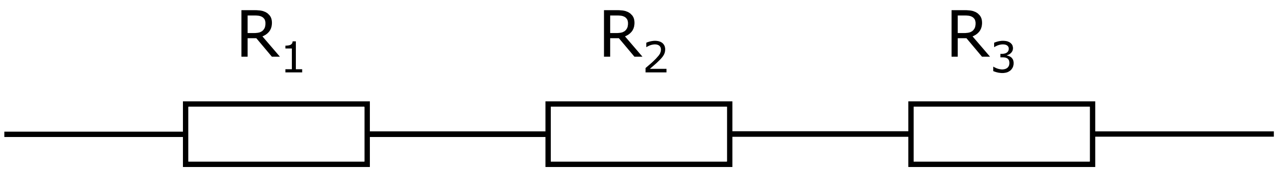


$$5V = I(1+3+4)k\Omega$$

$$I = \frac{5V}{8000\Omega} = 0.625mA$$

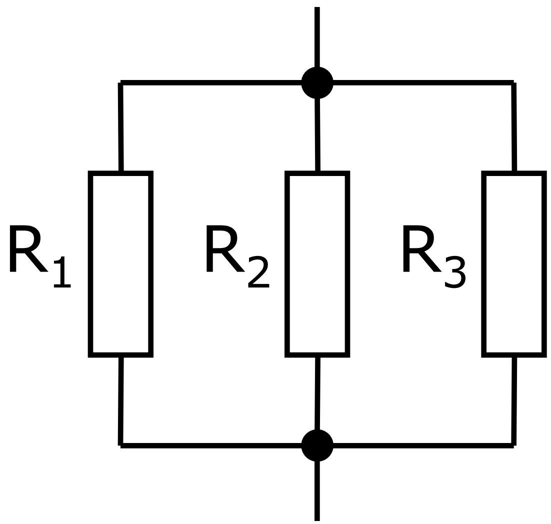
$$V_{R2} = 0.625mA \times 3k\Omega = 1.9V$$

Series / parallel circuits



$$R_T = \sum_n R_n$$

Resistors in series: $R_{Total} = R_1 + R_2 + R_3 \ldots$

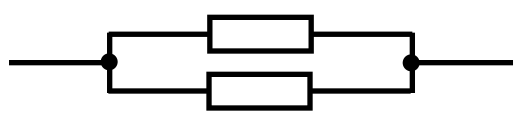


Resistors in parallel

$$\frac{1}{R_T} = \sum_n \frac{1}{R_n}$$

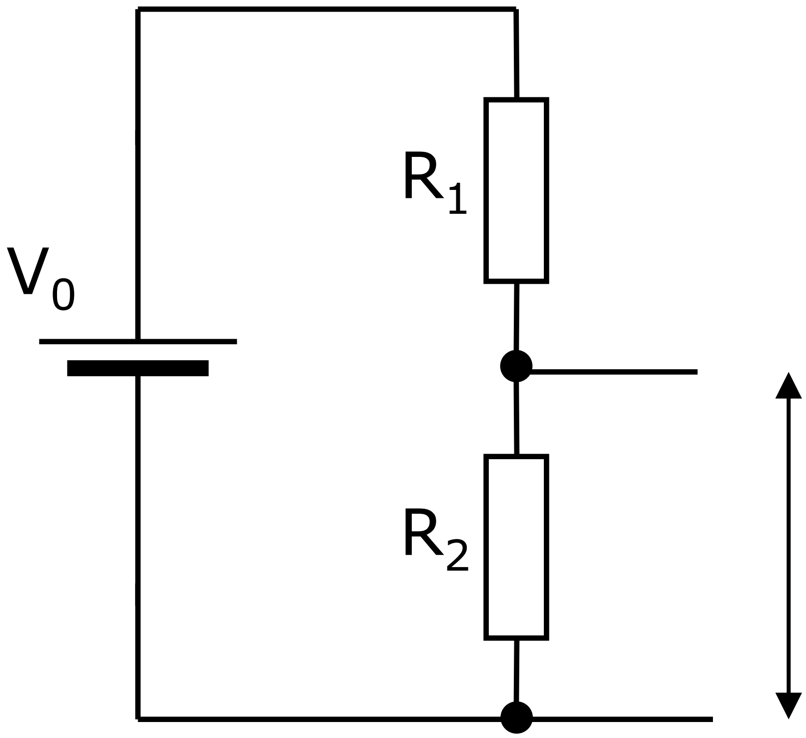
$$= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \ldots$$

Two parallel resistors:



$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Potential divider



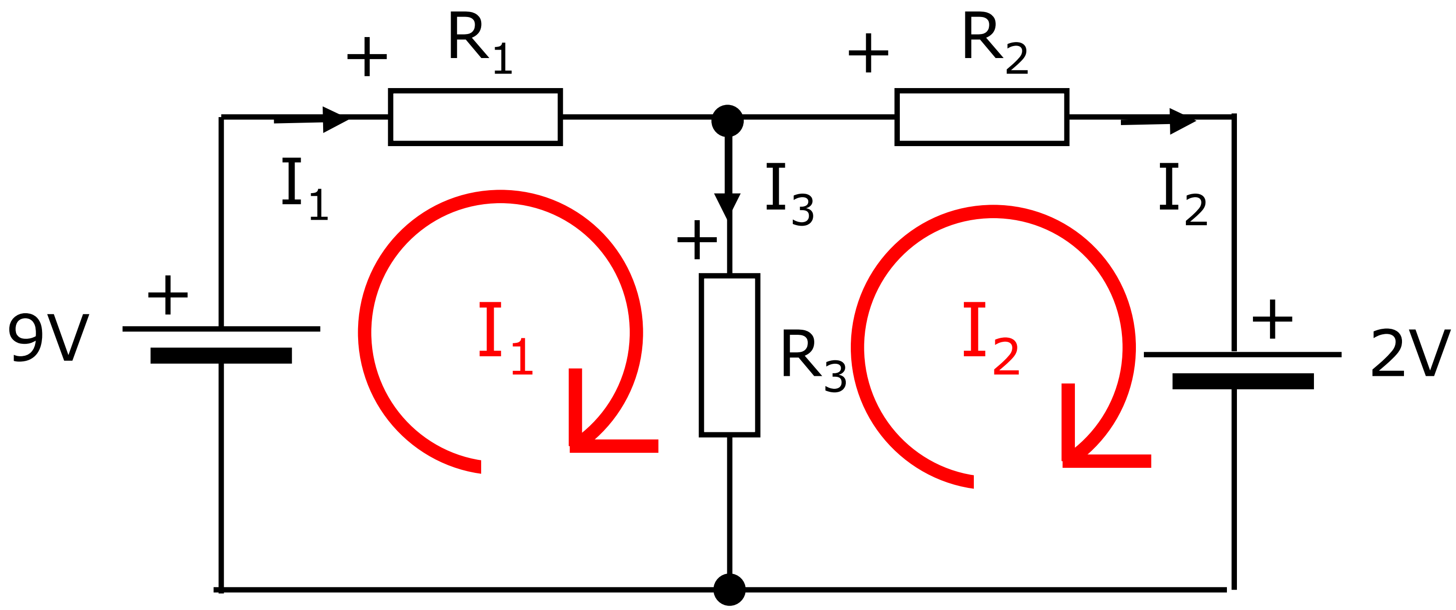
$$\frac{V_0 R_2}{R_1 + R_2}$$

USE PASSIVE SIGN CONVENTION!!!

Show on blackboard

Mesh currents

$R_1 = 3k\Omega$
$R_2 = 2k\Omega$
$R_3 = 6k\Omega$



First job: Label loop currents in all **interior** loops

Second job: USE PASSIVE SIGN CONVENTION!!!

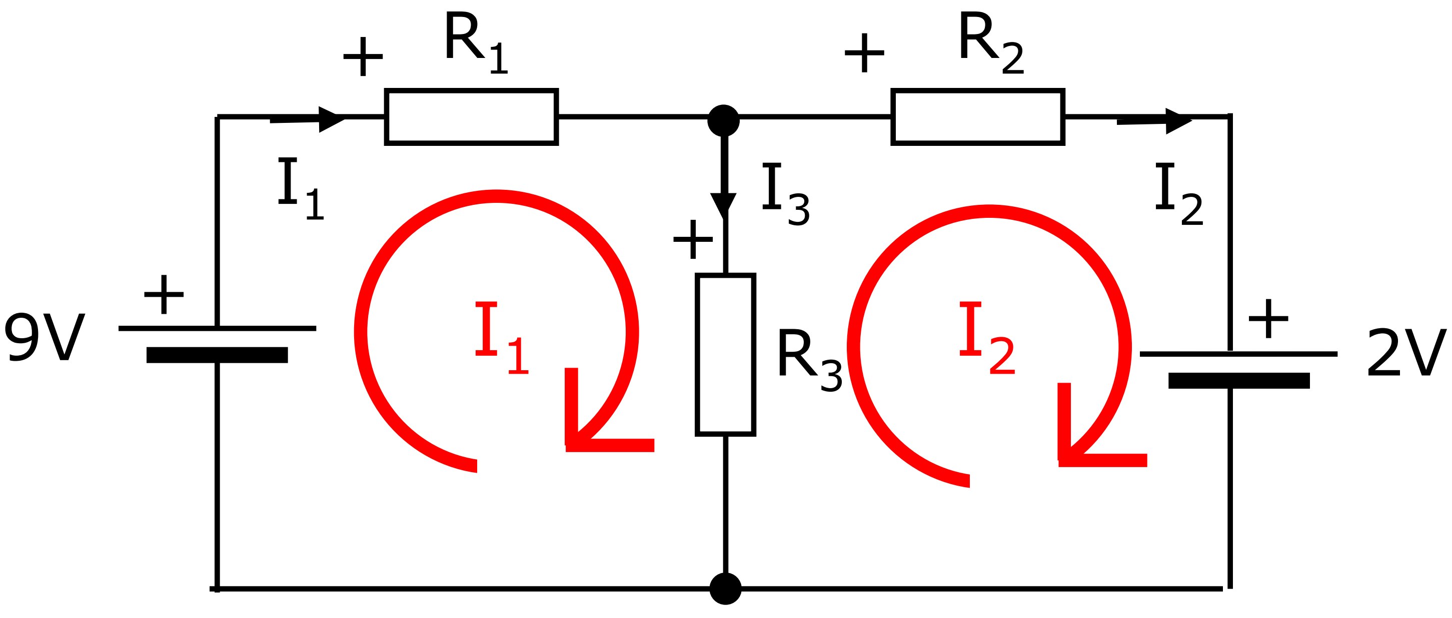
Third job: Apply KCL to nodes sharing loop currents

Define: Currents Entering Node are positive

$I_1 - I_2 - I_3 = 0 \rightarrow I_3 = I_1 - I_2$

Mesh currents

$R_1 = 3k\Omega$
$R_2 = 2k\Omega$
$R_3 = 6k\Omega$

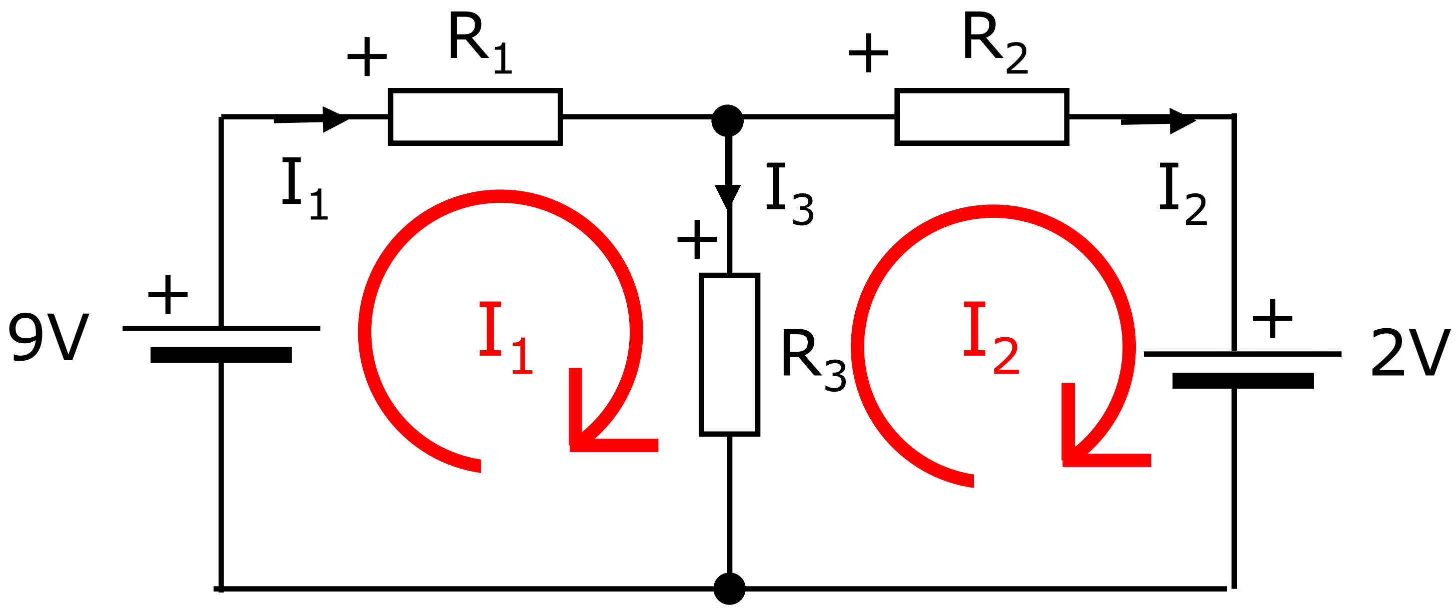


Fourth job: **Apply Kirchoff's Voltage law around each loop.**

Last job: **USE Ohm's law and solve equations.**

Mesh currents

$R_1 = 3k\Omega$
$R_2 = 2k\Omega$
$R_3 = 6k\Omega$



$-9V + I_1R_1 + I_3R_3 = 0$ $\quad I_3 = I_1 - I_2$

$-I_3R_3 + I_2R_2 + 2V = 0$

Solve simultaneous equations

$9V/k\Omega = 9I_1 - 6I_2$

$-2V/k\Omega = -6I_1 + 8I_2$

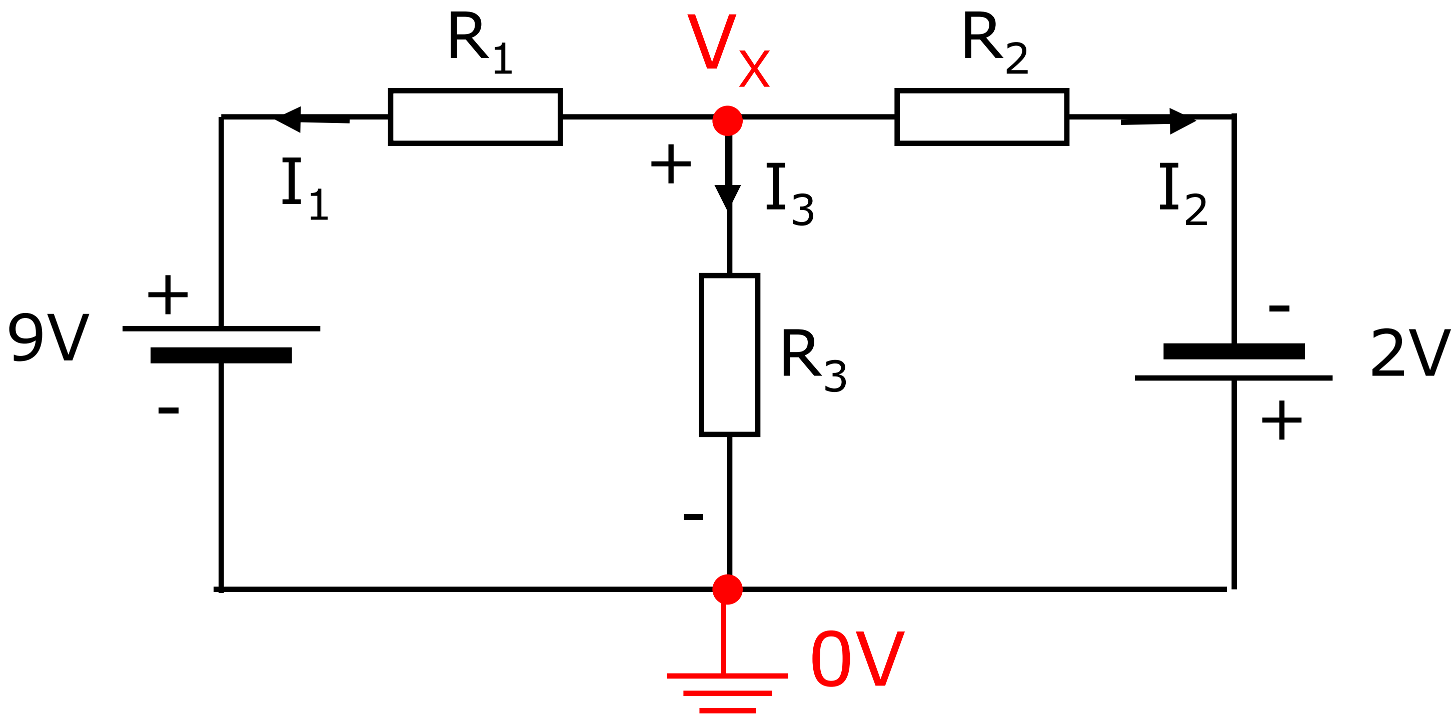
$\rightarrow I_2 = 1\ mA$

$I_1 = 5/3\ mA \qquad I_3 = 2/3\ mA$

$V_3 = R_3 I_3 = 4\ V$

Node voltages

$R_1 = 3k\Omega$
$R_2 = 2k\Omega$
$R_3 = 6k\Omega$



Step 1: Choose a ground node!

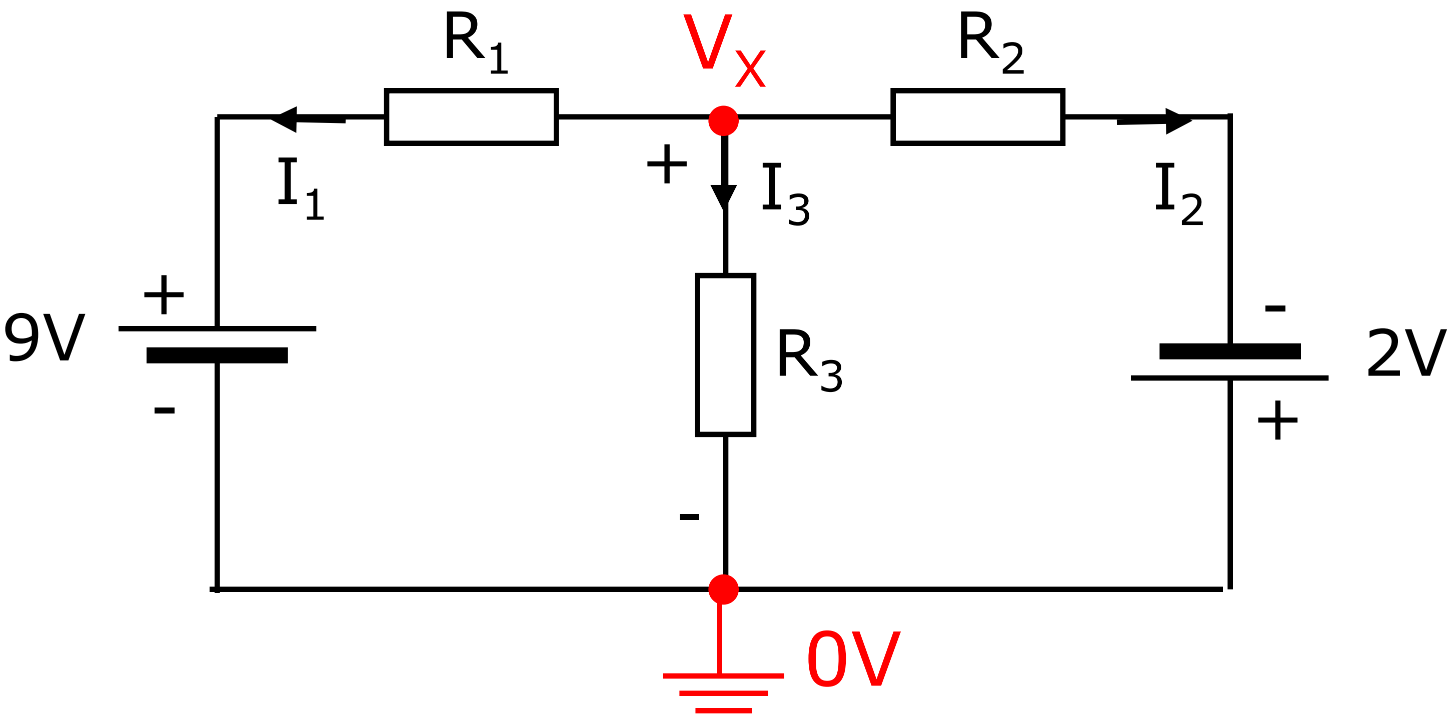
Step 2: Label V's and I's on all nodes

Step 3: Apply KVL to find ΔV across resistors

Step 4: Apply KCL and ohms law using the tricks

Node voltages

$R_1 = 3k\Omega$
$R_2 = 2k\Omega$
$R_3 = 6k\Omega$



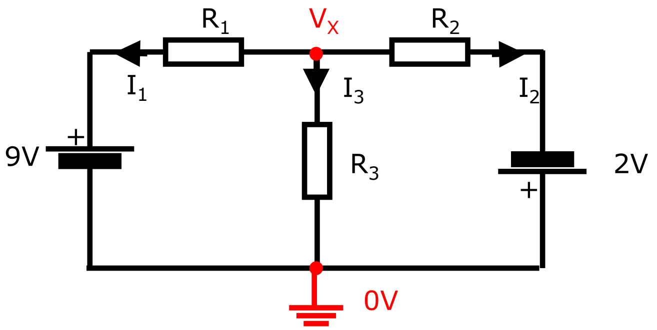
$$0 = I_2 + I_3 + I_1$$

$$0 = \frac{V_X - (-2V)}{R_2} + \frac{V_X}{R_3} + \frac{V_X - 9V}{R_1}$$

All currents leave all labeled nodes
And apply ΔV/R to each current.
Only one equation,
Mesh analysis would give two.

USE PASSIVE SIGN CONVENTION!!!

$$0 = I_2 + I_3 + I_1$$



$$0 = \frac{V_X - (-2V)}{R_2} + \frac{V_X}{R_3} + \frac{V_X - 9V}{R_1}$$

$$V_X\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_1}\right) = \frac{9V}{R_1} - \frac{2V}{R_2} = 3mA - 1mA = 2mA$$

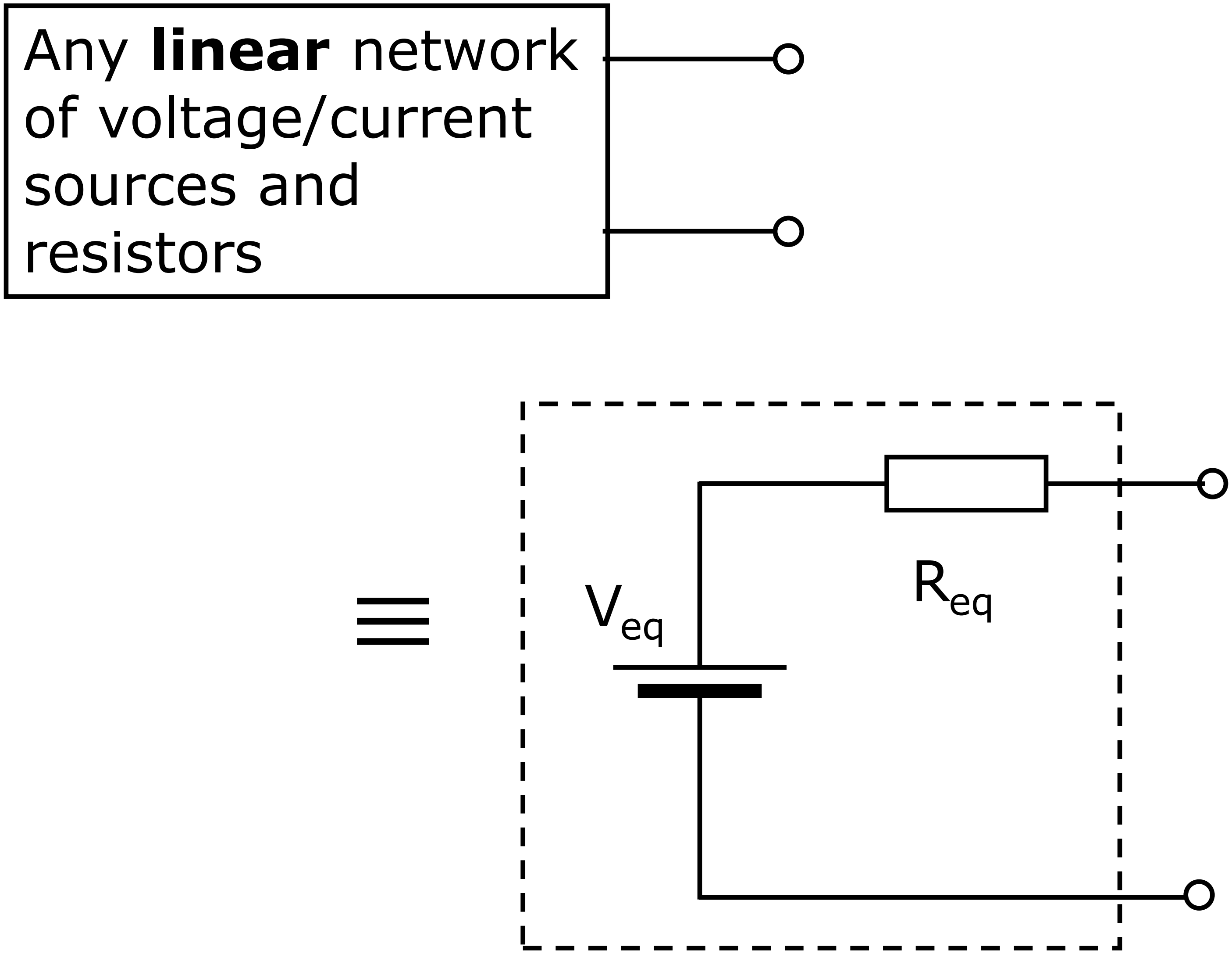
$$V_X\left(\frac{1}{2} + \frac{1}{6} + \frac{1}{3}\right)\frac{1}{k\Omega} = 2mA$$

$$\frac{V_X}{1k\Omega} = 2mA \quad \rightarrow \quad V_X = 2V$$

$$I_1 = \frac{2V - 9V}{3k\Omega} = -\frac{7}{3}\,mA \qquad I_2 = \frac{2V + 2V}{2k\Omega} = 2mA$$

$$I_3 = \frac{2V}{6k\Omega} = \frac{1}{3}\,mA$$

Thevenin's theorem



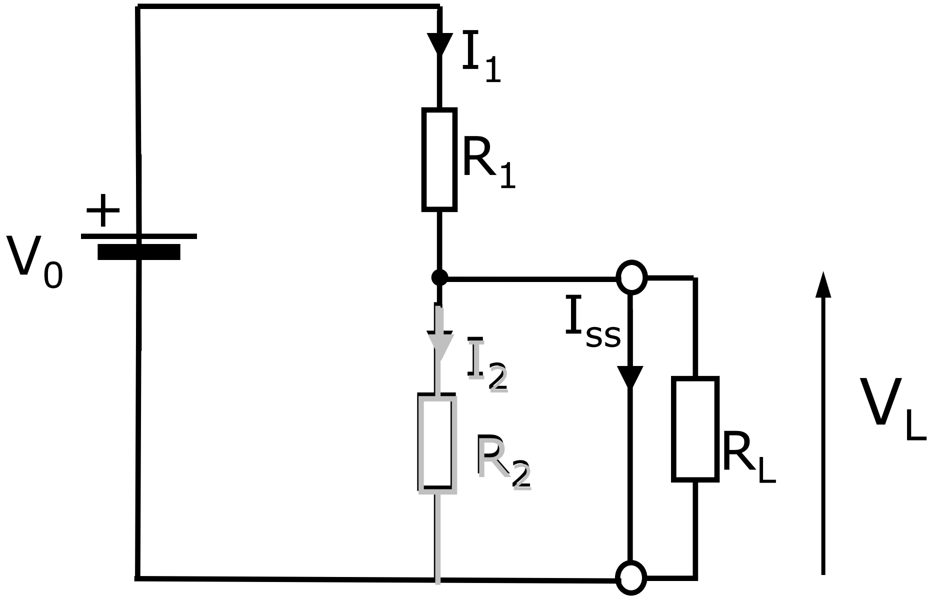
Equivalent circuit

In Practice, to find V_{eq}, R_{eq}...

$R_L \to \infty$ (open circuit) $\quad I_L \to 0 \quad V_{eq} = V_{OS}$

$R_L \to 0$ (short circuit) $\quad V_L \to 0 \quad R_{eq} = \dfrac{V_{OS}}{I_{SS}}$

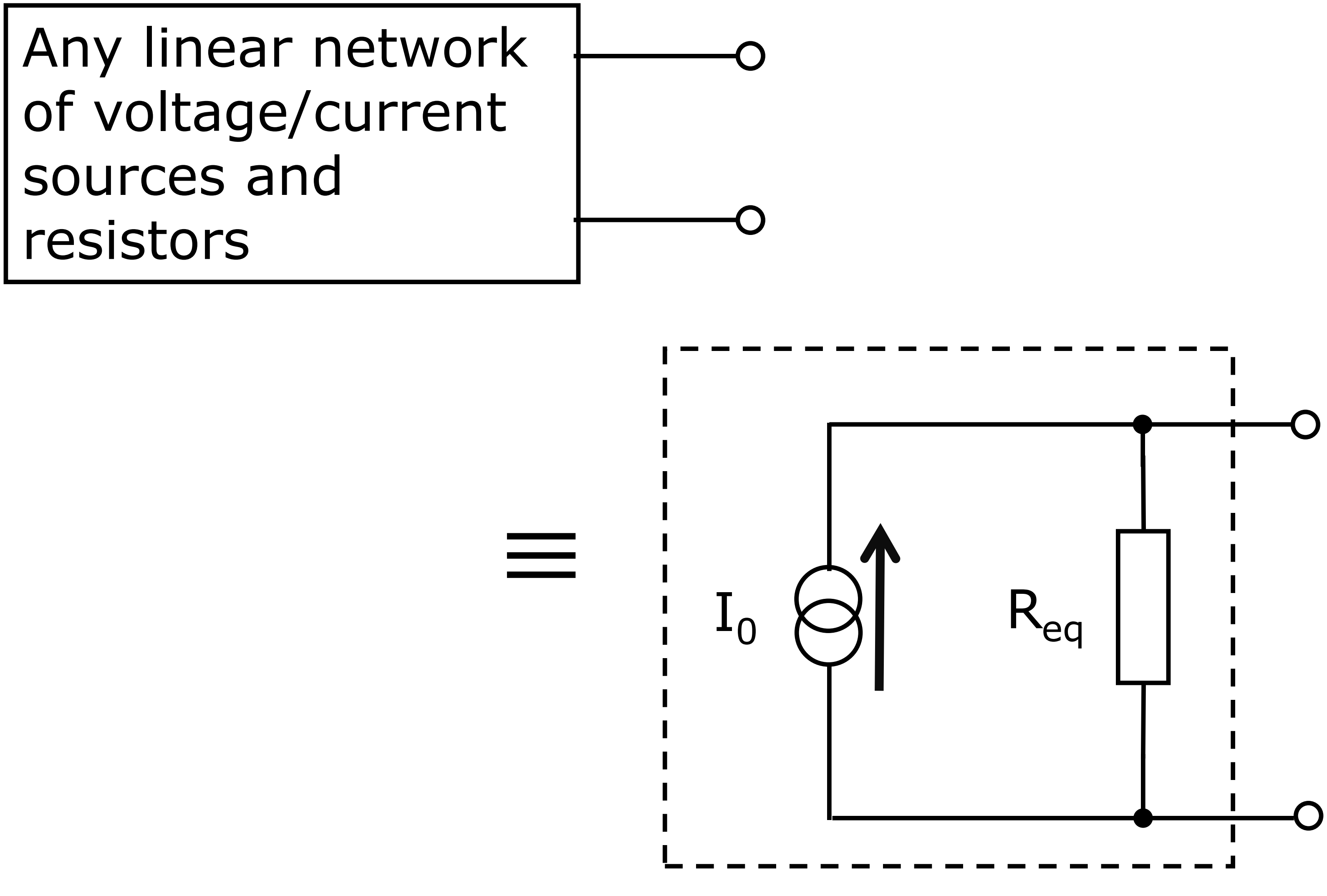
R_{eq} = resistance between terminals when all voltages sources shorted – Warning! This is not always obvious!



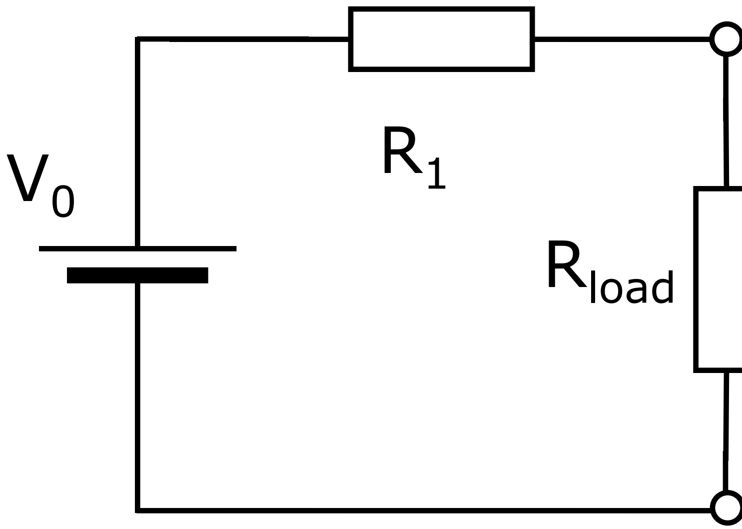
$$V_{os} = V_0 \frac{R_2}{R_1 + R_2} \qquad I_{ss} = \frac{V_0}{R_1}$$

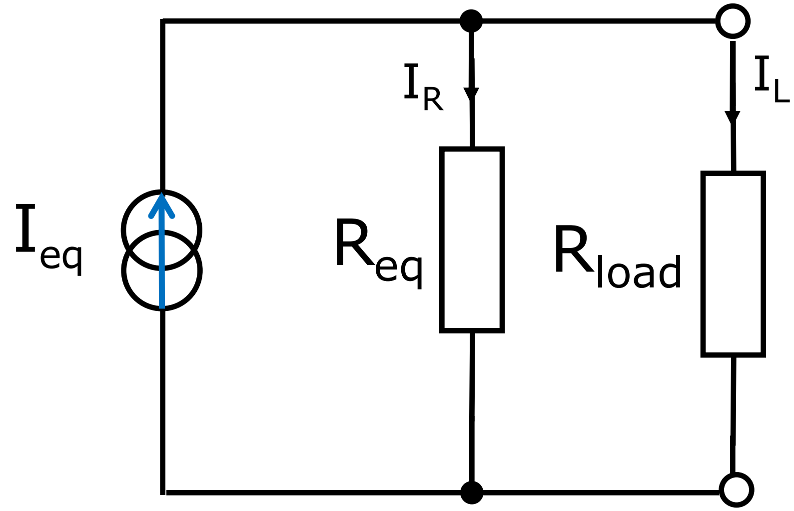
$$R_{eq} = \frac{V_0 \dfrac{R_2}{R_1 + R_2}}{\dfrac{V_0}{R_1}} = \frac{R_1 R_2}{R_1 + R_2}$$

Norton's theorem



Equivalent circuit





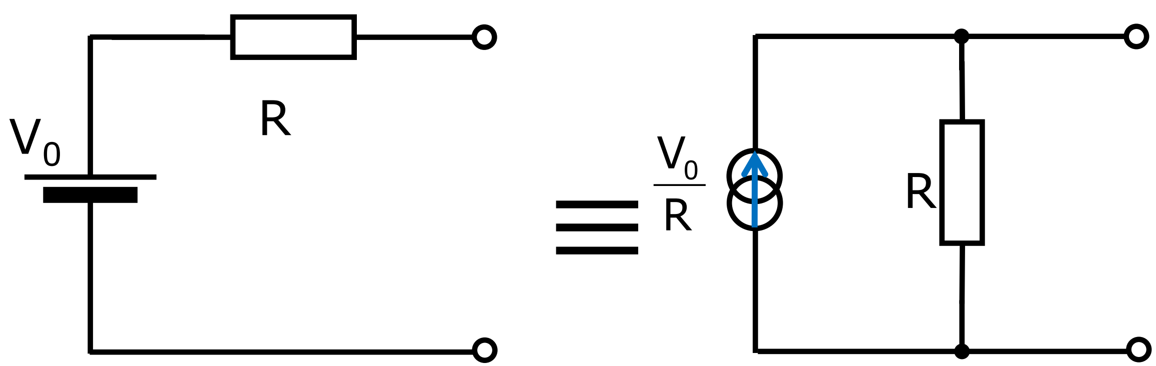
$$V_L = V_0 - I_L R_1$$

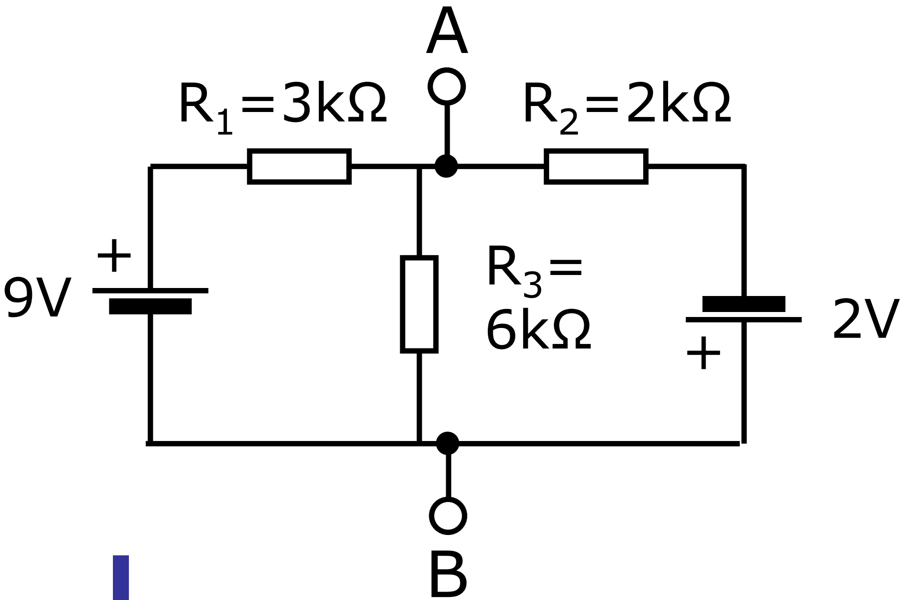
$$I_{eq} = I_R + I_L = \frac{V_L}{R_{eq}} + I_L$$

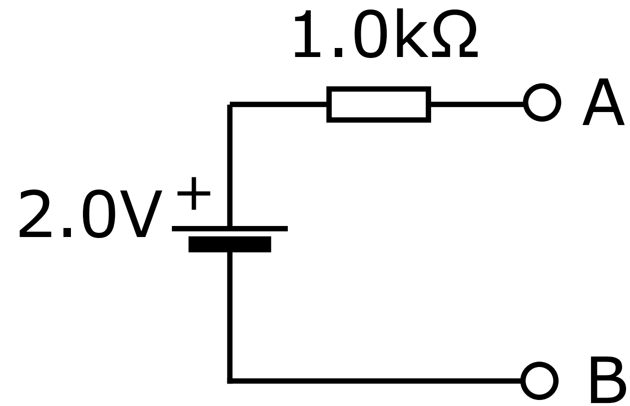
$$R_{eq} I_{eq} - R_{eq} I_L = V_L$$

$$\therefore I_{eq} = \frac{V_0}{R_{eq}}$$

$$R_{eq} = R_1$$

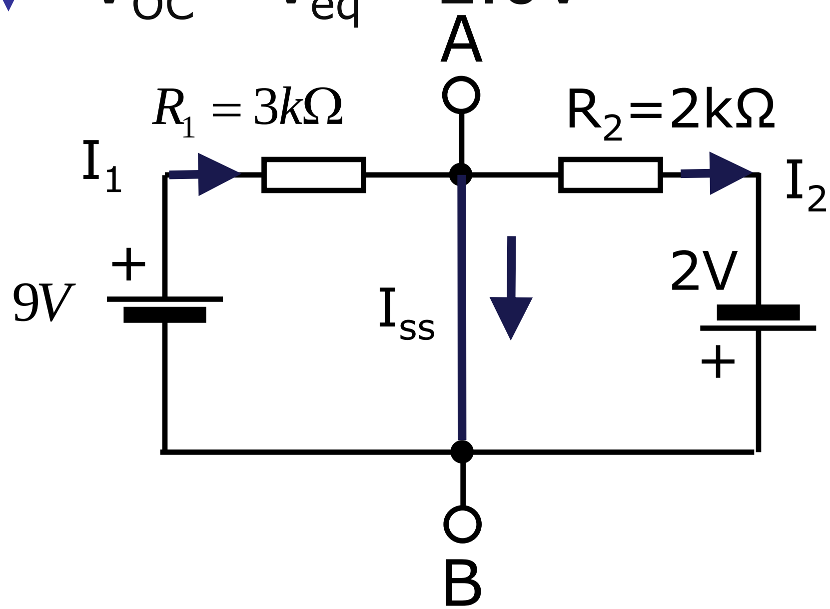






using Thevenin's theorem:

$V_{OC} = V_{eq} = 2.0V$

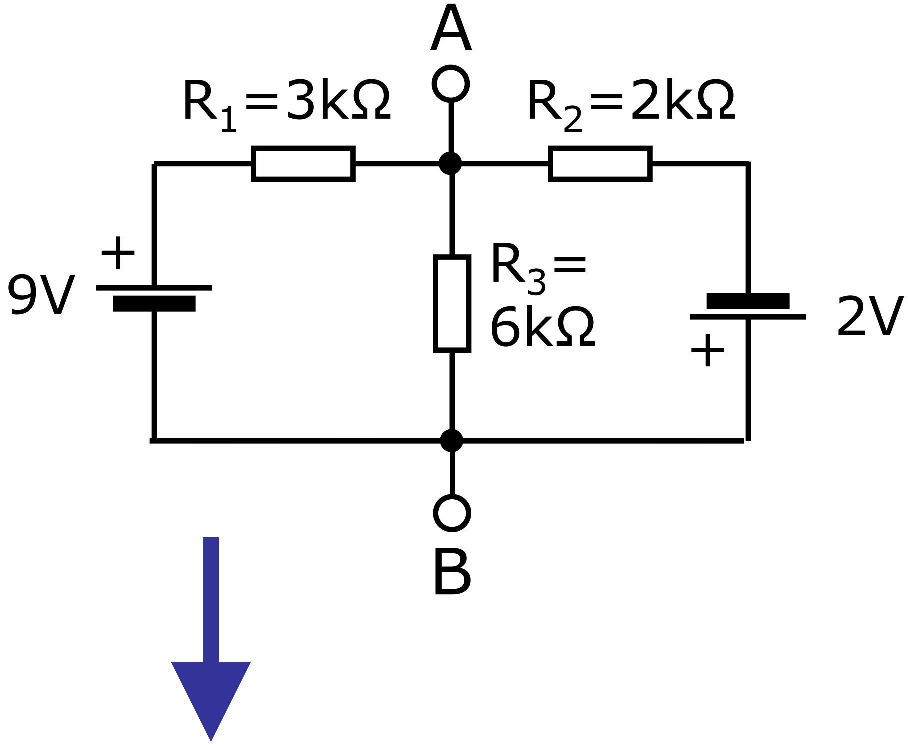


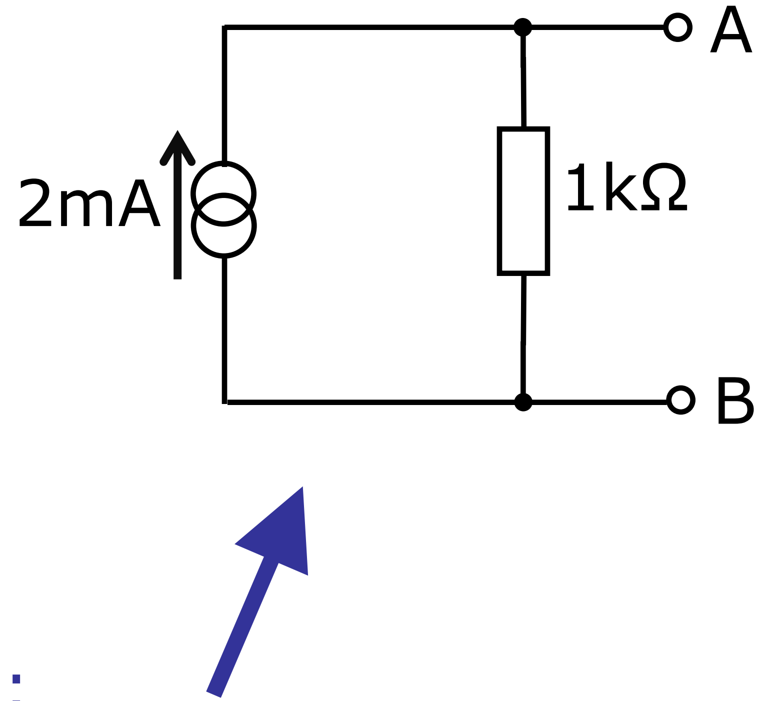
$$I_1 = \frac{9V}{3k\Omega} = 3ma$$

$$I_2 = \frac{2V}{2k\Omega} = 1ma$$

$$I_{SS} = I_1 - I_2 = 2ma$$

$$R_{eq} = \frac{V_{OC}}{I_{SS}} = 1.0\ k\Omega$$



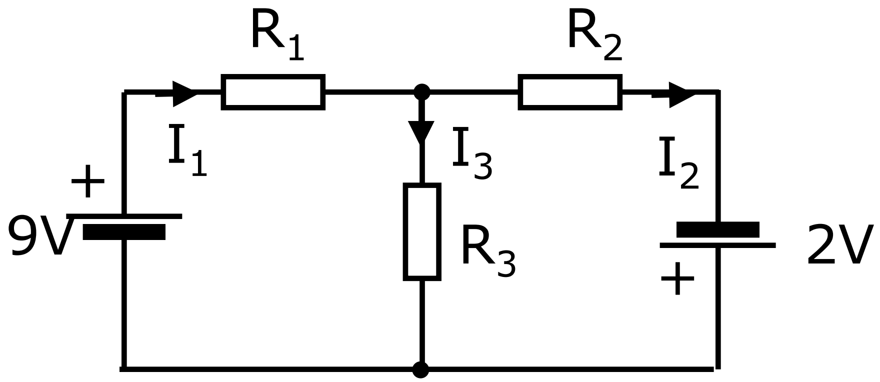


using Norton's theorem:

Same procedure: Find I_{SS} and V_{OC}

$I_{EQ} = I_{SS}$ and $R_{EQ} = V_{OC}/I_{SS}$

Superposition



Important: label ***Everything*** the same directions!

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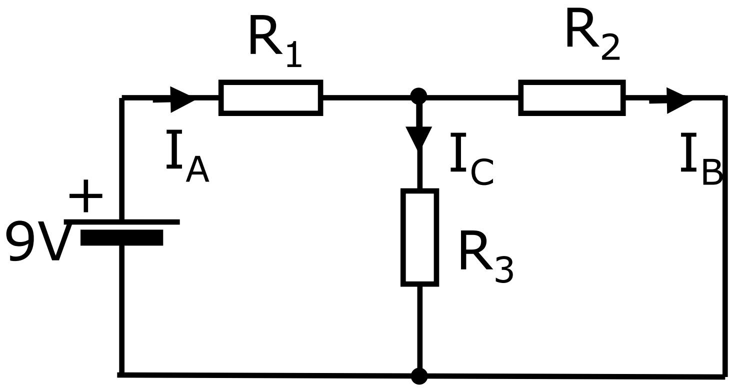

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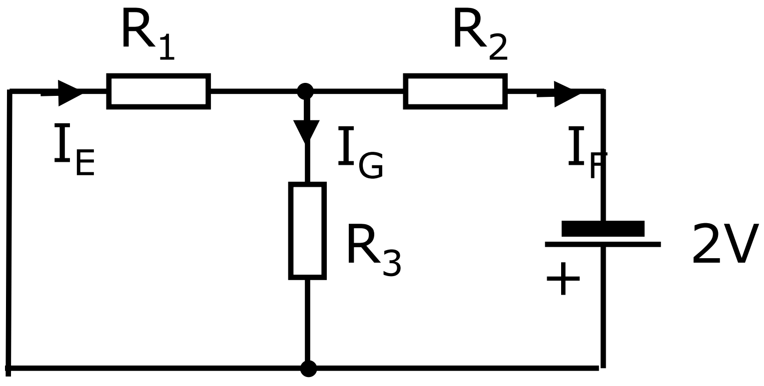
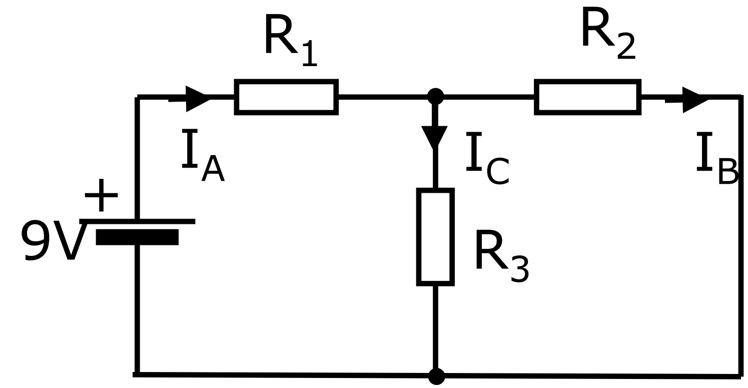
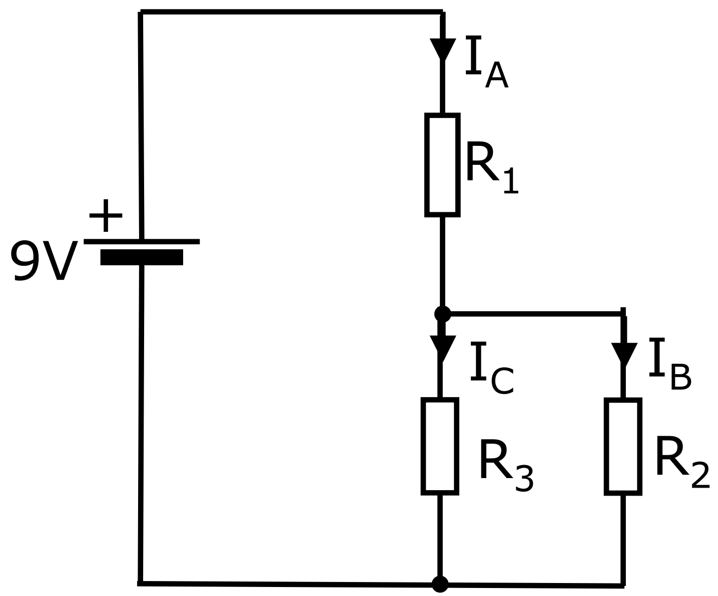


$I_1=I_A+I_E$ $\quad$ $I_2=I_B+I_F$ $\quad$ $I_3=I_C+I_G$



Example: Superposition

$R_1 = 3k\Omega$
$R_2 = 2k\Omega$
$R_3 = 6k\Omega$



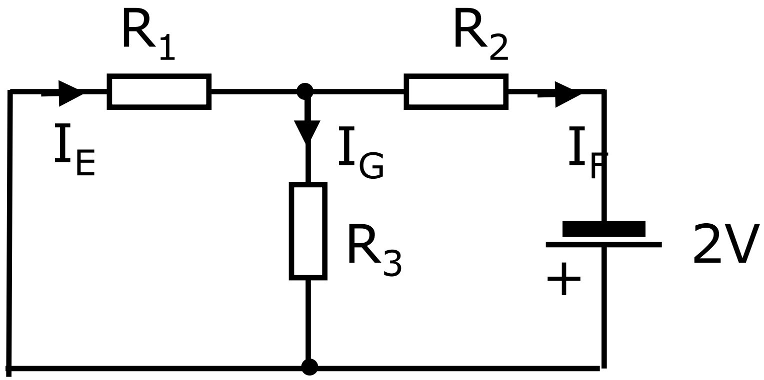
$$\frac{R_2 R_3}{R_2 + R_3} = 1.5k\Omega$$

$$R_1 + \frac{R_2 R_3}{R_2 + R_3} = 4.5k\Omega$$

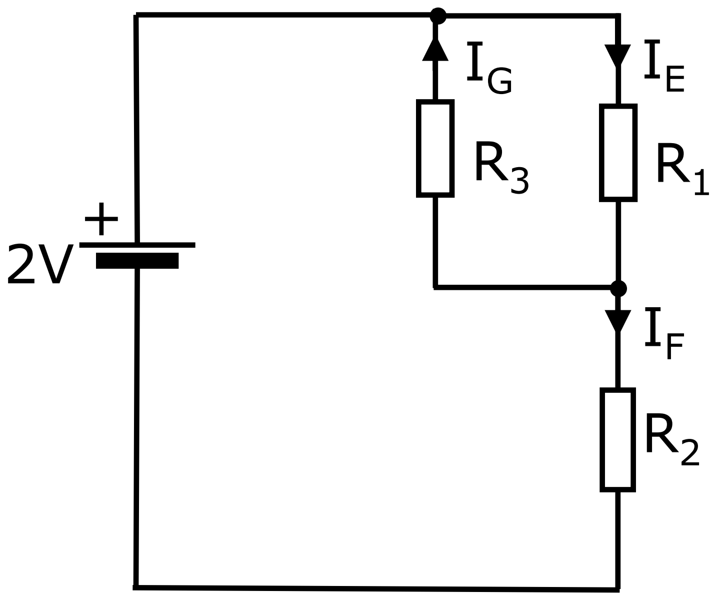
$$I_A = \frac{9V}{4.5k\Omega} = 2mA$$

$$I_C R_3 = I_B R_2 = 9V \frac{1.5}{4.5} = 3V$$

$$I_C = 0.5mA \qquad I_B = 1.5mA$$



$R_1 = 3k\Omega$
$R_2 = 2k\Omega$
$R_3 = 6k\Omega$



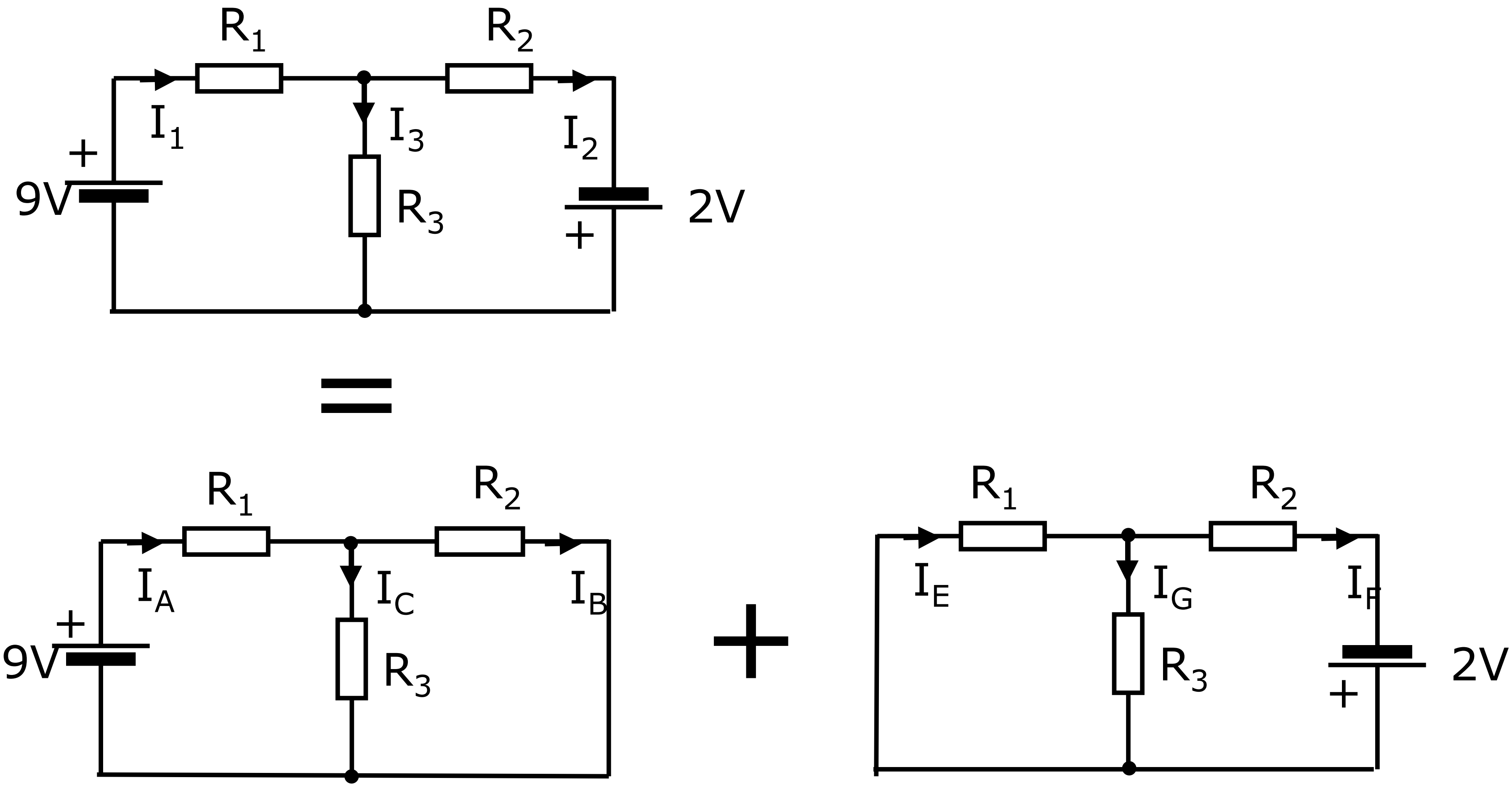
$$\frac{R_1 R_3}{R_1 + R_3} = 2k\Omega$$

$$R_2 + \frac{R_1 R_3}{R_1 + R_3} = 4k\Omega$$

$$-I_G R_3 = I_E R_1 = 2V\frac{2}{4} = 1V$$

$$I_F = \frac{2V}{4k\Omega} = 0.5mA$$

$$I_G = -\frac{1}{6}mA \qquad I_E = \frac{1}{3}mA$$



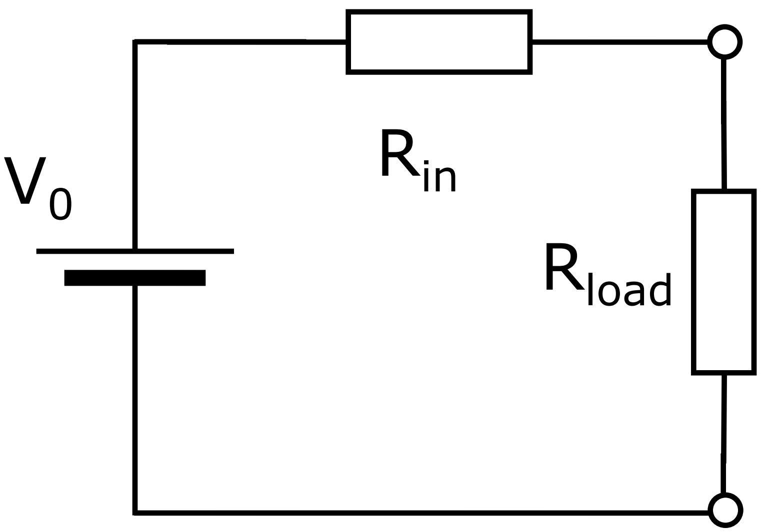
$$I_1 = I_A + I_E \qquad I_2 = I_B + I_F \qquad I_3 = I_C + I_G$$

$$I_1 = \left(2 + \frac{1}{3}\right)\text{mA} = \frac{7}{3}\,\text{mA}$$

$$I_2 = (1.5 + 0.5)\text{mA} = 2\text{mA}$$

$$I_3 = \left(0.5 - \frac{1}{6}\right)\text{mA} = \frac{1}{3}\,\text{mA}$$

Matching: maximum power transfer



Find R_L to give maximum power in load

$$P = \frac{V_L^2}{R_L} = V_0^2 \frac{R_L^2}{(R_{in} + R_L)^2} \frac{1}{R_L}$$

$$\frac{dP}{dR_L} = V_0^2 \frac{(R_{in} + R_L)^2 - 2R_L(R_{in} + R_L)}{(R_{in} + R_L)^4} = 0$$

$$R_{in} + R_L - 2R_L = 0$$

$$\therefore R_{in} = R_L$$

Maximum power transfer when $R_L = R_{in}$

Note – power dissipated half in R_L and half in R_{in}

Circuits have Consequences

- Problem:
  - My old speakers are 60W speakers.
  - Special 2-4-1 deal at El-Cheap-0 Acoustics on 120W speakers!!
    - (“Offer not seen on TV!”)
  - Do I buy them?
- Depends! 4Ω, 8Ω, or 16Ω speakers?
- **Why does this matter?**

And Now for Something Completely Different