First Year Physics: Prelims CP1

Classical Mechanics: Prof. Neville Harnew

Problem Set V Angular momentum & rotational dynamics

Questions 1-9 are "standard" examples. Questions 10-11 are additional questions that may also be attempted or left for revision. Problems with asterisks are either more advanced than average or require extensive algebra. All topics are covered in lectures 10-15 of Hilary term.

1. A turntable consists of a flat disk A of radius a and mass m. Calculate its moment of inertia about its centre. It is spinning freely with angular velocity ω. Give expressions for its angular momentum and its kinetic energy. A second identical disk B is held, horizontal and stationary, directly above A. B is dropped onto A and sticks to it. Find expressions for the angular velocity, the angular momentum and the kinetic energy of the assembly.

2. A cylinder of mass M, radius r, is set to rotate with angular velocity ω_0 about its own axis, which is fixed. If after time t the angular velocity is ω_1, find the frictional torque N on the cylinder at time t, assuming that:

(a) N is constant,

(b) N is proportional to ω, the instantaneous angular velocity.

[Ans: (a) $Mr^2(\omega_0 - \omega_1)/2t$, (b) $[Mr^2\omega/2t]\ln(\omega_0/\omega_1)$]

3. A cylindrical reel of thread of radius r and mass m is allowed to unwind under gravity, the upper end of the thread being fixed. Find (a) the initial acceleration of the reel (b) the initial tension in the thread. [The moment of inertia of a uniform cylinder of radius r and mass m about its axis is $\frac{1}{2}mr^2$.]
[Ans: (a) $2g/3$, (b) $mg/3$]

4. A space station is located in a gravity-free region of space. It consists of a large diameter, hollow thin-walled cylinder which is rotating freely about its axis. It is spinning at a speed such that the apparent gravity on the inner surface is the same as that on earth. The cylinder is of radius r and mass M.

(a) What is the minimum total work which had to be done to get the cylinder spinning up to speed.

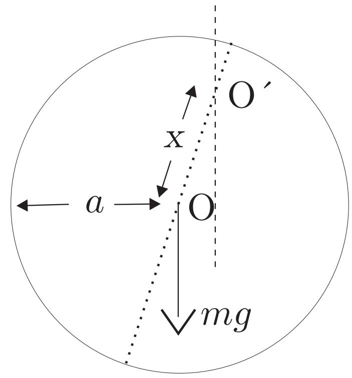
(b) Radial spokes, of negligible mass, connect the cylinder to the centre of rotation. An astronaut, of mass m, climbs a spoke from the inner surface of the cylinder to the centre. What will be the fractional change in the apparent gravity on the surface of the cylinder?

(c) If the astronaut now climbs halfway up a spoke and lets go, how far along the cylinder circumference from the base of the spoke will the astronaut hit the cylinder?
Assume throughout that the astronaut is point-like. [Ans: $(\sqrt{3} - \pi/3)r$]

5. A disk of radius a and mass m is suspended at its centre by a vertical torsion wire which exerts a couple $-c\theta$ on the disk when it is twisted through an angle θ from its equilibrium position. Show that oscillations of the disk are simple harmonic, and obtain an expression for the period.

A wire ring of mass m and radius $a/2$ is dropped concentrically onto the disk and sticks to it. Calculate the changes to (a) the period (b) the amplitude (c) the energy of the oscillations for the two cases where the ring is dropped on (i) at the end of the swing when the disk is instantaneously at rest (ii) at the midpoint of the swing when the disk is moving with its maximum angular velocity.

6. A compound pendulum consists of a uniform circular disk of radius a and mass m with a series of small holes drilled at regular intervals along a diameter. A horizontal axis about which the disk is free to turn is placed through the hole O', a distance x from the centre of the disk (O).



(a) Find the period T of small-angle oscillations using the torque-angular momentum equation,
(b) Sketch the behaviour of T^2 as a function of x for $0 < x < a$. For what values of x is T a minimum? Outline how you would use the measurement of $T(x)$ to measure g.

[Ans: Minimum value of T is at $x = a/\sqrt{2}$.]

7. State and prove the theorem of parallel axes. Find by integration the moment of inertia of a uniform rod of mass m, length b about (i) one end (ii) the centre of gravity. Show that your results are consistent with the theorem.

The rod (one end A, the other B) lies at rest on a smooth horizontal table. A horizontal force F is applied to its centre, perpendicular to its length. Find the initial acceleration of A. The experiment is repeated with the point of application of the force now at B (but still perpendicular to its length). Find the initial acceleration of A. In which direction does it start to move? Find the distance from A at which the force must be applied for the initial acceleration of A to be zero.

[Ans: $mb^2/3$; $mb^2/12$; F/m; $-2F/m$; $2b/3$.]

8. Show that the moment of inertia of a spherical shell of radius R and mass M about an axis through its centre is $\frac{2}{3}MR^2$. Show also that the moment of inertia of a uniform solid sphere of radius R and mass M is $\frac{2}{5}MR^2$. The spheres are allowed to roll (from rest), without slipping a distance L down a plane inclined at a angle θ to the horizontal. Find expressions for the speeds of the spheres at the bottom of the incline and show that $\frac{\Delta v}{\langle v \rangle} = 8.7\%$ where Δv is the difference in the speeds and $\langle v \rangle$ is the mean of the two speeds. Which sphere has the larger speed?

9. An interacting system is composed of two bodies of masses m_1, m_2 at positions $\mathbf{r}_1$, $\mathbf{r}_2$ with respect to an origin at O. Show that the angular momentum of the system about an axis through O may be written as

$$\mathbf{L} = (m_1 + m_2)\mathbf{r}_{cm} \times \dot{\mathbf{r}}_{cm} + \mu \mathbf{r} \times \dot{\mathbf{r}},$$

where $\mathbf{r}_{cm}$ is the position of the centre-of-mass of the system (wrt O), $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$ is the relative coordinate and μ is the reduced mass $1/\mu = 1/m_1 + 1/m_2$.

Additional questions

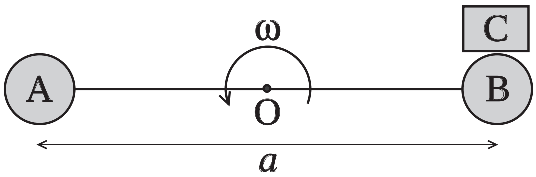
10. A thin uniform bar of length l and mass M is suspended horizontally at rest. It is suddenly released and immediately it is struck by a sharp blow vertically upwards at one end (the duration of the impulse is negligible). The impulse causes the bar to rotate at an angular velocity ω_0 about the centre of mass.

(a) Describe the motion of the bar as a combination of translation of the centre of mass (CM) and rotation about the CM.

(b) Find the maximum height reached by the CM and the time taken to reach it.

(c) Assuming that the bar passes through its original position after a time t show that $t^2 = 2\pi nl/3g$ where n is an integer.

11. Two objects A and B, each of mass M are fixed to the ends of a rigid rod of negligible mass and length a. The centre of mass, O, of the system is stationary in space, and the system is rotating with angular velocity ω about an axis through O perpendicular to AB. One of the rotating masses (B) strikes a third stationary object C, also of mass M, and the two stick together. The dimensions of the objects themselves may be neglected.



(a) Find the position and velocity of the CM, O′, of the three-particle system just prior to the collision. [Hint: in an obvious notation, $\mathbf{r}_{\mathrm{CM}} = (\mathbf{r}_{\mathrm{A}} + \mathbf{r}_{\mathrm{B}} + \mathbf{r}_{\mathrm{C}})/3$; differentiate this to find the velocity of the CM, using the facts that O and C are at rest before the collision. Note that O′ does *not* move as if it were attached to the rod.]

(b) Find the total angular momentum of the system about O′ just before and just after the collision. Hence show that the angular velocity after the collision is $3\omega/4$.

(c) Show that the initial KE of the system is $Ma^2\omega^2/4$ and the final KE is $3Ma^2\omega^2/16$.

(d) How would your results change if the collision between B and C was totally elastic rather than totally inelastic as above? (Assume that A does not aquire instantaneous momentum from the elastic collision of B & C.)