First Year Physics: Prelims CP1

Classical Mechanics: Prof. Neville Harnew

Introductory Problems

These problems do not require knowledge beyond A-Level physics and maths. They are intended for revision before or early in MT.

1. A rocket is fired vertically upwards with constant acceleration from the earth's surface. After one minute it reaches a height of 36 km, and the motor is shut off. Neglect air resistance and variation of g with height.

(a) Calculate the maximum height reached.
(b) Calculate the total time of flight.
(c) Draw a $v - t$ diagram for the entire flight.
[Ans: (b) 109.4 km, (c) 5.53 min]

2. Consider the motion of a object able to move along a line under constant acceleration a. At time $t = 0$, its initial position and velocity are x_0 and v_0 respectively. Show that at a later time t,

$$v = v_0 + at, \quad x = x_0 + v_0 t + \frac{1}{2}at^2.$$

Eliminate the time between these two relations to show that

$$a(x - x_0) = \frac{1}{2}(v^2 - v_0^2).$$

Use this to find the minimum stopping distance for a car travelling at $100\,\mathrm{kmh}^{-1}$ if its maximum deceleration is $10\,\mathrm{ms}^{-2}$. [Ans 38.6 m]

3. Estimates of inertia and forces:

(a) A cabin cruiser of 15 tonnes (1 tonne = 10^3 kg) drifts in towards a dock at a speed of 0.3 $\mathrm{m\,s}^{-1}$ after its engines have been stopped. A woman on the dock is able to reach the boat with a boathook when it is 2 m from the dock and thereafter can push with a constant force of 400 N. Can she bring the boat to rest before it hits the dock?

(b) A man of mass 80 kg jumps down on a paved road from a height of 0.5 m. He does not bend his knees on landing so his motion is arrested in about 2 cm. What is average force on his bone structure?

(c) If the man now tries jumping from a height of 1.5 m but does bend his knees on impact so that his centre of gravity descends a further distance h after his feet touch the ground, what must h be so that the average force exerted on him by the impact is only five times his normal weight?

[Ans: (a) Yes, (b) 1.96×10^4 N, (c) 0.375 m.]

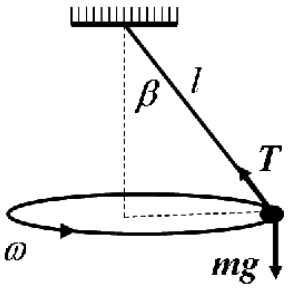
4. When a car corners at speed one is relying on the friction between the tyres and the road to counter the centrifugal force tending to throw the car outwards. One way to reduce the dependence on friction is to 'bank' the corner. For a car travelling with a speed v entering a corner of radius r, show that the angle of the bank, θ, (measured from the horizontal) should satisfy $\tan\theta = v^2/(gr)$ if there is to be no reliance on friction.
In the light of this result discuss the design of the 'Wall of Death' fairground attraction in which a motorbike is ridden at speed around the inner surface of a large vertical cylinder, with the bike and rider at a small angle to the horizontal plane. What role does friction play and why is a small angle necessary?

5. Calculate the following centripetal accelerations as fractions or multiples of g and comment on the results: (a) The acceleration towards the Earth's axis of an object resting on the Earth's surface at 45° latitude. The Earth's radius is 6378 km. (b) The acceleration of the Moon towards the Earth. The radius of the Moon's orbit is 384399 km and its period is 27.3 days. (c) The acceleration of an electron moving around a proton at a speed of $0.007c$ where c is the speed of light in an orbit of radius 0.05 nm (the first Bohr orbit of the H atom).

[Ans: (a) 2.4×10^{-3}, (b) 2.8×10^{-4}, (c) 9×10^{21}.]

6. An object fixed with respect to the surface of a planet identical in mass and radius to the Earth, experiences zero apparent gravity at the equator. What is the length of a day on that planet? What apparent gravity would be experienced by an object at the planet's poles? [Apparent gravity is the force on a stationary object taking into account both gravitational attraction and rotation.]

7. The muzzle velocity of a gun is 60 $\mathrm{m\,s^{-1}}$. A woman shoots one shot each second straight up into the air, which may be considered frictionless. How many bullets will be in the air at any time and at what height above the ground will they pass each other?



8. A conical pendulum consists of a mass m suspended by a massless string of length l as shown. The mass rotates in a horizontal circle at fixed angular velocity ω so that the string makes a constant angle β with the vertical. Show that the angular velocity of rotation is given by $\omega = \sqrt{g/l\cos\beta}$.

9. A ladder leans against a frictionless wall while the bottom rests on a horizontal floor with coefficient of friction μ as shown. Show that the smallest angle that the ladder can make with the floor without slipping is given by $\tan\theta_{min} = 1/2\mu$.