Classical Mechanics

LECTURE 8:

TRANSFORMATIONS FROM LAB TO CM FRAMES

Prof. N. Harnew

University of Oxford

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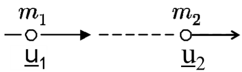
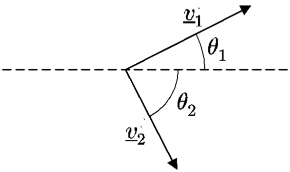
OUTLINE : 8. TRANSFORMATIONS FROM LAB TO CM FRAMES

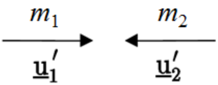
8.1 Elastic collisions in the CM frame

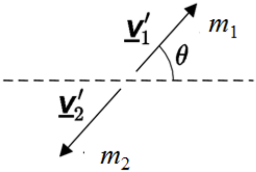
Lab frame:





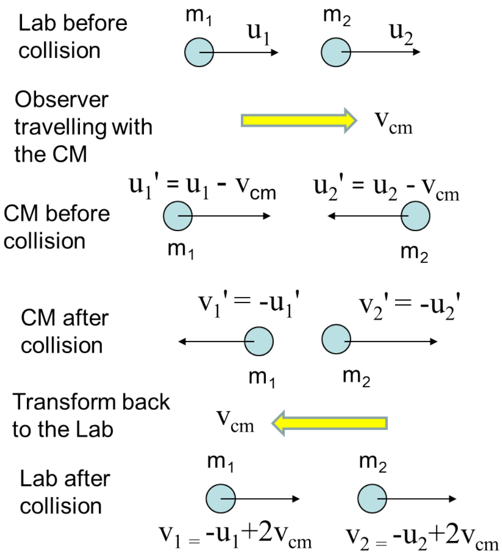
Centre of mass frame (zero momentum frame)





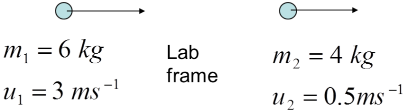
- Conservation of momentum in CM:
  $m_1\underline{\mathbf{u}}'_1 + m_2\underline{\mathbf{u}}'_2 = 0$; $m_1\underline{\mathbf{v}}'_1 + m_2\underline{\mathbf{v}}'_2 = 0$
- Conservation of energy in CM:
  $\frac{1}{2}m_1\underline{\mathbf{u}}'^2_1 + \frac{1}{2}m_2\underline{\mathbf{u}}'^2_2 = \frac{1}{2}m_1\underline{\mathbf{v}}'^2_1 + \frac{1}{2}m_2\underline{\mathbf{v}}'^2_2$

8.2 Lab to CM : 2-body 1D elastic collision



- $v_{cm} = \frac{(m_1 u_1 + m_2 u_2)}{(m_1 + m_2)}$
- Before in CM :
  $m_1 u_1' + m_2 u_2' = 0$
- After in CM :
  $m_1 v_1' + m_2 v_2' = 0$
- From last lecture
  $u_1' - u_2' = v_2' - v_1'$
- Sub for u_2', v_2' :
  $u_1'(1 + m_1/m_2) =$
  $-v_1'(1 + m_1/m_2)$
- $v_1' = -u_1'$
  $v_2' = -u_2'$

8.2.1 Collision in 1D : numerical example

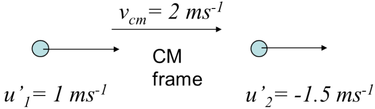


1) Find CM velocity relative to laboratory frame :

   $v_{cm} = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2} = \frac{6 \times 3 + 0.5 \times 4}{10} = 2\,\text{ms}^{-1}$

2) Transform initial velocities into CM : $u' = u - v_{cm}$

   $u'_1 = 3 - 2 = 1\,\text{ms}^{-1}$; $u'_2 = 0.5 - 2 = -1.5\,\text{ms}^{-1}$



3) Conservation of energy : $v'_1 = -u'_1$: $v'_2 = -u'_2$

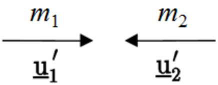
   $v'_1 = -1\,\text{ms}^{-1}$; $v'_2 = 1.5\,\text{ms}^{-1}$ after collision

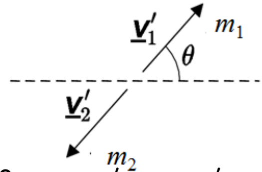
4) Transform final velocities back to Laboratory frame :

   $v = v' + v_{cm}$

   $v_1 = -1 + 2 = 1\,\text{ms}^{-1}$; $v_2 = 1.5 + 2 = 3.5\,\text{ms}^{-1}$

8.3 Relationship between speeds in CM in 2D



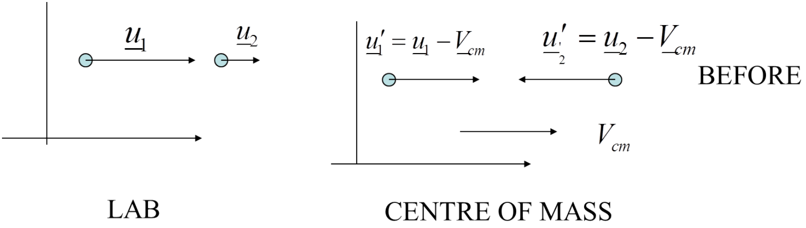


- Momentum : $m_1\underline{\mathbf{u}}_1' + m_2\underline{\mathbf{u}}_2' = 0$; $m_1\underline{\mathbf{v}}_1' + m_2\underline{\mathbf{v}}_2' = 0$ (1)
- Dot products : $m_1 u_1'^2 = -m_2\underline{\mathbf{u}}_1' \cdot \underline{\mathbf{u}}_2'$; $m_2 u_2'^2 = -m_1\underline{\mathbf{u}}_1' \cdot \underline{\mathbf{u}}_2'$

  $m_1 v_1'^2 = -m_2\underline{\mathbf{v}}_1' \cdot \underline{\mathbf{v}}_2'$; $m_2 v_2'^2 = -m_1\underline{\mathbf{v}}_1' \cdot \underline{\mathbf{v}}_2'$
- Energy : $\frac{1}{2}m_1 {u_1'}^2 + \frac{1}{2}m_2 u'^2_2 = \frac{1}{2}m_1 v'^2_1 + \frac{1}{2}m_2 v'^2_2$

  Hence $-(m_1 + m_2)\underline{\mathbf{u}}_1' \cdot \underline{\mathbf{u}}_2' = -(m_1 + m_2)\underline{\mathbf{v}}_1' \cdot \underline{\mathbf{v}}_2'$
- $\underline{\mathbf{u}}_1' \cdot \underline{\mathbf{u}}_2' = \underline{\mathbf{v}}_1' \cdot \underline{\mathbf{v}}_2'$: magnitudes $u_1' u_2' = v_1' v_2' \rightarrow$ back-to-back
- From (1), magnitudes $u_2' = -\frac{m_1}{m_2}u_1'$; $v_2' = -\frac{m_1}{m_2}v_1'$
- Hence $u_1'(-\frac{m_1}{m_2}u_1') = v_1'(-\frac{m_1}{m_2}v_1') \rightarrow u_1'^2 = v_1'^2$; $u_2'^2 = v_2'^2$
- $|v_1'| = |u_1'|$; $|v_2'| = |u_2'|$ $\rightarrow$ Speeds before = speeds after Back-to-back in direction as shown in diagrams.

8.4 Lab to CM : 2-body 2D elastic collision



1) Find centre of mass velocity v_{CM}

- $(\underline{\mathbf{u}}_1 - \underline{\mathbf{v}}_{CM})m_1 + (\underline{\mathbf{u}}_2 - \underline{\mathbf{v}}_{CM})m_2 = 0$
- $\rightarrow \underline{\mathbf{v}}_{CM} = \frac{m_1\underline{\mathbf{u}}_1 + m_2\underline{\mathbf{u}}_2}{m_1+m_2}$

2) Transform initial Lab velocities to CM

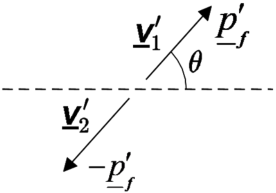
- $\underline{\mathbf{u}}_1' = \underline{\mathbf{u}}_1 - \underline{\mathbf{v}}_{CM}$, $\underline{\mathbf{u}}_2' = \underline{\mathbf{u}}_2 - \underline{\mathbf{v}}_{CM}$

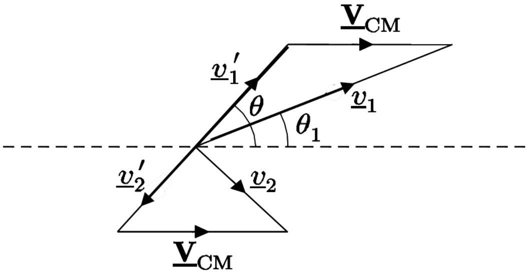
3) Get final CM velocities

- $|\underline{\mathbf{v}}_1'| = |\underline{\mathbf{u}}_1'|$; $|\underline{\mathbf{v}}_2'| = |\underline{\mathbf{u}}_2'|$

4) Transform vectors back to the Lab frame

- $\underline{\mathbf{v}}_1 = \underline{\mathbf{v}}_1' + \underline{\mathbf{v}}_{CM}$; $\underline{\mathbf{v}}_2 = \underline{\mathbf{v}}_2{}' + \underline{\mathbf{v}}_{CM}$





5) Can then use trigonometry to solve