Classical Mechanics

LECTURE 1:

INTRODUCTION TO CLASSICAL MECHANICS

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MT 2016

OUTLINE : 1. INTRODUCTION TO MECHANICS

1.1 Outline of lectures

Two groups of lectures

- 10 in MT - mostly 1D & 2D linear motion.
- 19 in HT - 3D full vector treatment of Newtonian mechanics, rotational dynamics, orbits, introduction to Lagrangian dynamics

Info on the course is on the web:
http://www.physics.ox.ac.uk/users/harnew/lectures/

- Synopsis and suggested reading list
- Problem sets
- Lecture slides

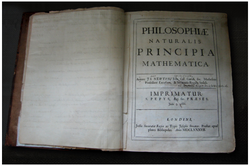
1.2 Book list

- Introduction to Classical Mechanics A P French & M G Ebison (Chapman & Hall)
- Introduction to Classical Mechanics D. Morin (CUP) (good for Lagrangian Dynamics and many examples).
- Classical Mechanics : a Modern Introduction, M W McCall (Wiley 2001)
- Mechanics Berkeley Physics Course Vol I C Kittel et al. (McGraw Hill)
- Fundamentals of Physics Halliday, Resnick & Walker (Wiley)
- Analytical Mechanics 6th ed, Fowles & Cassidy (Harcourt)
- Physics for Scientists & Engineers, (Chapters on Mechanics) P.A Tipler & G. Mosca (W H Freeman)
- Classical Mechanics T W B Kibble & F H H Berkshire (Imperial College Press)

1.3 What is Classical Mechanics?

Classical mechanics is the study of the motion of bodies in accordance with the general principles first enunciated by Sir Isaac Newton in his Philosophiae Naturalis Principia Mathematica (1687). Classical mechanics is the foundation upon which all other branches of Physics are built. It has many important applications in many areas of science:

- Astronomy (motion of stars and planets)
- Molecular and nuclear physics (collisions of atomic and subatomic particles)
- Geology (e.g., the propagation of seismic waves)
- Engineering (eg structures of bridges and buildings)



Classical Mechanics covers:

- The case in which bodies remain at rest
- Translational motion– by which a body shifts from one point in space to another
- Oscillatory motion– e.g., the motion of a pendulum or spring
- Circular motion–motion by which a body executes a circular orbit about another fixed body [e.g., the (approximate) motion of the earth about the sun]
- More general rotational motion–orbits of planets or bodies that are spinning
- Particle collisions (elastic and inelastic)

Forces in mechanics

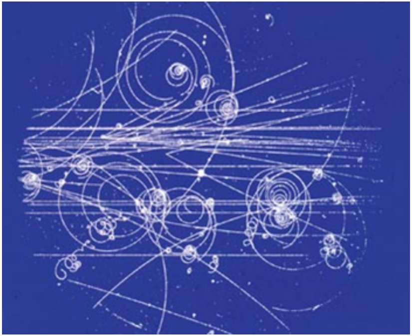
Relative magnitude of forces:

- Strong force - nuclear : ~ 1
- Electromagnetism - charged particles : $\frac{1}{137}$
- Weak force - β decay : $\sim 10^{-5}$
- Gravitational - important for masses, relative strength : $\sim 10^{-39}$

Not too fast!

Classical Mechanics valid on scales which are:

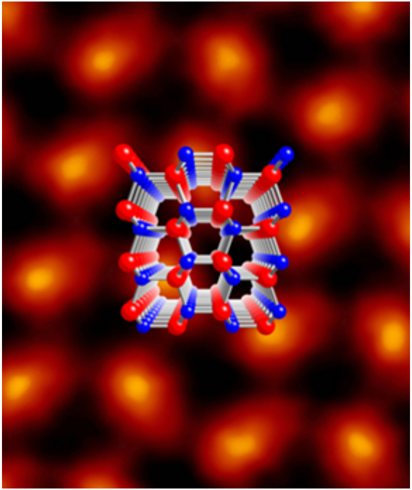
- Not too fast
- eg. high energy particle tracks from CERN
- $v << c$ [speed of light in vacuo]
- If too fast, time is no longer absolute - need special relativity.



Not too small!

Classical Mechanics valid on scales which are:

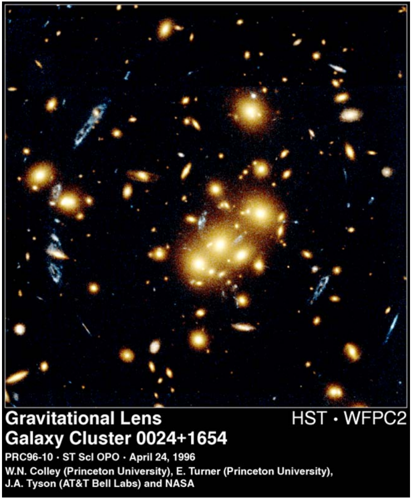
- Not too small!
- Images of atom planes in a lattice by scanning tunneling electron microscope
- Particles actually have wave-like properties : $\lambda = \frac{h}{p}$ $\quad$ ($h = 6.6 \times 10^{-34}$ Js)
- Hence for scales $>> \lambda$, wave properties can be ignored



Not too large!

Classical Mechanics valid on scales which are:

- Not too large!
- Gravitational lens produced by a cluster of galaxies
- Space is “flat” in classical mechanics - curvature of space is ignored
- Also in Newtonian mechanics, time is absolute



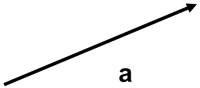
1.4 Vectors in mechanics

The use of vectors is essential in the formalization of classical mechanics.

- A scalar is characterised by magnitude only: energy, temperature.
- A vector is a quantity characterised by magnitude and direction: eg. Force, momentum, velocity.

Notation:

- Vector: **a** (bold); in components $\underline{\mathbf{a}} = (a_x, a_y, a_z)$
- Magnitude of $\underline{\mathbf{a}}$ is $|\underline{\mathbf{a}}|$ or simply a.

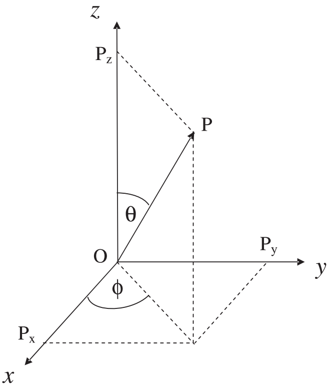


- Two vectors are equal if they have the same magnitude and direction (i.e. parallel)
  $\underline{\mathbf{a}} = \underline{\mathbf{b}}$ gives $a_x = b_x,\ \ a_y = b_y,\ \ a_z = b_z$

1.4.1 Vector components in 3D

Projecting the components:

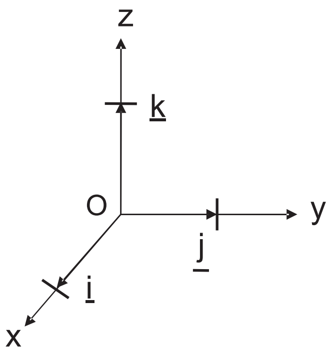
$$\underline{\mathbf{p}} = (p_x, p_y, p_z)$$



- x-component
  $p_x = |\underline{\mathbf{p}}| \sin(\theta) \cos(\phi)$
- y-component
  $p_y = |\underline{\mathbf{p}}| \sin(\theta) \sin(\phi)$
- z-component
  $p_z = |\underline{\mathbf{p}}| \cos(\theta)$
- Magnitude $|\underline{\mathbf{p}}| = \sqrt{}(p_x^2 + p_y^2 + p_z^2)$
- Direction $\tan(\phi) = (p_y/p_x)$
  $\cos(\theta) = (p_z/|\underline{\mathbf{p}}|)$

1.4.2 Unit vectors

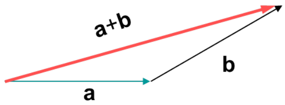
- A unit vector is a vector with magnitude equal to one.
- e.g. Three unit vectors defined by orthogonal components of the Cartesian coordinate system:
  - $\underline{\mathbf{i}}$ = (1,0,0), obviously $|\underline{\mathbf{i}}|$ = 1
  - $\underline{\mathbf{j}}$ = (0,1,0), $|\underline{\mathbf{j}}|$ = 1
  - $\underline{\mathbf{k}}$ = (0,0,1), $|\underline{\mathbf{k}}|$ = 1
- A unit vector in the direction of general vector $\underline{\mathbf{a}}$ is written $\underline{\hat{\mathbf{a}}} = \underline{\mathbf{a}}/|\underline{\mathbf{a}}|$
- $\underline{\mathbf{a}}$ is written in terms of unit vectors $\underline{\mathbf{a}} = a_x\underline{\mathbf{i}} + a_y\underline{\mathbf{j}} + a_z\underline{\mathbf{k}}$



1.4.3 Vector algebra

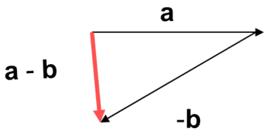
Sum of two vectors

- To calculate the sum of two vectors
  $\underline{\mathbf{c}} = \underline{\mathbf{a}} + \underline{\mathbf{b}}$
  Triangle rule: Put the second vector nose to tail with the first and the resultant is the vector sum.
- $\underline{\mathbf{c}} = \underline{\mathbf{a}} + \underline{\mathbf{b}}$: in (x, y, z) components
  $(c_x, c_y, c_z) = (a_x+b_x, a_y+b_y, a_z+b_z)$
- Alternatively $\underline{\mathbf{c}} = \underline{\mathbf{a}} + \underline{\mathbf{b}}$
  $c_x\underline{\mathbf{i}} + c_y\underline{\mathbf{j}} + c_z\underline{\mathbf{k}} =$
  $(a_x + b_x)\underline{\mathbf{i}} + (a_y + b_y)\underline{\mathbf{j}} + (a_z + b_z)\underline{\mathbf{k}}$



Vector algebra laws

- $\underline{\mathbf{a}} + \underline{\mathbf{b}} = \underline{\mathbf{b}} + \underline{\mathbf{a}}$: commutative law
- $\underline{\mathbf{a}} + (\underline{\mathbf{b}} + \underline{\mathbf{c}}) = (\underline{\mathbf{a}} + \underline{\mathbf{b}}) + \underline{\mathbf{c}}$: associative law
- Can treat vector equations in same way as ordinary algebra
  $\underline{\mathbf{a}} + \underline{\mathbf{b}} = \underline{\mathbf{c}} \quad \Rightarrow \quad \underline{\mathbf{a}} = \underline{\mathbf{c}} - \underline{\mathbf{b}}$
- Note that vector $-\underline{\mathbf{b}}$ is equal in magnitude to $\underline{\mathbf{b}}$ but in the opposite direction.
  so $\underline{\mathbf{a}} - \underline{\mathbf{b}} = \underline{\mathbf{a}} + (-\underline{\mathbf{b}})$



a - b = a + (-b)

Multiplication of a vector by a scalar

- This gives a vector in the same direction as the original but of proportional magnitude.
- For any scalars α and β and vectors $\underline{\mathbf{a}}$ and $\underline{\mathbf{b}}$
  - $(\alpha\beta)\ \underline{\mathbf{a}} = \alpha(\beta\ \underline{\mathbf{a}}\ ) = \beta(\alpha\ \underline{\mathbf{a}}\ ) = \underline{\mathbf{a}}\ (\alpha\beta)$: associative & commutative
  - $(\alpha+\beta)\underline{\mathbf{a}} = \alpha\underline{\mathbf{a}} + \beta\underline{\mathbf{a}}$: distributive
  - $\alpha(\underline{\mathbf{a}} + \underline{\mathbf{b}}) = \alpha\underline{\mathbf{a}} + \alpha\underline{\mathbf{b}}$