Synopsis for First Year CP3: Vectors and Matrices

Michaelmas Term 2012

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The purpose of the course is to provide an elementary introduction to vectors and matrices. The lectures will fully cover the syllabus and will assume very little previous knowledge of the subject. Loosely, I plan to distribute the material as follows:-

Lecture 1

Introduction to vectors and scalars. Unit vectors in Cartesian coordinates. Magnitude and projections of a vector. Introduction to vector equations and algebra. Simple addition of vectors. Multiplication of vector by a scalar.

Lecture 2

Multiplication of two vectors. Dot product; commutative, associative and distributive properties. Vector (cross) product; non-commutative, associative and non-distributive properties. Dot and cross product in spatial components. Geometrical interpretation of cross product: area as a vector. Examples of cross product: angular momentum, torque, Lorentz force.

Lecture 3

Scalar triple product and its geometrical interpretation. Vector triple product. Lagrange's identity. Elementary vector geometry of lines in 2D and 3D. Cartesian and vector representation.

Lecture 4

Shortest distance between a point and a line. Minimum distance between two lines. Angle between two crossing lines. Shortest distance between skew lines. Examples.

Lecture 5

Vector representation of a plane. Angle between two planes. Shortest distance between a point and plane. Examples. Intercept of line with a plane. Distance from a line to a plane. Coplanar vectors. Three planes crossing. Examples. Vector equation of a sphere.

Lecture 6

From vectors to matrices. Linear vector spaces. Linearly dependent and linearly independent vectors. Examples. Basis vectors in N-dimensions.

Lecture 7

Inner product. Orthogonal bases. Linear operators. Dual vectors. Examples. Introducing the concepts of a matrix.

Lecture 8

The dimensions of a matrix. Simple matrix algebra addition, multiplication by a scalar, matrix multiplication. Examples. Functions of a matrix.

Lecture 9
Special matrices and matrix operations. The inverse and transpose of a matrix. Matrix equations. The null and unit (identity) matrices. Hermitian matrices. Orthogonal , unitary and normal matrices. The trace of a matrix. Other special matrices: diagonal, symmetric, orthogonal. (All restricted to 2D and 3D matrices).

Lecture 10
The determinant of a matrix. Minors, cofactors, adjoints. Finding the value of a determinant by row and column manipulation: the Laplace development. Properties of determinants. Examples. The adjugate matrix.

Lecture 11
The inverse and rank of a matrix. Using determinants to find the inverse of a matrix. Examples.

Lecture 12
Application of matrix methods to the solution of simultaneous linear equations. Unique solutions and geometrical interpretation. Inverse matrix method and Cramer's method. Examples.

Lecture 13
Gauss reduction method. Lower and upper triangle (ethalon) matrices. Examples. Gauss reduction method for finding the inverse of a matrix.

Lecture 14
Simultaneous linear equations: cases in which solutions are not unique or do not exist; geometric interpretation of these cases.

Lecture 15
Change of basis and matrix transformations. Matrices as operators. Rotations of coordinate systems. Matrix transformation of vectors in 2D and 3D. Examples.

Lecture 16
Matrices and quadratic forms. Eigenvectors and eigenvalues and their determination. Examples.

Lecture 17
Diagonalization of matrices. Similarity transformations. Eigenvalues and eigenvectors of a Hermitian matrix and their orthogonality.

Lecture 18
Putting together all the concepts. Worked example: changing the basis of a hyperboloid. Summary of the course.