CP2 ELECTROMAGNETISM

https://users.physics.ox.ac.uk/∼harnew/lectures/

LECTURE 9:

CAPACITANCE

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$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\frac{1}{\mu_0}\nabla \times \mathbf{B} = \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

[1] With thanks to Prof Laura Herz

OUTLINE : 9. CAPACITANCE

9.1 Capacitance

9.2 Cylindrical capacitor

9.3 Spherical capacitor

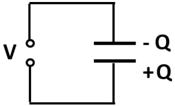
9.4 Capacitance networks

9.5 Energy stored in a capacitor

9.6 Changing C at constant V

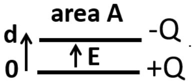
9.1 Capacitance

- Capacitors store electrostatic energy, by keeping two opposite charge accumulations on different metallic surfaces.



- Capacitance is defined as the charge that is stored per unit voltage applied between the two surfaces.

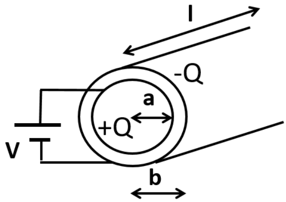
  Capacitance definition $C = \frac{\textit{Stored charge } Q}{\textit{Voltage applied}}$



- The charge is equal and opposite on both surfaces.
  - Simple example : Parallel plate capacitor

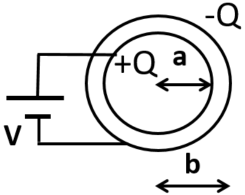
9.2 Cylindrical capacitor

- Example : coaxial cable. Battery supplies $+Q$ on the inner surface, $-Q$ is induced on the outer (Gauss)



9.3 Spherical capacitor

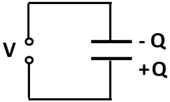
- Example : spherical capacitor with concentric hollow spheres. Battery supplies $+Q$ on the inner sphere, $-Q$ is induced on the outer (Gauss).



Capacitors summary

Capacitance: Storage of energy through separation of two oppositely poled charge accumulations

$$\text{Capacitance } C = \frac{\text{charge } Q}{\text{voltage } V \text{ applied}}$$



(i) parallel-plate capacitor:

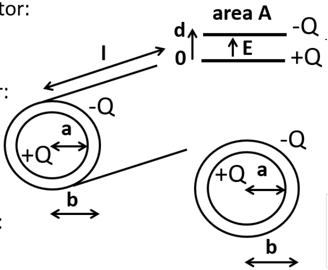
$$C = \varepsilon_0 \frac{A}{d}$$

(ii) cylindrical capacitor:

$$\frac{C}{l} = \frac{2\pi\varepsilon_0}{\ln(b/a)}$$

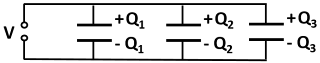
(iii) spherical capacitor:

$$C = 4\pi\varepsilon_0 \frac{a\,b}{b-a}$$

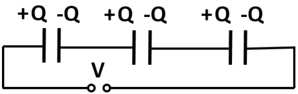


9.4 Capacitance networks

1. Capacitors in parallel
    - Voltage is the same across each capacitor.



2. Capacitors in series
    - Charge is the same on each capacitor plate (inner plates are isolated from the outside world, with $Q_{tot} = 0$).

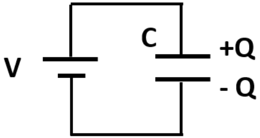


9.5 Energy stored in a capacitor

- Capacitor is initially uncharged : add a small amount of charge.
- Further charge will have to be brought up against the potential created by the existing charge :

  Work done $\rightarrow$ $dW = V(q)\,dq$

9.6 Changing C at constant V



- Battery maintains capacitor at constant V. What happens if C changes ?
- Energy stored in capacitor : $U_C = \frac{1}{2}\, C\, V^2$

  Change in capacitor energy : $dU_C = \frac{1}{2}\, V^2\, dC$
- Hence if C increases, U_C increases
- Since $Q = CV$, if C increases (ie. dC is positive), battery has to *supply* charge to maintain the same V. Hence charge on capacitor *increases*, and energy stored in battery *decreases*.
- Battery supplies dQ at constant $V \;\rightarrow\;$ energy change of battery is $dU_B = -V\,dQ$ (minus because battery *loses* stored energy in providing $+dQ$ to the plates of the capacitor)
- $Q = CV$, $dQ = V\,dC$, hence $dU_B = -V^2\,dC$
- This is a general result. If U_C *increases* at constant V, this is matched by a factor 2 *decrease* in battery energy.
- Cons. of energy : $dU_{total} = dU_B + dU_C = dW$, where

  $dW = -\frac{1}{2}\, V^2\, dC$ is the work done to change $C^{(*)}$.

(*) Note dC is negative if plates are pulled apart, since C decreases.