CP2 ELECTROMAGNETISM

https://users.physics.ox.ac.uk/~harnew/lectures/

LECTURE 6:

GAUSS LAW EXAMPLES

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$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

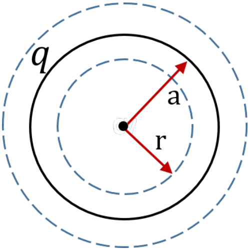
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

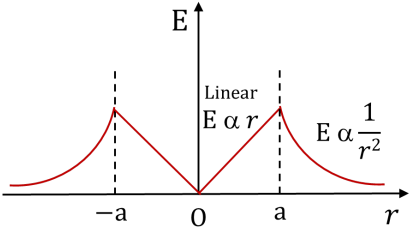
$$\frac{1}{\mu_0} \nabla \times \mathbf{B} = \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

[1] With thanks to Prof Laura Herz

OUTLINE : 6. GAUSS LAW EXAMPLES

6.1 Gauss theorem : uniform volume charge



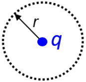


Summary Gauss Law : spherical symmetry

Spherically symmetric charge distributions.

$$\oint_S \mathbf{E} \, . \, \mathbf{da} = E_r \times 4\pi r^2 = \frac{1}{\varepsilon_0} \int_V \rho \, dV \longrightarrow E_r = \frac{1}{4\pi\varepsilon_0 r^2} \int_V \rho \, dV$$

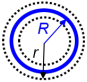
(i) point charge q:



$$E_r = \frac{q}{4\pi\varepsilon_0 r^2}$$

for any *r*

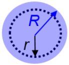
(ii) hollow sphere with q spread evenly across surface:



For 0 < r < R (inside sphere): $E_r = 0$

For R < r (outside sphere): $E_r = \dfrac{q}{4\pi\varepsilon_0 r^2}$

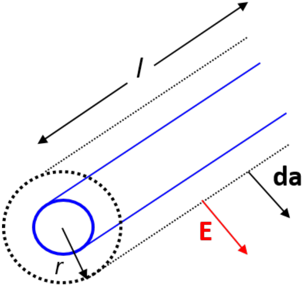
(iii) Sphere carrying uniform volume charge ρ:



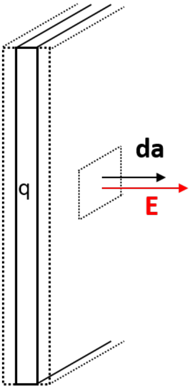
For 0 < r < R (inside sphere): $E_r = \dfrac{q}{4\pi\varepsilon_0 R^2} \dfrac{r}{R}$

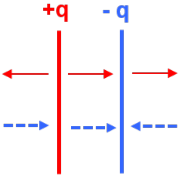
For R < r (outside sphere): $E_r = \dfrac{q}{4\pi\varepsilon_0 r^2}$

6.2 Gauss Theorem : Long, uniformly charged rod



6.3 Uniformly charged infinite plate





6.4 Electric field inside a conductor

Inside a conductor, one or more electrons per atom are free to move throughout the material (copper, gold, and other metals). We are considering electroSTATICS (static charge). As a result:

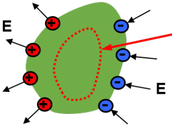
(i) **E**=0 inside a conductor (free charge moves to surface until the internal electric field is cancelled).

(ii) ρ =0 inside a conductor (from Gauss' law: **E**=0 hence ρ=0).

(iii) Therefore any net charge resides on the surface.

(iv) A conductor is an equipotential (since **E**=0, $V(\mathbf{r}_1)=V(\mathbf{r}_2)$).

(v) At the surface of a conductor, **E** is perpendicular to the surface (otherwise charges will flow until the tangential component becomes zero when equilibrium is reached).



$$\oint_S \mathbf{E} \cdot d\mathbf{a} = 0$$

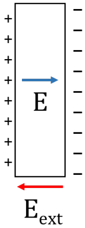
ρ=0

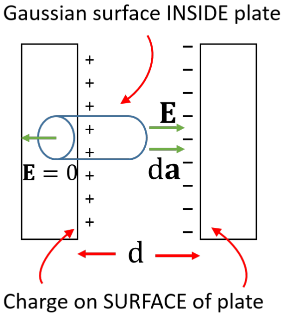
inside *V*

Properties of conductors

1. $\underline{\mathbf{E}} = 0$ inside a conductor

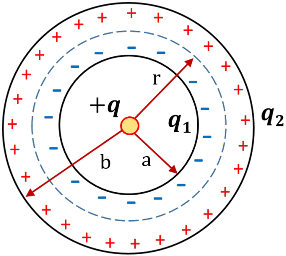
   - We are dealing with electroSTATICS - charges can move in an $\underline{\mathbf{E}}$-field !
   - They will move to the surface, creating surface charge which opposes applied field.
   - Equilibrium reached with $\underline{\mathbf{E}} = 0$ inside conductor.

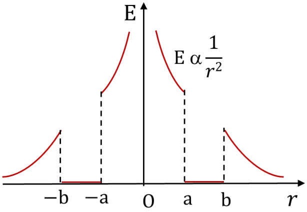




6.5 Revisit the electric field inside a hollow sphere

Consider an uncharged hollow metal sphere of finite thickness, with point charge $+q$ at its centre.





$$\rightarrow \quad E_r = \frac{q}{4\pi\epsilon_0 \, r^2}$$