# CP2 ELECTROMAGNETISM

*https://users.physics.ox.ac.uk/~harnew/lectures/*

# LECTURE 4:

# CONTINUOUS CHARGE DISTRIBUTIONS


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$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\frac{1}{\mu_0}\nabla \times \mathbf{B} = \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$


[1] With thanks to Prof Laura Herz

# *OUTLINE : 4. CONTINUOUS CHARGE DISTRIBUTIONS*

# 4.1 Continuous Charge Distributions

- Reminder : the potential at $P$ , position vector $\underline{\mathbf{r}}$, due to assembly of charges :

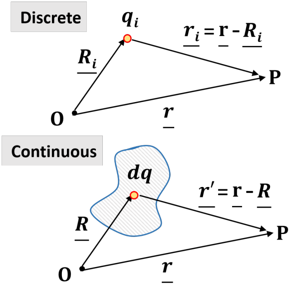
  $V(\underline{\mathbf{r}}) = \sum_i V_i(q_i) = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{|\underline{\mathbf{r_i}}|}$

  (where $\underline{\mathbf{r_i}} = \underline{\mathbf{r}} - \underline{\mathbf{R_i}}$).

- And the field : $\underline{\mathbf{E}}(\underline{\mathbf{r}}) = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{(\underline{\mathbf{r_i}})^2} \frac{\underline{\mathbf{r_i}}}{|\underline{\mathbf{r_i}}|}$

- For a continuous charge distribution $\sum_i V_i \rightarrow \int dV$

  Hence $V(\underline{\mathbf{r}}) = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{|\underline{\mathbf{r}} - \underline{\mathbf{R}}|}$




- Integrate over all infinitesimal $dq$ over the charge distribution, noting that $q \equiv q(\underline{\mathbf{R}})$

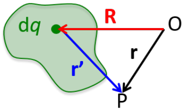
- Alternatively $V(\underline{\mathbf{r}}) = \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} \frac{\rho(\underline{\mathbf{R}})\, d\mathcal{V}}{|\underline{\mathbf{r}} - \underline{\mathbf{R}}|}$ over volume $\mathcal{V}$, where $\rho(\underline{\mathbf{R}})$ is the charge density.

- Similarly for the electric field $\underline{\mathbf{E}}(\underline{\mathbf{r}}) = \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} \frac{\rho(\underline{\mathbf{R}})(\underline{\mathbf{r}} - \underline{\mathbf{R}})}{|\underline{\mathbf{r}} - \underline{\mathbf{R}}|^3} d\mathcal{V}$

# *Continuous Charge Distributions*

Continuous charge distribution:

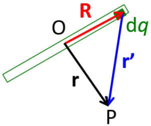
$$\sum V_i \rightarrow \int dV$$




$$V = \int \frac{dq}{4\pi\varepsilon_0 |\mathbf{r} - \mathbf{R}|}$$

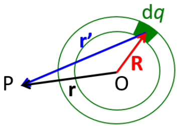
Line charge:

$dq = \lambda \, dl$



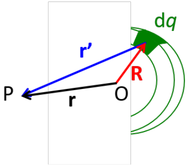

Surface charge:

$dq = \sigma \, dA$




Volume charge:

$dq = \rho \, dV$

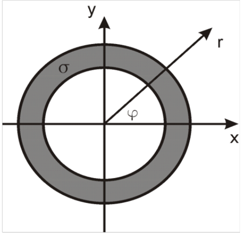


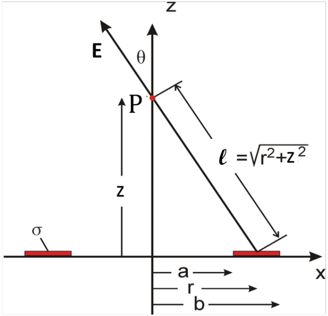

Choose a convenient origin O suiting the geometry of the charge distribution!

Adopt the notation: $\lambda$ = charge density for 1D distribution, $\sigma$ = charge density for 2D, $\rho$ = charge density for 3D

# *4.2 Example 1 : uniformly charged annulus*

**Uniformly charged ring.**



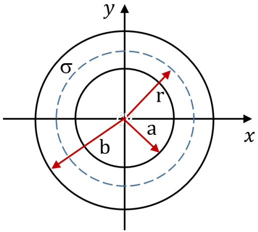



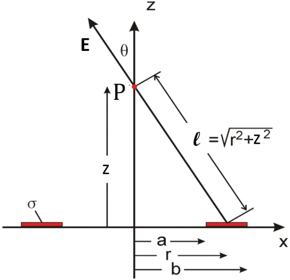

$$V = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{b^2+z^2} - \sqrt{a^2+z^2}\right]$$

$$\mathbf{E} = \frac{\sigma}{2\varepsilon_0}\left[\frac{z}{\sqrt{a^2+z^2}} - \frac{z}{\sqrt{b^2+z^2}}\right]\hat{\mathbf{z}}$$

# *Uniformly charged annulus*

Annulus contains charge $q$. Calculate the potential V on the annulus axis at a distance $z$ above its centre. Note the radial symmetry.



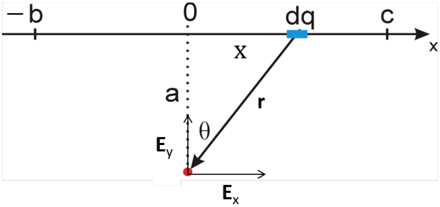




$$\underline{\mathbf{E}} = \frac{\sigma}{2\epsilon_0}\left\{\frac{z}{\sqrt{a^2+z^2}} - \frac{z}{\sqrt{b^2+z^2}}\right\}\underline{\hat{\mathbf{z}}}$$

# *Special cases*

# 4.3 Example 2 : uniformly charged rod

**Uniformly charged rod.**

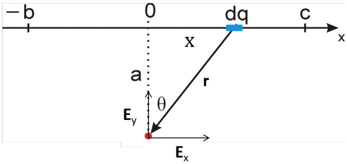



$$\mathbf{E}_x = \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{a^2+c^2}} - \frac{1}{\sqrt{a^2+b^2}}\right]$$

$$\mathbf{E}_y = \frac{-\lambda}{4\pi\varepsilon_0 a}\left[\frac{c}{\sqrt{a^2+c^2}} + \frac{b}{\sqrt{a^2+b^2}}\right]$$

# *Uniformly charged rod*

- Calculate the field $\underline{\mathbf{E}}$ at a distance $a$ from a uniformly charged rod, with length between coordinates $-b$ and $c$.



# *Special cases*