CP2 ELECTROMAGNETISM

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LECTURE 19:

MOTION IN E & B FIELDS, DISPLACEMENT CURRENT

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$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\frac{1}{\mu_0}\nabla \times \mathbf{B} = \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

[1] With thanks to Prof Laura Herz

OUTLINE : 19. MOTION IN E & B FIELDS, DISPLACEMENT CURRENT

19.1 Motion of charged particles in E and B fields

19.2 Example : the mass spectrometer

19.3 Example : magnetic lenses

19.4 Electrodynamics "before Maxwell"

19.5 Revisit Ampere's Law

19.6 Fixing Ampere's Law : displacement current

19.1 Motion of charged particles in E and B fields

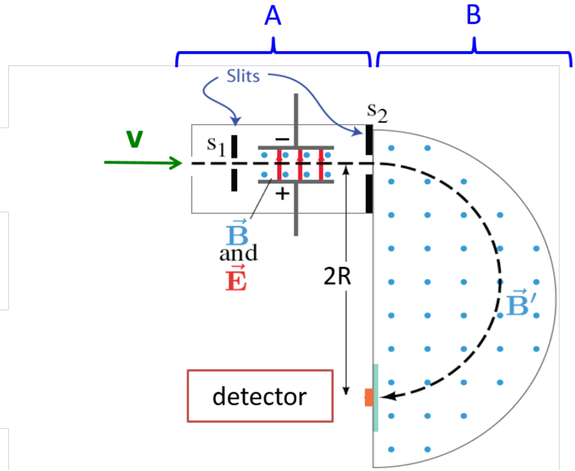
- Force on a charged particle in an $\underline{\mathbf{E}}$ and $\underline{\mathbf{B}}$ field :

- Newton second law provides equation of motion :

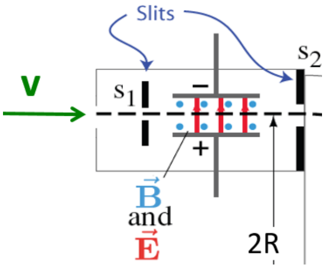
- Will demonstrate with 2 examples :
  1. Mass spectrometer
  2. Magnetic lens

19.2 Example : the mass spectrometer

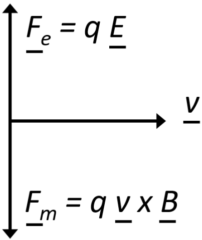
Used for detecting small charged particles (molecules, ions) by their mass *m* .



Stage A : The velocity filter



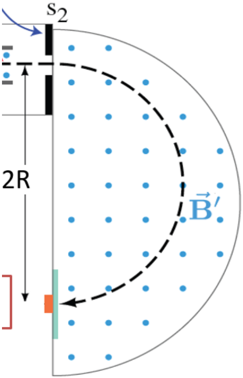
- The particle will pass through both slits if it experiences no net force inside the filter
- The region has both $\underline{\mathbf{E}}$ and $\underline{\mathbf{B}}$ fields



- Will filter particles with $v = \frac{|\underline{\mathbf{E}}|}{|\underline{\mathbf{B}}|}$ and the spread $\pm \Delta v$ is given by the slit width

Stage B : The mass filter

- This region has only a $\underline{\mathbf{B}}$ field



Mass spectrometer summary

In the presence of both E- and B-fields, a charge experiences the force:

$$\mathbf{F}_{\text{EM}} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$

Mass Spectrometer.

A. velocity filter:

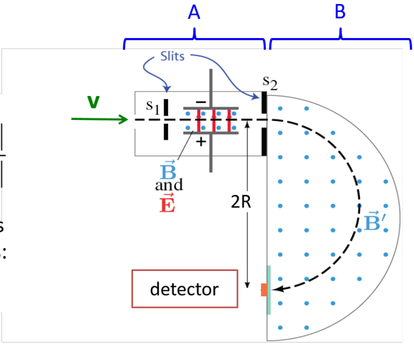
E&B-fields present. Charged particles pass through Stage A if their velocity equals the amplitude ratio:

$$v = \frac{|\mathbf{E}|}{|\mathbf{B}|}$$

B. Filter stage:

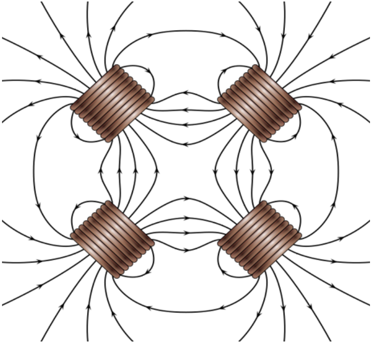
Only B-field present. Charged particles are forced on circular path with radius:

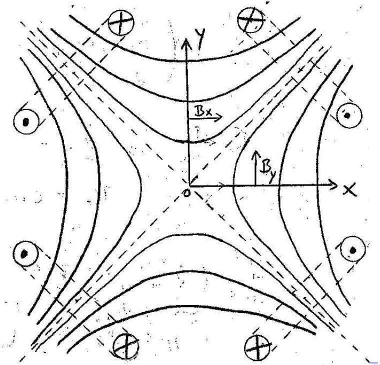
$$R = \frac{mv}{qB}$$



19.3 Example : magnetic lenses

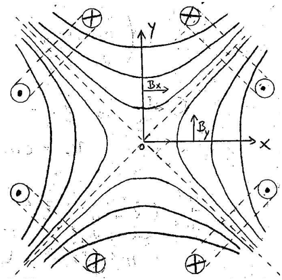
- Magnetic lenses are used for focusing and collimating charged particle beams. Used in electron microscopes, particle accelerators etc.
- Quadrupole lens : four identical coils aligned in z-direction.
- Sum of 4 dipole fields : for small values of x, y close to the axis of symmetry, $B_x \propto y$, $B_y \propto x$





Quadrupole lens

- Along x-axis : only B_y component
- Along y-axis : only B_x component
- No z-component (symmetry)



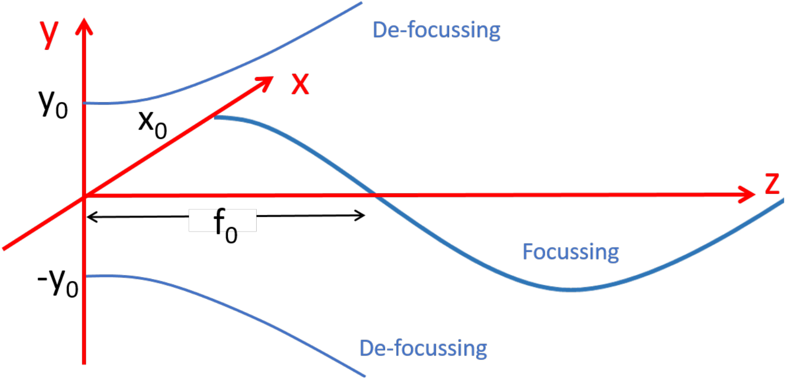
Quadrupole lens continued

- Equ. of motion : $\ddot{x} = -\alpha^2 x$ & $\ddot{y} = \alpha^2 y$, where $\alpha = \sqrt{\frac{q\,k\,v}{m}}$

- Focal points in z direction (x=0) at $f_n = \frac{\pi}{2}\sqrt{\frac{m\,v}{q\,k}} + n\,\pi\sqrt{\frac{m\,v}{q\,k}}$
- Use lens pair with 90° angle for collimating a charged beam

Quadrupole lens continued

The lens pulls the beam on-axis in x and removes particles deviating in y

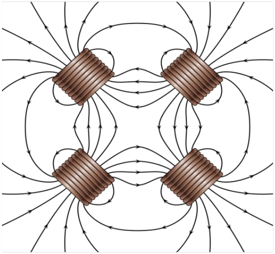


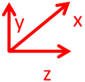
$$f_n = \frac{\pi}{2}\sqrt{\frac{m\,v}{q\,k}} + n\pi\sqrt{\frac{m\,v}{q\,k}}$$

Magnetic lens summary

Magnetic Lens.

$$\mathbf{B} = (\mathrm{k}\, y, \mathrm{k}\, x, 0)$$





Equation of Motion: $m\ddot{\mathbf{r}} = q\dot{\mathbf{r}} \times \mathbf{B}$

Solutions:

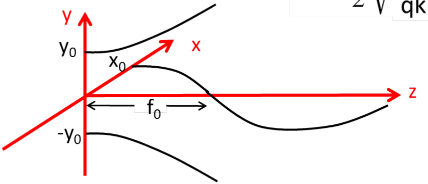
$$y(z) = y_0 \cosh \sqrt{\frac{q\,\mathrm{k}}{vm}}\, z$$

de-focusing

$$x(z) = x_0 \cos \sqrt{\frac{q\,\mathrm{k}}{vm}}\, z$$

focusing with

$$f_0 = \frac{\pi}{2}\sqrt{\frac{vm}{q\mathrm{k}}}$$



19.4 Electrodynamics "before Maxwell"

Time-varying B-fields generate E-fields. *However*, time-varying E-fields do not seem to create B-fields in this version.
Is there something wrong ?

19.5 Revisit Ampere's Law

- Therefore Ampere's Law in its current form violates the continuity equation (and hence charge conservation) !
- But this is not surprising since we derived Ampere's Law assuming that $\frac{\partial}{\partial t}(\rho) = 0$

$\rightarrow$ We have to "fix" Ampere's Law !

19.6 Fixing Ampere's Law : displacement current

- Add a term to Ampere's Law to make it compatible with the continuity equation :

  $\left(\epsilon_0 \frac{\partial \underline{\mathbf{E}}}{\partial t}\right)$ is called the *displacement current* $\underline{\mathbf{J}}_D$ (but is actually a time-varying electric field)

- Time-varying $\underline{\mathbf{E}}$ fields now generate $\underline{\mathbf{B}}$ fields and vice versa. Also satisfies charge conservation.