CP2 ELECTROMAGNETISM

https://users.physics.ox.ac.uk/∼harnew/lectures/

LECTURE 17:

SELF & MUTUAL INDUCTANCE

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$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\frac{1}{\mu_0}\nabla \times \mathbf{B} = \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

[1] With thanks to Prof Laura Herz

OUTLINE : 17. SELF & MUTUAL INDUCTANCE

17.1 Example : self inductance of two parallel wires

17.2 Mutual inductance

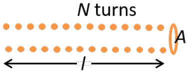
17.3 Mutual induction of two coaxial solenoids

Self inductance summary

Self-inductance L is the ratio of the voltage (emf) produced in a circuit by self-induction, to the rate of change in current causing the induction.

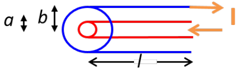
$$L = \frac{\frac{d\Phi}{dt}}{\frac{dI}{dt}} = \frac{d\Phi}{dI} = \frac{-\varepsilon}{\dot{I}}$$

Self-inductance of a long coil.



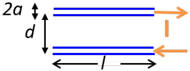
$$L = \frac{d\Phi}{dI} = \mu_0 \frac{N^2}{l} A$$

Self-inductance of a coaxial cable.



$$L = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right) l$$

Self-inductance of two parallel wires.

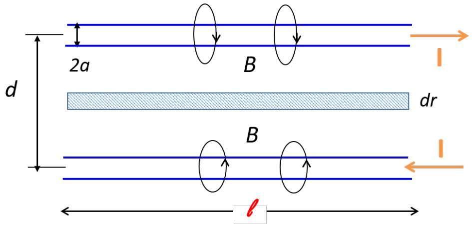


$$L = \frac{\mu_0}{\pi} \ln\left(\frac{d-a}{a}\right) l$$

Units of self inductance : the Henry $[H] \equiv [kg\, m^2 s^{-2} A^{-2}]$.
When the current changes at one ampere per second ($A\, s^{-1}$), an inductance of 1 H results in the generation of one volt (1 V) of potential difference.

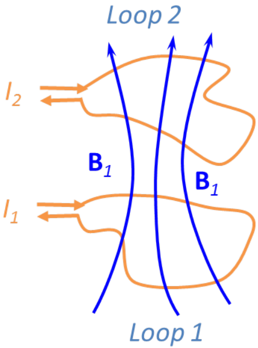
17.1 Example : self inductance of two parallel wires

Calculate the self inductance of two parallel wires,radius *a*, separation to the centres *d*, and of length ℓ



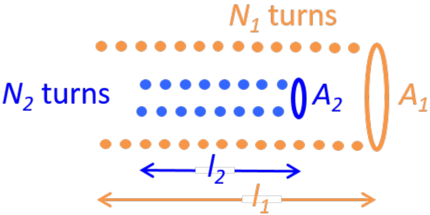
17.2 Mutual inductance

- Current I_1 through circuit loop 1 generates magnetic field density B_1 which penetrates circuit loop 2
- A change in current I_1 will induce an EMF in circuit loop 2



17.3 Mutual induction of two coaxial solenoids

1. Current through coil 1 creates magnetic field through coil 2.

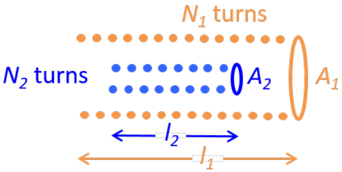


Mutual induction of two coaxial solenoids continued

2. Current through coil 2 creates magnetic field through coil 1.

- $\Phi_1 = \int B_2 \, da_1'$ (da_1' is "effective" area)

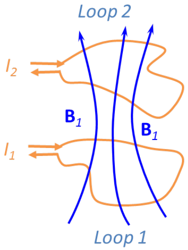
  Now it's more complicated
  as B_2 is not uniform through coil 1 !



- $M_{12} = M_{21}$ This is Neumann's theorem. (It turns out even if we had done the exact calculation the result would have been the same)

Mutual inductance summary

Mutual Inductance M: is the ratio of the voltage (emf) produced in a circuit by self-induction, to the rate of change in current causing the induction.

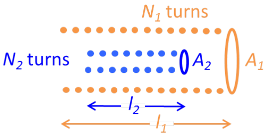


$$M_{12} = \frac{d\phi_1}{dI_2} \quad \longleftrightarrow \quad M_{21} = \frac{d\phi_2}{dI_1}$$

Neumann formula

$$M_{12} = M_{21}$$

Mutual inductance of two coaxial solenoids.



$$M_{12} = \mu_0 \frac{N_1 N_2}{l_1} A_2$$

Units of mutual inductance :
again the Henry $[H] \equiv [kg\, m^2 s^{-2} A^{-2}]$.