CP2 ELECTROMAGNETISM

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LECTURE 16:

INDUCTION EXAMPLES & SELF INDUCTION

Neville Harnew[1]

University of Oxford

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$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\frac{1}{\mu_0}\nabla \times \mathbf{B} = \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

[1] With thanks to Prof Laura Herz

OUTLINE : 16. INDUCTION EXAMPLES & SELF INDUCTION

16.1 Example : the Homopolar Generator (Faraday's disk)

16.2 Example : coil rotating in a B-field

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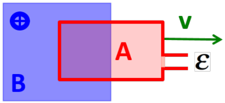
16.5 Example : long coil in varying B with resistive load

16.6 Example : self induction of a coaxial cable

Faraday's and Lenz's Laws summary

Faraday's Law of electromagnetic induction:



The induced electromotance $\mathcal{E}$ in any closed circuit is equal to the negative of the time rate of change of the magnetic flux Φ through the circuit.

$$\mathcal{E} = \frac{d\Phi}{dt} = -\frac{d}{dt}\oint_S \boldsymbol{B}\,.\,\boldsymbol{da}$$

In terms of E- and B-fields:

Integral form:

$$\oint \boldsymbol{E}\,.\,\boldsymbol{d\ell} = -\frac{d}{dt}\oint_S \boldsymbol{B}\,.\,\boldsymbol{da}$$

Differential form:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Lenz's Law:

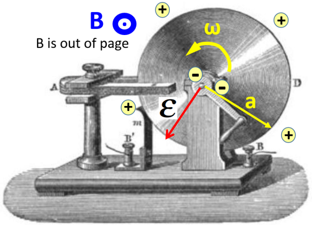
An induced electromotance always gives rise to a current whose magnetic field opposes the original change in magnetic flux.

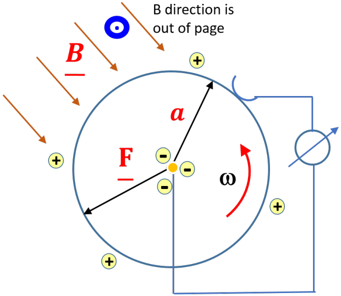
Unit of magnetic flux Weber $[Wb] = [Tm^2] = [kg\, m^2 s^{-2} A^{-1}]$

16.1 Example : the Homopolar Generator (Faraday's disk)

1. Determine voltage using Lorenz force

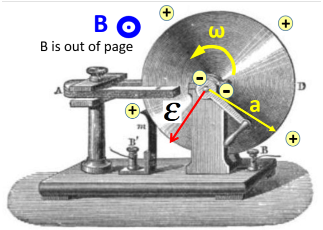
- Metal disk mechanically rotated (performing work)
- A *B*-field is present with $\underline{\mathbf{B}}$ perpendicular to the disk area.
- Voltage pick-up between the centre and rim of disk.
- EMF is *radial*, with identical potential along each circumference element, radius *r*

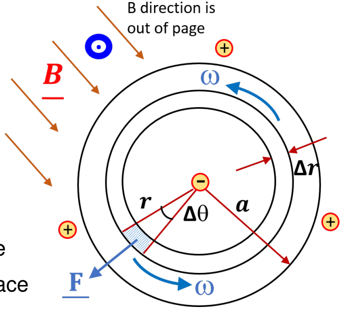




The Homopolar Generator continued

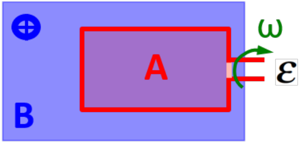
2. Determine using Faraday's Law

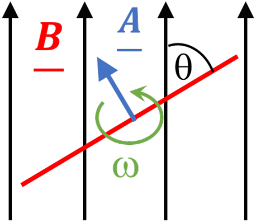




* Strictly speaking, this method from Faraday's Law is not entirely sensible since the current is continuous across the disk and $\int_S \underline{\mathbf{B}} \cdot \underline{\mathbf{da}}$ is in principle only applicable for a surface bounding a closed current path (see for example Griffiths).

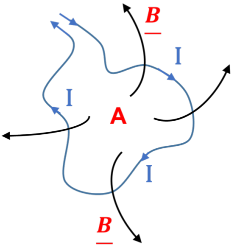
16.2 Example : coil rotating in a B-field





- This is a generator/dynamo (incorporated into most aspects of electrical power generation).

16.3 Self inductance



- Take a closed-loop circuit through which current flows
- The current I has an associated magnetic field which penetrates the circuit, $B \propto I$
- If the current changes, there will be a changing B-field through the loop.

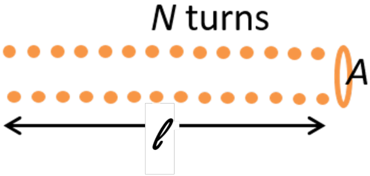
  Faraday : The changing magnetic flux Φ induces an EMF (voltage) in the loop *itself* :

  Lenz : This EMF will act in a direction so as to oppose the change in flux which caused it

  - L depends solely on the geometry of the circuit. (Compare with circuit theory : $V = L\,\frac{dI}{dt}$)

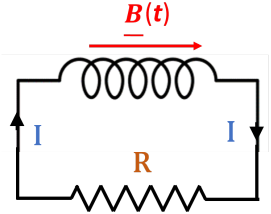
16.4 Example : self induction of a long coil

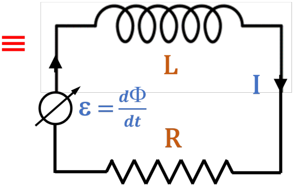
Calculate the self inductance of a long coil, area A, length ℓ, with N turns



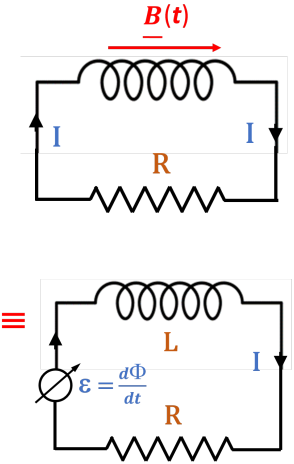
16.5 Example : long coil in varying B with resistive load

- Consider a long coil, area A, length ℓ, with N turns.
- Coil is immersed in axial time-varying magnetic field : $B(t) = B_0 \cos \omega t$
- EMF is induced in coil, coil is connected across a resistor $\rightarrow$ current will flow





Long coil in varying B with resistive load, continued



16.6 Example : self induction of a coaxial cable

Calculate the self inductance of a coaxial cable, inner/outer radii *a* & *b*, length ℓ