CP2 ELECTROMAGNETISM

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LECTURE 14:

AMPERE'S CIRCUITAL LAW, CHARGE CONSERVATION

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$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

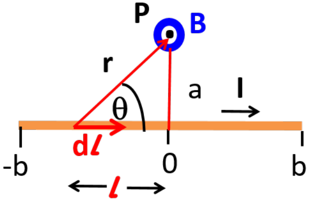
$$\frac{1}{\mu_0} \nabla \times \mathbf{B} = \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

[1] With thanks to Prof Laura Herz

OUTLINE : 14. AMPERE'S CIRCUITAL LAW & CHARGE CONSERVATION

14.1 Ampere's Circuital Law

- Ampere's Circuital Law can be derived formally from the Biot-Savart Law and vector calculus but is beyond the scope of this course.
- But for a special case, we return to the *B*-field due to an infinite straight wire with current I, previously derived.



Note Ampere's Law needs to be amended in the presence of any time-varying electric field (see later).

Ampere's Circuital Law continued

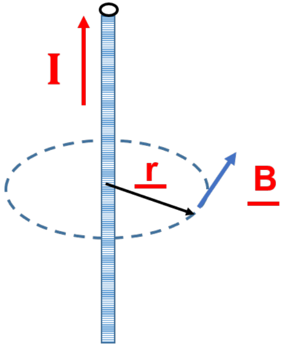
- Ampere's Circuital Law in integral form

  $\oint \underline{\mathbf{B}} \cdot \underline{\mathbf{d}\ell} = \mu_0 I = \mu_0 \int_S \underline{\mathbf{J}} \cdot \underline{\mathbf{da}}$ for current density $\underline{\mathbf{J}}$

- Ampere's Law : an integral of magnetic flux density $\underline{\mathbf{B}}$ over a closed loop bounding a surface equals the current flowing through the surface.
- Allows straightforward calculations of *B*-fields along loops where *B* is constant.

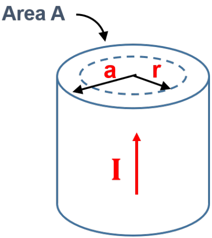
14.2 Example : B-field inside and outside a cylindrical wire

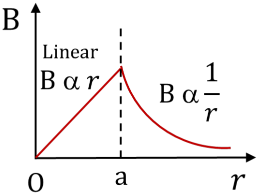
1. Outside the wire (this should be obvious $\cdots$)



Cylindrical wire continued

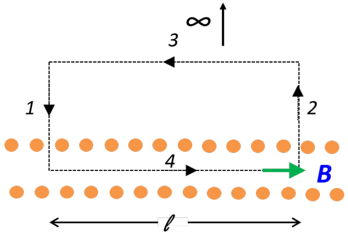
2. Inside the wire





14.3 Example : B-field of a long solenoid

- Solenoid carrying current I
- Amperean path is a rectangle inside and outside the solenoid

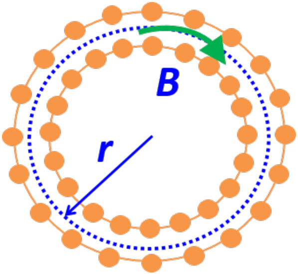


- B is uniform inside and zero outside the solenoid (if "infinite")

(*) Note that if the coil is not "infinite", end effects will need to be taken into account and here the field will not be uniform, i.e. Ampere's Law will not be as useful as presented here.

14.4 Example : B-field of a toroidal coil

- Toroid has N windings of wire carrying current I



- B- field is uniform in toroid and follows circular path
- B-field is zero outside the confines of the toroid

Ampere's Law summary

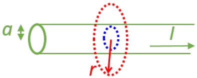
Ampere's law: $$\oint \boldsymbol{B}\,.\,\boldsymbol{dl} = \mu_0 I$$

Electric currents generate magnetic fields whose field lines form closed loops.

"Gauss' law of Magnetism": $$\oint_S \boldsymbol{B}\,.\,\boldsymbol{da} = 0$$

There are no magnetic monopoles.

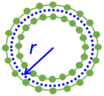
Infinite straight wire.



outside: $r > a$ $$\oint \boldsymbol{B}\,.\,\boldsymbol{dl} = B.\,2\pi r = \mu_0 I \rightarrow B = \frac{\mu_0 I}{2\pi r}$$

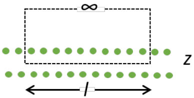
inside: $0 < r < a$ $$\oint \boldsymbol{B}\,.\,\boldsymbol{dl} = B.\,2\pi r = \mu_0 I = \frac{\pi r^2}{\pi a^2} \rightarrow B = \frac{\mu_0 I}{2\pi a^2} r$$

Toroidal coil.



$$\oint \boldsymbol{B}\,.\,\boldsymbol{dl} = B.\,2\pi r = \mu_0 N I \rightarrow B = \frac{\mu_0 N I}{2\pi r}$$

Infinite solenoid.

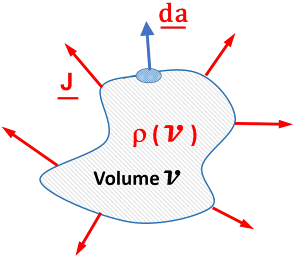


$$\oint \boldsymbol{B}\,.\,\boldsymbol{dl} = B.\,l = \mu_0 N I \rightarrow B = \mu_0 I \frac{N}{l}$$

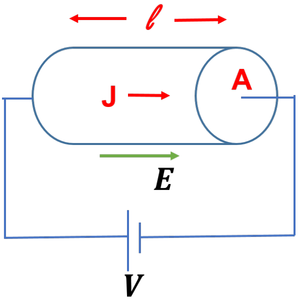
14.5 Conservation of charge

- Consider a volume $\mathcal{V}$ bounded by a surface S.

- The integral of current density flowing out (or into) the surface $\underline{\mathbf{J}} \cdot \underline{\mathbf{da}}$ is equal to the charge lost by the volume [per unit time].



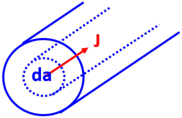
14.6 Current density and Ohm's Law



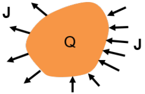
Summary : charge conservation & the continuity equation

Define current density **J**:

$$\mathbf{J} = \frac{d\mathbf{I}}{da_{\perp}}$$



For a closed volume, the net current entering must be equal to the rate in change of charge inside the volume (charge conservation):



Continuity Equation:

$$\oint_S \mathbf{J} \cdot d\mathbf{a} = I = -\frac{\partial Q}{\partial t} \longleftrightarrow \nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$$

In the limit of electro/magneto-statics:

$\frac{\partial \mathbf{J}}{\partial t} = 0$ steady currents — constant B-fields

$\frac{\partial \rho}{\partial t} = 0$ stationary charges — constant E-fields

$\xrightarrow{\text{C E}}$ $\nabla \cdot \mathbf{J} = 0$