CP2 ELECTROMAGNETISM

https://users.physics.ox.ac.uk/~harnew/lectures/

LECTURE 12:

BIOT SAVART LAW & THE MAGNETIC DIPOLE

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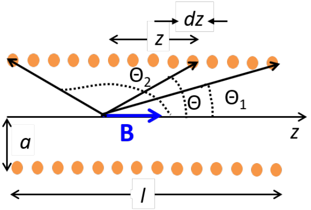
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\frac{1}{\mu_0}\nabla \times \mathbf{B} = \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

[1] With thanks to Prof Laura Herz

OUTLINE : 12. BIOT SAVART LAW & THE MAGNETIC DIPOLE

12.4 Example : B-field of a solenoid

Calculate the B-field due to a solenoid with current I, radius a, length ℓ with N turns. Sum over all contributions from all loops at a distance z (integrate from θ_1 to θ_2).



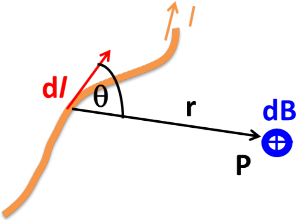
- Hence $B = \frac{\mu_0 I N}{2\ell}(\cos\theta_2 - \cos\theta_1)$
- For a long coil $\theta_1 = 0,\ \theta_2 = \pi \ \rightarrow$ $B = -\mu_0 I \frac{N}{\ell}$

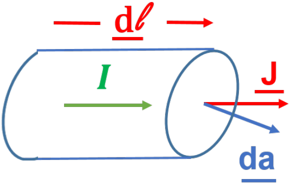
  (sign depends on direction of current $\rightarrow$ RH screw rule)

12.5 Biot-Savart Law in terms of current density

- The Biot-Savart Law :

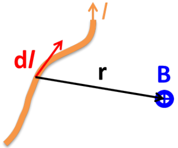
$$\underline{\mathbf{dB}} = \mu_0 \, I \, \frac{\underline{\mathbf{d}\boldsymbol{\ell}} \times \underline{\hat{\mathbf{r}}}}{4\pi \, r^2}$$





Biot-Savart Law in terms of current density $\underline{\mathbf{J}}$ integrated over volume $\mathcal{V}$

Biot-Savart Law summary

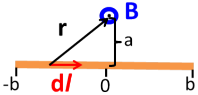


The magnetic flux density **B** created by a current loop is given by:

$$\mathbf{B} = \frac{\mu_0}{4\pi}\int I\,\frac{\mathrm{d}l \times \hat{\mathbf{r}}}{r^2} \quad \text{or} \quad \mathbf{B} = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}\times\mathbf{r}}{r^3}\,\mathrm{d}^3 r$$

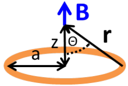
Biot-Savart Law

Straight wire.



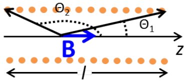
$$\mathbf{B} = \frac{\mu_0 I}{2\pi a}\frac{1}{(a^2/b^2+1)^{1/2}}$$

Circular loop.



$$\mathbf{B} = \frac{\mu_0 I a^2}{2\sqrt{z^2+a^2}^3} = \frac{\mu_0 I}{2a}\sin^3\theta$$

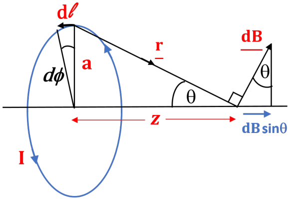
Solenoid.

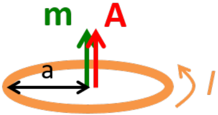


$$\mathbf{B} = \frac{\mu_0 I N}{2l}(\cos\theta_2 - \cos\theta_1)$$

12.3 The magnetic dipole

A small current loop defines a *magnetic dipole*





Magnetic dipole moment $\underline{\mathbf{m}} = I\,\underline{\mathbf{A}}$
= [Current] $\times$ [Area bounded by the loop]