CP2 ELECTROMAGNETISM

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LECTURE 11:

MAGNETOSTATICS & THE BIOT-SAVART LAW

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$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\frac{1}{\mu_0}\nabla \times \mathbf{B} = \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

[1] With thanks to Prof Laura Herz

OUTLINE : 11. MAGNETOSTATICS & THE BIOT-SAVART LAW

11.1 The Biot-Savart Law for calculating magnetic fields

11.2 Example : the B-field of a straight wire

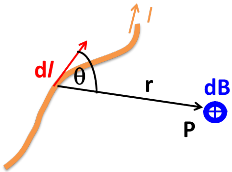
11.3 Example : force between 2 current-carrying wires

11.4 Example : B-field of a circular current loop

11.1 The Biot-Savart Law for calculating magnetic fields

The Biot-Savart is here taken as an empirical starting point for calculation of magnetic fields, but can be derived from Maxwell's equations and the magnetic potential (see later).

- The Biot-Savart Law states the field at point P:



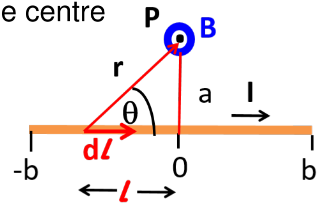
$$\underline{\mathbf{dB}} = \mu_0 \, I \, \frac{\underline{\mathbf{d}\ell} \times \underline{\hat{\mathbf{r}}}}{4\pi \, r^2}$$

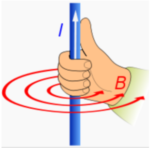
  - $\mu_0 = 4\pi \times 10^{-7} \quad \mathrm{N\,A^{-2}}$ permeability of free space
  - $\underline{\mathbf{dB}}$ is the magnetic flux density contribution at P
  - I is the current flowing through element $\underline{\mathbf{d}\ell}$
  - $\underline{\mathbf{r}}$ is the vector connecting $\underline{\mathbf{d}\ell}$ and P
  - $\underline{\mathbf{dB}}$ is oriented perpendicular to $\underline{\mathbf{r}}$ and the current

Then integrate $\underline{\mathbf{dB}}$ to get *total* field from a circuit which has current

11.2 Example : the B-field of a straight wire

Calculate the B-field due to a straight wire with current I, length $2b$, at a distance a from the centre

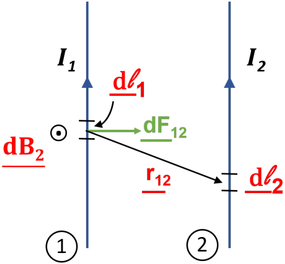


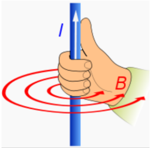


- For an infinite straight wire ($b \to \infty$) $$B = \frac{\mu_0\, I}{2\pi\, a}$$

11.3 Example : force between 2 current-carrying wires

Two wires: force on small element of wire 1 from magnetic field of small element of wire 2



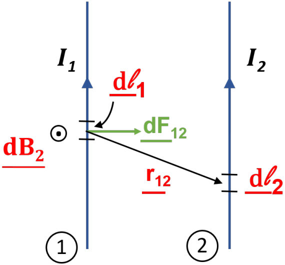


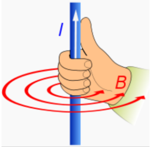
- Force between 2 current-carrying wires :

$$\underline{\mathbf{F}}_{12} = \int_{\ell_1} \int_{\ell_2} \frac{\mu_0\, I_1\, I_2}{4\pi\, r_{12}^3}\, [\underline{\mathbf{d}\ell}_2 \times (\underline{\mathbf{d}\ell}_1 \times \underline{\mathbf{r}}_{12})]$$

$\cdots$ *and if the wires are parallel and infinite*

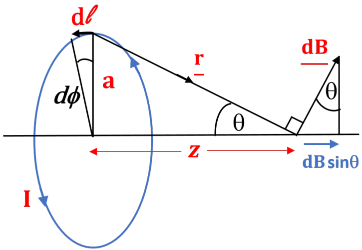
If wires are infinite, separated by distance *a*, currents I_1 and I_2





11.4 Example : B-field of a circular current loop

Calculate the B-field due to a circular wire with current I, radius a, at a distance z along its axis from the centre



- Hence $$B = \frac{\mu_0\, I\, a^2}{2(z^2+a^2)^{\frac{3}{2}}}$$
- Or since $\sin\theta = \frac{a}{\sqrt{(z^2+a^2)}}$, $B = \frac{\mu_0\, I}{2a}\sin^3\theta$