

CP2 ELECTROMAGNETISM

<https://users.physics.ox.ac.uk/~harnew/lectures/>

LECTURE 10:

CAPACITANCE, ENERGY & MAGNETOSTATICS



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$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\frac{1}{\mu_0} \nabla \times \mathbf{B} = \mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

OUTLINE : 10. CAPACITANCE, ENERGY & MAGNETOSTATICS

10.1 Force between capacitor plates (2 cases)

10.2 Energy density of the electric field

10.3 Example : hollow spherical shell

10.4 Principle of superposition for energy density

10.5 Origins of magnetism

10.6 Magnetostatics terminology

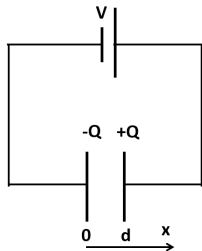
10.7 Forces on current-carrying wires in magnetic fields

10.8 The Lorenz force

10.9 Example : measuring B field

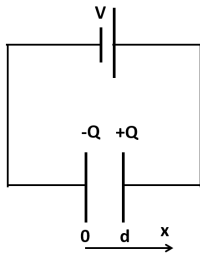
10.1 Force between capacitor plates (2 cases)

- ▶ Capacitor plates are oppositely charged \rightarrow an attractive force F exists between them.
- ▶ By pulling the plates apart we perform work on the capacitor / battery system



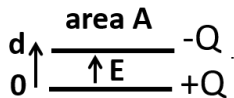
Force between capacitor plates continued

2. Plates pulled apart at constant voltage (which is supplied by the battery)



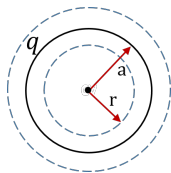
- ▶ Pulling plates apart leaves the capacitance lowered, charge returns to the battery, work is performed on the capacitor/battery system.

10.2 Energy density of the electric field



10.3 Example : hollow spherical shell

Example : Energy of hollow spherical shell carrying charge q



- ▶ Alternative approach : energy required to bring up charge dq from infinity against potential $V(q)$ is $dW = V(q) dq$

$$W = \int_0^q V(q') dq' = \int_0^q \frac{q'}{4\pi\epsilon_0} \frac{1}{a} dq' = \frac{q^2}{8\pi\epsilon_0} \frac{1}{a}$$

which is the same result as above.

10.4 Principle of superposition for energy density

Question: does the principle of superposition apply to energy density?

MAGNETOSTATICS - OVERVIEW

1. Introduction: Origins of Magnetism
2. Forces on Current-Carrying Wires in Magnetic Fields
3. The Biot-Savart Law (B-fields of Wires, Solenoids, etc.)
4. Magnetic Dipoles
5. Ampere's Law & Gauss' Law of Magnetostatics
6. Current Density and the Continuity Equation



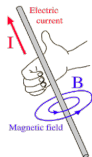
Problem
Set 3

10.5 Origins of magnetism



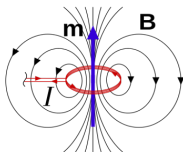
Minerals found in ancient Greek city Magnesia (“Magnetite”, Fe_3O_4) attract small metal objects.

Materials containing certain atoms such as Iron (Fe), Cobalt (Co), Nickel (Ni) can exhibit “permanent” magnetic dipoles.



Forces exist between pairs of current-carrying wires (attractive for current flowing in the same, repulsive for current flowing in opposite directions).

An electric current through a wire creates a magnetic field whose field lines loop around the wire.



Magnetic field lines form closed loops. They do not originate from “magnetic monopoles”.

The fundamental generators of magnetic fields are dipoles that may result from electrical current loops or inherent material properties such as aligned angular momenta of charged particles.

10.6 Magnetostatics terminology

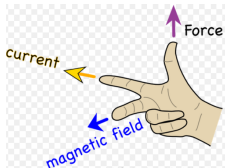
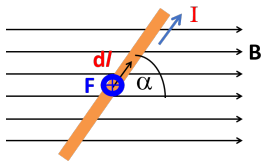
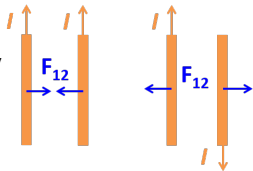
- ▶ Magnetic flux density $\underline{\mathbf{B}}$ (“B-field”) $[\mathbf{B}] = \text{T}$ (Tesla)
- ▶ Magnetic field (strength) $\underline{\mathbf{H}} = \frac{1}{\mu_0} \underline{\mathbf{B}}$ (in non magnetic materials) $[\mathbf{H}] = \text{A m}^{-1}$

where $\mu_0 = 4\pi \times 10^{-7} \text{ N A}^{-2}$ (permeability of free space)

10.7 Forces on current-carrying wires in magnetic fields

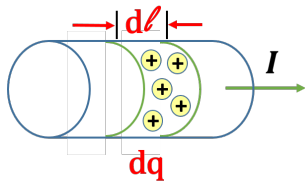
Experimental observations :

1. Two wires attract (repel) one another if they carry current in the same (opposite) directions.
2. A current-carrying wire in a magnetic field, flux density B , experiences a force :



10.8 The Lorentz force

- ▶ Force on current-carrying wire in a B-field:
 $\underline{dF} = I \underline{d\ell} \times \underline{B}$
- ▶ Zoom into a wire segment, assume it's the (+) charge moving (“conventional” current)



- ▶ Any charge q moving with velocity \underline{v} in a magnetic flux density \underline{B} experiences a Lorentz force $\underline{F} = q \underline{v} \times \underline{B}$ perpendicular to both
- ▶ Work done on the moving charge
 $dW = -\underline{F} \cdot \underline{d\ell} = -q(\underline{v} \times \underline{B}) \cdot \underline{v} dt = 0$
- ▶ Magnetic fields do no work

10.9 Example : measuring \underline{B} field

From torque on a wire loop carrying current I in field \underline{B} :

