CP2 ELECTROMAGNETISM

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LECTURE 1:

INTRODUCTION TO THE COURSE

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$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\frac{1}{\mu_0} \nabla \times \mathbf{B} = \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

[1] With thanks to Prof Laura Herz

OUTLINE : 1. INTRODUCTION TO THE COURSE

1.1 Syllabus of the Course

1.2 Structure of the Course

1.3 Book list

1.4 Electromagnetism through the years

1.5 Summary of the properties of charge

1.6 Properties of charge: Millikan Experiment

1.7 Properties of charge: Coulomb's Law

1.8 The Principle of Superposition

1.1 Syllabus of the Course

1. Electrostatics

Coulomb's law. The electric field E and potential due to a point charge and systems of point charges, including the electric dipole. The couple and force on, and the energy of, a dipole in an external electric field. Energy of a system of point charges; energy stored in an electric field. Gauss' Law; the E field and potential due to surface and volume distributions of charge (including simple examples of the method of images), no field inside a closed conductor. Force on a conductor. The capacitance of parallel-plate, cylindrical and spherical capacitors, energy stored in capacitors.

2. Magnetostatics

The forces between wires carrying steady currents. The magnetic field B, Ampere's law, Gauss' Law ("no magnetic monopoles"), the Biot-Savart Law. The B field due to currents in a long straight wire, in a circular loop (on axis only) and in straight and toroidal solenoids. The magnetic dipole; its B field. The force and couple on, and the energy of, a dipole in an external B field. Energy stored in a B field. The force on a charged particle in E and B fields.

3. Induction

Electromagnetic induction, the laws of Faraday and Lenz. EMFs generated by an external, changing magnetic field threading a circuit and due to the motion of a circuit in an external magnetic field, the flux rule. Self and mutual inductance: calculation for simple circuits, energy stored in inductors. The transformer.

4. Electromagnetic waves

Charge conservation, Ampere's law applied to a charging capacitor, Maxwell's addition to Ampere's law ("displacement current"). Maxwell's equations for fields in a vacuum (rectangular coordinates only). Plane electromagnetic waves in empty space: their speed; the relationships between **E**, **B** and the direction of propagation.

1.2 Structure of the Course

1. Electrostatics

Charges create "electric fields" which represent the resulting force experienced by a small test charge.

$$\oint_S \mathbf{E} \, . \, \mathbf{da} = \frac{Q}{\varepsilon_0} \quad \text{(closed surface)}$$

GAUSS LAW

2. Magnetostatics

Electrical currents create "magnetic fields" which create forces on moving test charges. There are no magnetic monopoles.

AMPERE'S CURRENT LAW

$$\frac{1}{\mu_0} \oint \boldsymbol{B} \, . \, \boldsymbol{dl} = I$$

$$\oint_S \boldsymbol{B} \, . \, \boldsymbol{da} = 0 \quad \text{(closed surface)}$$

3. Induction

A time-varying magnetic flux through an area creates an electromotive force along the area's rim.

$$\oint \mathbf{E} \, . \, \boldsymbol{dl} = -\frac{\mathrm{d}}{\mathrm{dt}} \int_S \mathbf{B} \, . \, \mathbf{da}$$

FARADAY / LENS LAW

4. Electromagnetic waves

A time-varying electric flux through an area creates an magnetic field along the area's rim.

$$\frac{1}{\mu_0} \oint \mathbf{B} \, . \, \mathbf{dl} = I + \varepsilon_0 \frac{\mathrm{d}}{\mathrm{dt}} \int_S \mathbf{E} \, . \, \mathbf{da}$$

Electromagnetic wave propagation

1.3 Book list

Introductory undergraduate textbooks on electromagnetism:

D. J. Griffiths, *Introduction to Electromagnetism*
Pearson, 4th edition, ISBN: 978 0 321 84781 2

I. S. Grant and W. R. Phillips, *Electromagnetism*
John Wiley, 2nd edition, ISBN: 978 0 471 92712 9

E. M. Purcell and D. J. Morin, *Electricity and Magnetism*
Pearson, 4th edition, ISBN: 978 1 107 01402 2

P. Lorrain, D. R. Corson and F. Lorrain, Fundamentals of Electromagnetic Phenomena
Freeman, ISBN: 978 0 716 73568 7

Also of interest:

W. J. Duffin, *Electricity and Magnetism*
Duffin Publishing (out of print)

Feynman, Leighton, Sands, *The Feynman Lectures on Physics, Vol II*
ISBN: 978 0 465 02382 0

W. G. Rees, *Physics by Example*
Cambridge University Press, ISBN: 978 0 521 44975 5

Electrostatics - problem sheets

1.1. Introduction: Properties of charge; Coulomb's law

1.2. The Principle of Superposition

1.3. The Electric Field and Electrostatic Potential

1.4. Assemblies of discrete charges; multipoles

1.5. Continuous charge distributions

→ Problem Set 1 (1.1–1.5)

1.6. Gauss' law

1.7. Poisson and Laplace equations

1.8. The Method of Image Charges

→ Problem Set 2 (1.6–1.8)

1.9. Capacitance and Energy of the Electric Field

→ Problem Set 3 (1.9)

1.4 Electromagnetism through the years

Electrostatics

Ancient Greece: rubbing amber against fur allows it to attract other light substances such as dust or papyrus

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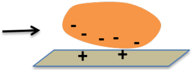


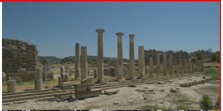
Greek word for “amber”: *ἤλεκτρον* (*elektron*)

Magnetostatics

Magnesia (ancient Greek city in Ionia, today in Turkey): Naturally occurring minerals were found to attract metal objects (first references ~600BC).

Crystals are referred to as: Iron ore, Lodestone, Magnetite, Fe_3O_4

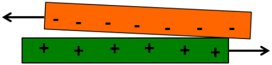
Use of Lodestone compass for navigation in medieval China



Electromagnetism through the years

17th century AD to mid 18th century:

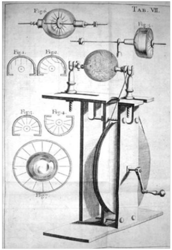
Dominated by "frictional electrostatics" :



- When two different materials are brought into contact, charge flows to equalize their electro-chemical potentials, bonds form across surface
- Separating them may lead to charge remaining unequally distributed when bonds are broken
- Rubbing enhances effect through repeat contact

Focus on "electrostatic generators" – today's van de Graaff Generators:

Machines involved frictional passage of "positive" materials such as hair, silk, fur, leather against "negative" materials such as amber, sulfur

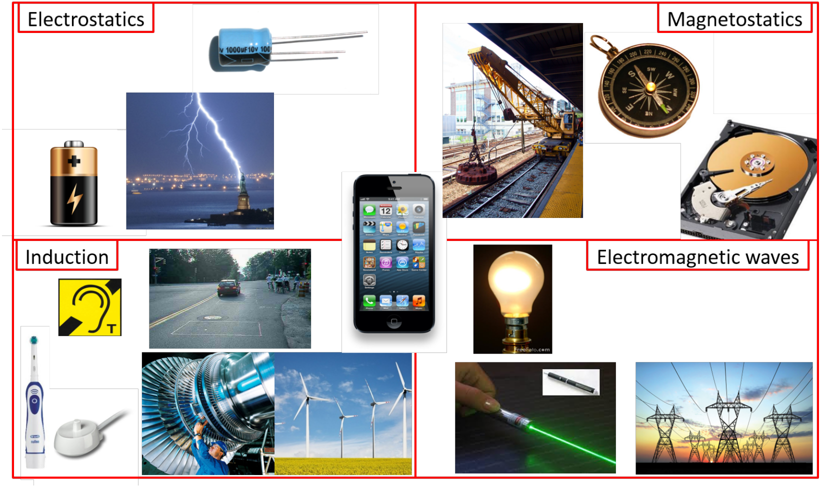


Electromagnetism through the years

From late 18th century: *Rapid progress on both fundamental science and technology:*

- 1784: Charles-Augustin de Coulomb uses “torsion balance” to show that forces between two charged spheres vary with the square of the inverse distance between them.
- 1800: Alessandro Volta constructs the first electrochemical battery (zinc/copper/sulfuric acid) allowing high-density electrical energy storage
- 1821: André-Marie Ampère investigates attractive and repulsive forces between current-carrying wires
- 1831-55: Michael Faraday discovers electromagnetic induction by experimenting with two co-axial coils of wire, wound around the same bobbin.
- 1830ies: Heinrich Lenz shows that induced currents have a direction that opposes the motions that produce them
- 1831: first commercial telegraph line, from Paddington Station to West Drayton
- 1864: James Clerk Maxwell introduces unified theory of electromagnetism, including a link to light waves
- 1887: Heinrich Hertz demonstrates the existence of electromagnetic waves in space
- Late 19th century: development of “wireless telegraphy” – radio!

Electromagnetism in everyday life



1.5 Summary of the properties of charge

- Both positive and negative charge exists (triboelectric experiments showed electrostatic attraction and repulsion)

- Charge is quantized (Millikan experiment, 1913): e=1.602×10^{-19} As

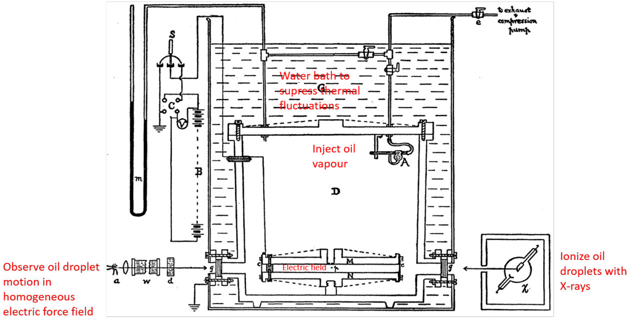
- Coulomb's law (1785): the force between two point charges varies with the square of their inverse distance:

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

- Superposition: The force between two point charges varies linearly with the amount of each charge, hence the forces resulting from individual charges superimpose in an assembly of charges: $\mathbf{F} = \Sigma\, \mathbf{F}_i$

1.6 Properties of charge: Millikan Experiment

Millikan experiment, Phys. Rev. 2, 109–143 (1913):



- Millikan oil drop experiment : observe small oil drops inside a parallel plate capacitor.
- Oil drops became electrically charged through friction with the nozzle as they are sprayed (or alternatively ionize with X-rays).
- Oil drop soon reaches terminal velocity due to friction with air.

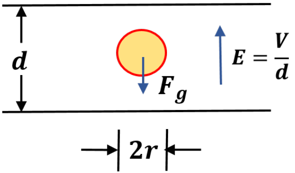
Properties of charge: quantization

Stokes' Law : retarding frictional force on sphere moving in viscous fluid $\rightarrow$ $F_{Stokes} = 6\pi\eta r v_T$

[η = dynamic viscosity, r = sphere radius, v_T = terminal velocity]

1. No E-field $F_g = F_{stokes}$: particle moving with V_T (measure)
   - $mg = \frac{4\pi}{3} r^3 (\rho_{oil} - \rho_{air}) g = 6\pi\eta r v_T$
   - Determine $r = \sqrt{\frac{9\eta v_T}{2(\rho_{oil} - \rho_{air})g}}$ and

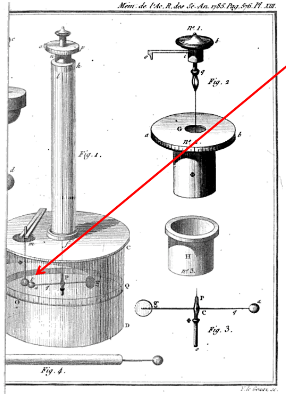
     hence $F_g = 18\pi\eta v_T \sqrt{\frac{\eta v_T}{2(\rho_{oil} - \rho_{air})g}}$



2. Ramp E-field until particle levitates ($v = 0$, $F_{total} = 0$)
   - $F_g = qE$ $\rightarrow$ determine q
   - Millikan found : $q = Ne$ (N an integer) with $e = 1.592 \times 10^{-19}$C
   - Charge is quantized

1.7 Properties of charge: Coulomb's Law

Coulomb's Torsion Balance experiment, *Histoire de l'Academie Royale des Science*, p. 569-577 (1785):



Measure force between two charged spheres through torsion force on wire:

He found: $$F \propto \frac{1}{r^2}$$

Coulomb's law: $$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

$$\varepsilon_0 = 8.854 \times 10^{-12}\frac{As}{Vm}$$

The relative strength of the Coulomb force

- Coulomb 1785 : Magnitude of the force between two point charges q_1, q_2

  $F_{elec} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$
- Newton 1665 : Magnitude of the force between two point masses m_1, m_2

  $F_{grav} = G\frac{m_1 m_2}{r^2}$
- For two protons :

  $\frac{F_{grav}}{F_{elec}} = G \times 4\pi\epsilon_0(\frac{m_p}{e})^2$

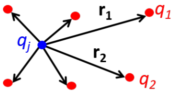
  $= 8 \times 10^{-37}$!!
- The electrostatic force is many magnitudes stronger than the gravitational force.

1.8 The Principle of Superposition

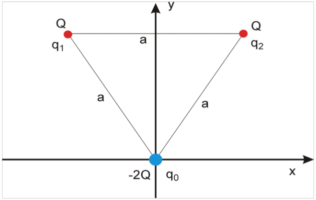
The principle of superposition states that, for all linear systems, the net response caused by two or more stimuli is the sum of the responses that would have been caused by each stimulus individually.

The force on charge q_j originating from all other charges q_i is given by:

$$\mathbf{F_j} = \frac{q_j}{4\pi\varepsilon_0} \sum_{i \neq j} \frac{q_i}{r_{ji}^2} \hat{\mathbf{r}}_{\mathbf{ji}}$$



Example 1: force on charge -2Q resulting from two charges Q in the corners of a triangle:

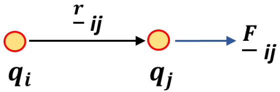


$$\mathbf{F} = \frac{\sqrt{3}Q^2}{2\pi\varepsilon_0 a^2} \hat{\mathbf{e}}_{\mathbf{y}}$$

Principle of Superposition

- Start with two charges q_i and q_j separated by $\underline{\mathbf{r}}_{\mathbf{ij}}$
  $\underline{\mathbf{F}}_{\mathbf{ij}}$ is the force on q_j due to q_i

  $\rightarrow \; \underline{\mathbf{F}}_{\mathbf{ij}} = \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}^2} \, \hat{\underline{\mathbf{r}}}_{\mathbf{ij}}$ where $\hat{\underline{\mathbf{r}}}_{\mathbf{ij}} = \underline{\mathbf{r}}_{\mathbf{ij}} / |\underline{\mathbf{r}}_{\mathbf{ij}}|$



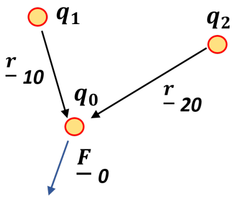
- Next go to three charges : total force on charge q_0

  $\underline{\mathbf{F}}_{\mathbf{0}} = \underline{\mathbf{F}}_{\mathbf{10}} + \underline{\mathbf{F}}_{\mathbf{20}}$

  $\underline{\mathbf{F}}_{\mathbf{0}} = \frac{q_0 q_1}{4\pi\epsilon_0 r_{10}^2} \, \hat{\underline{\mathbf{r}}}_{\mathbf{10}} + \frac{q_0 q_2}{4\pi\epsilon_0 r_{20}^2} \, \hat{\underline{\mathbf{r}}}_{\mathbf{20}}$

- In general :

  $\underline{\mathbf{F}}_{\mathbf{j}} = \frac{1}{4\pi\epsilon_0} \sum_{i(i \neq j)} \frac{q_i q_j}{r_{ij}^2} \, \hat{\underline{\mathbf{r}}}_{\mathbf{ij}}$

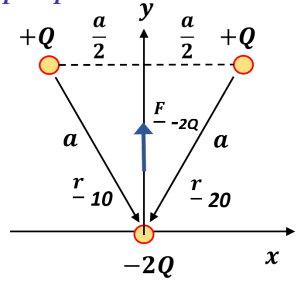


- Principle of Superposition also works for $\frac{\underline{\mathbf{F}}_{\mathbf{j}}}{q_j} = \frac{1}{4\pi\epsilon_0} \sum_{i(i \neq j)} \frac{q_i}{r_{ij}^2} \, \hat{\underline{\mathbf{r}}}_{\mathbf{ij}}$

  This is a vector field that only depends on the distribution of other charges : the *electic field* generated by the other charges.

Example 1 : principle of superposition

- Three charges arranged at corners of an equilateral triangle, $+Q$ and $+Q$ on top, $-2Q$ on the bottom. Calculate the force on $-2Q$.



- $\underline{\mathbf{F}}_{-2\mathbf{Q}} = \frac{-2Q\cdot Q}{4\pi\epsilon_0 r_{10}^2}\,\hat{\underline{\mathbf{r}}}_{\mathbf{10}} + \frac{-2Q\cdot Q}{4\pi\epsilon_0 r_{20}^2}\,\hat{\underline{\mathbf{r}}}_{\mathbf{20}}$

  $\rightarrow\ \underline{\mathbf{F}}_{-2\mathbf{Q}} = \frac{-Q^2}{2\pi\epsilon_0 a^2}(\hat{\underline{\mathbf{r}}}_{\mathbf{10}} + \hat{\underline{\mathbf{r}}}_{\mathbf{20}})$

- Now $\hat{\underline{\mathbf{r}}}_{\mathbf{10}} = \frac{\mathbf{1}}{\mathbf{a}}\begin{pmatrix} +a/2 \\ -\sqrt{a^2 - a^2/4} \end{pmatrix}$; $\hat{\underline{\mathbf{r}}}_{\mathbf{20}} = \frac{\mathbf{1}}{\mathbf{a}}\begin{pmatrix} -a/2 \\ -\sqrt{a^2 - a^2/4} \end{pmatrix}$

- $\hat{\underline{\mathbf{r}}}_{\mathbf{10}} + \hat{\underline{\mathbf{r}}}_{\mathbf{20}} = \frac{\mathbf{1}}{\mathbf{a}}\begin{pmatrix} +a/2 - a/2 \\ -\sqrt{3}a/2 - \sqrt{3}a/2 \end{pmatrix} = \begin{pmatrix} 0 \\ -\sqrt{3} \end{pmatrix}$

- Hence $\underline{\mathbf{F}}_{-2\mathbf{Q}} = \frac{\sqrt{3}Q^2}{2\pi\epsilon_0 a^2}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

- This is as expected : there is only a y component of $\underline{\mathbf{F}}$ due to symmetry.