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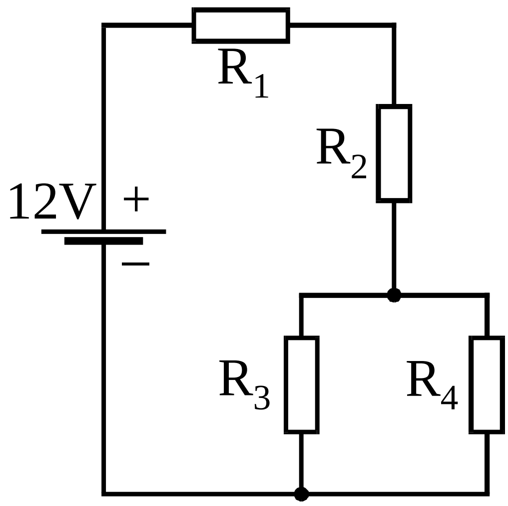
Michaelmas Term 2003

Problem Set for CP2 : Circuit Theory

I. Introduction to Simple Circuits of Resistors

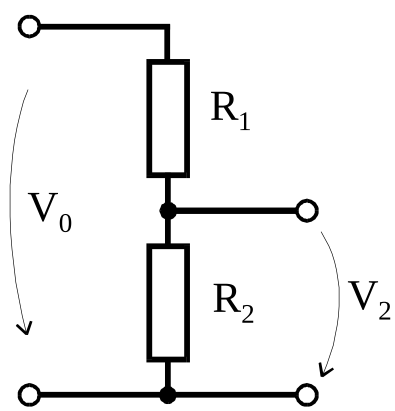
1. For the following circuit calculate the currents through and voltage drops across all resistors. The resistances are: $R_1 = 1k\Omega$, $R_2 = 3k\Omega$, $R_3 = 3k\Omega$ and $R_4 = 6k\Omega$.



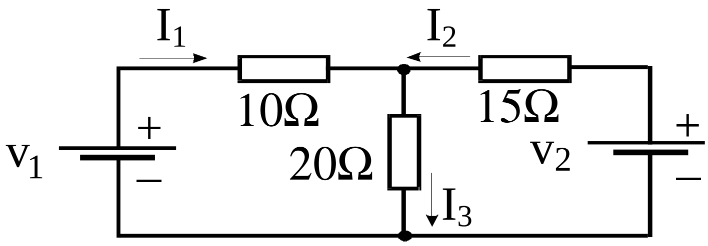
2. The structure of a cube is soldered by using resistors R for its edges. What is the resistance between two most distant corners (diametrically opposite) ?

3. Harder (for the enthusiast) : In problem 2 above, what is the resistance between two adjacent corners and between two corners on the same face but not adjacent ?

4. For the voltage divider, where $R_1 = 1k\Omega$, $R_2 = 4k\Omega$ and $V_0 = 5V$ find the voltage drop V_2 across R_2. When a load resistor R_L is fitted in parallel with R_2, what minimum value must R_L have in order not to change V_2 by more than 5%?

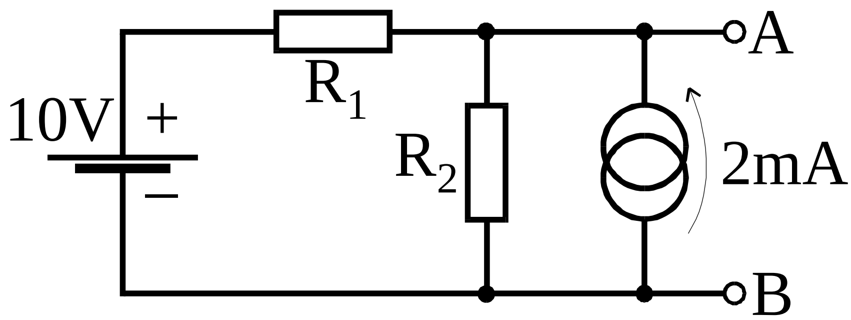


5. Consider the following circuit:



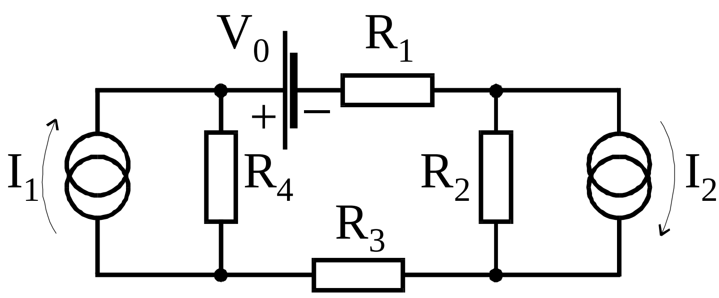
Given V_1 =5V and V_2 =10V, find I_1, I_2 and I_3. Hint : you may wish to tackle the problem using mesh currents.

6. Consider the following circuit with resistors $R_1 = R_2 = 1k\Omega$:



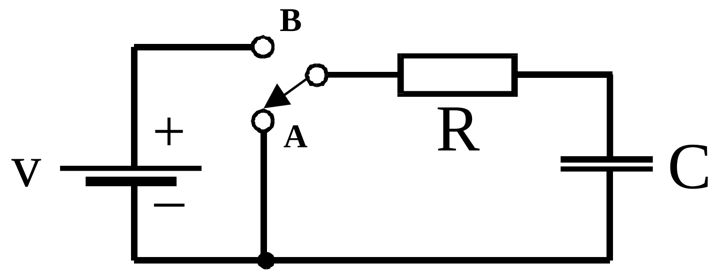
Find the voltage between A and B and the currents I_1 and I_2 through the resistors R_1 and R_2. Considering Thevenin's theorem, what are V_{eq} and R_{eq} in the equivalent circuit? What is the equivalent circuit according to Norton's theorem? Find I_{eq} and R_{eq} in this case.

7. Find the currents through resistors R_1 to R_4. Give magnitudes and directions. The values are: $V_0 = 2V$, $I_1 = 1mA$, $I_2 = 4mA$, $R_1 = 1k\Omega$, $R_2 = 1.5k\Omega$, $R_3 = 0.5k\Omega$ and $R_4 = 2k\Omega$.

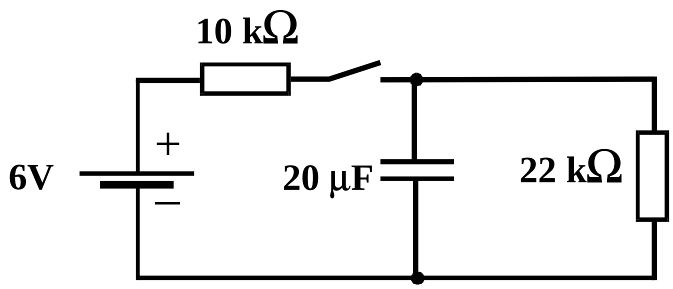


II. Response of Linear Circuits to Transients.

8. The capacitor C is initially uncharged. At time t=0 the switch is moved from position A to position B. Derive an expression for the current flowing through R at time t.



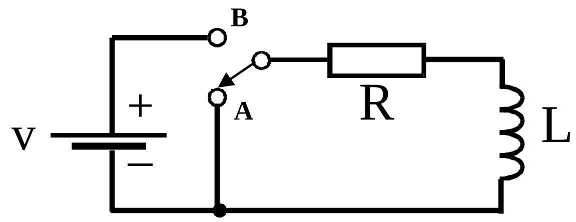
By performing an integration over time, derive an expression for the total energy dissipated in the resistor. What is the final energy stored in the capacitor ? Hence show that the total energy supplied by the battery ic CV^2.

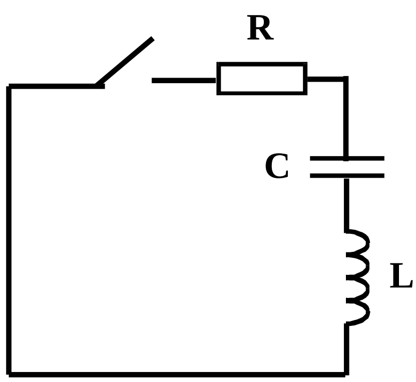


9. Initially the switch in the circuit below is open and the capacitor is uncharged. At time t=0 the switch is closed. Show that the voltage $V_C(t)$ across the capacitor as a function of time goes as: 4.125 x (1 - exp(-t/0.1375)) Volts.

When the steady state has been reached, what is i) the power dissipated in each resistor and ii) the energy stored in the capacitor ?

10. At time t=0 the switch is moved from position A to position B. Derive an expression for the current flowing through R at time t.



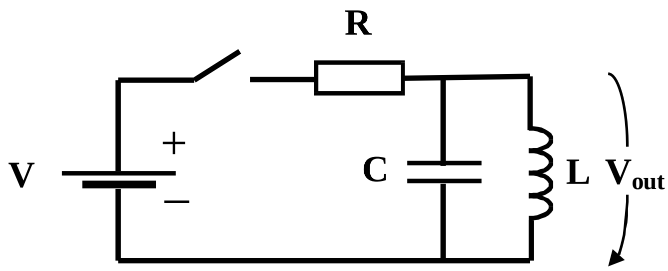


11. At times t < 0 the switch is open and the capacitor is charged with 1V across its terminals. At t =0 the switch is closed. Show that the subsequent time response of the circuit is oscillatory and damped, and sketch (quantitatively) the response of the circuit for t > 0, given R=150Ω, L=10 mH, and C=10 nF.

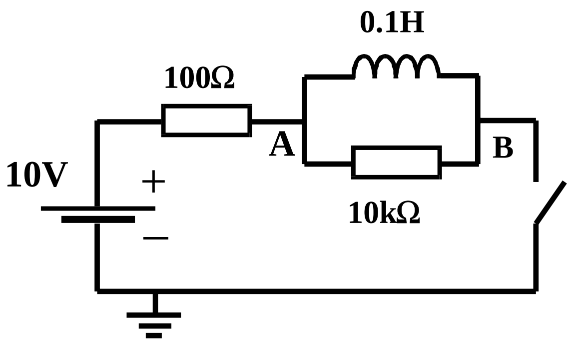
12. In the circuit of question 11, by making any necessary approximations (i.e. verify very light damping), calculate the energy dissipated in the circuit over a cycle, the energy stored in the circuit for the same cycle, and hence show that the Q- value of the circuit has a value of 6.7.

(Hint : If you don't approximate for light damping (small R), the problem may get tedious. When calculating the energy lost per cycle, you can (for example) integrate the power (V_R^2/R) over the first cycle, ignoring the damping term (i.e. the exponential decay). The energy stored at the beginning of the first cycle should be obvious to you).

13. In the RLC circuit shown, at time t=0 the switch is closed. By solving the appropriate differential equation, show that the voltage V_{out} is oscillatory and damped with an exponential decay-time constant given by t_0=2CR. If the component values are R=100 kΩ, C=0.01 μF and L=4 mH, show that the resonant frequency, ω_0, is equal to 1.6 x 10^5 rad/s.



14. The switch in the figure has been closed for a long time, ie. a steady state has been reached, it is then suddenly opened. Demonstrate that $V_A = V_B$ = 0V before the switch is opened, V_A = 10V, V_B = 1010V immediately after the switch is opened, then that V_B falls exponentially to 10V with a time constant of 10^{-5}s.



After many time constants have elapsed, the switch is again closed. Show that the voltage across the 100Ω resistor rises exponentially with time with a time constant of 1 ms.

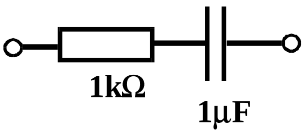
III. Complex Impedances and Response of Linear Circuits to AC.

15. RMS-values of voltages:

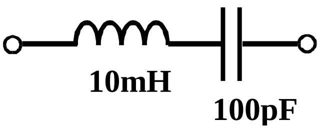
a) What is the RMS-voltage of a constant voltage $V(t) = V_C$?
b) What is the RMS-voltage of a square wave between 0 and V volts ?
c) What is the RMS-voltage of a square wave between –V/2 and +V/2 volts ? Why is the answer to b) and c) different ?
d) Calculate the RMS-value of a saw-tooth voltage $V(t) = V_0 \times t/T$ of period 2T.

16.

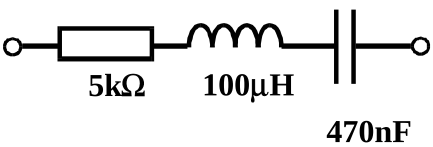
a)



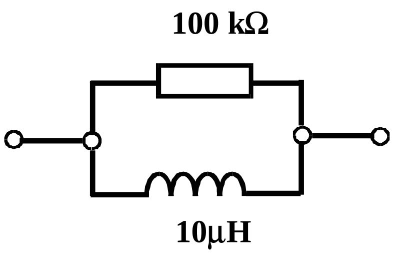
b)



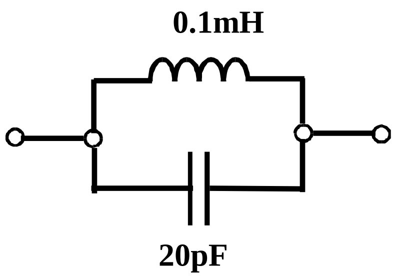
c)



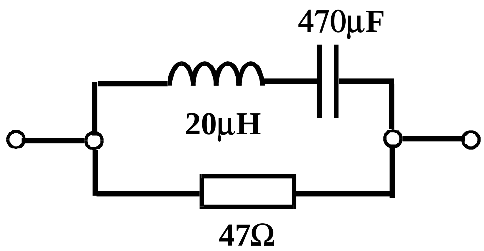
d)



e)



f)



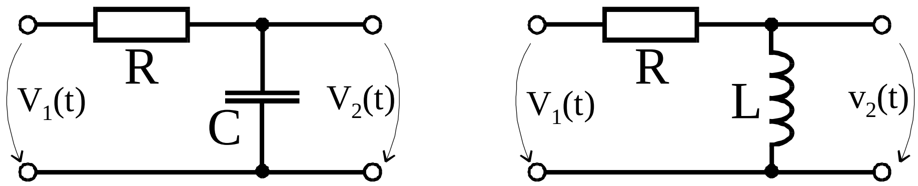
For each of the above networks:

i) Calculate the (complex) impedances;

ii) A voltage of the form $V=V_0\sin(\omega t)$ is applied across the network. Evaluate the phase shift between the voltage and the current flowing through the network. State whether the current leads or lags;

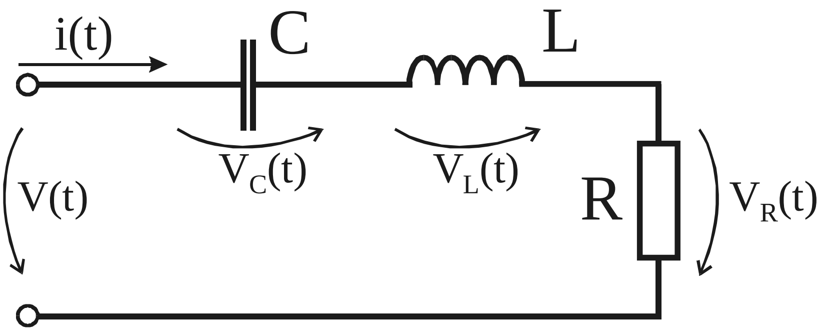
iii) Calculate the peak voltage drop across the resistors in circuits a) and d), the capacitors in circuits b) and e), and the inductors in circuits c) and f). Take the driving voltage to be of the form $V=V_0\sin(\omega t)$ with V_0=10V and frequency $(\omega/2\pi)$ =10 kHz.

17. The two circuits below are driven by sinusoidal input voltages $V_1 = V_0 \sin \omega t$.



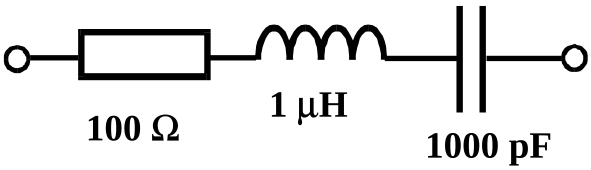
a) Draw the phasor diagrams of all voltages.
b) With the help of the phasor diagrams determine the ratio of the amplitudes of voltages V_2 and V_1 as well as the phases of the output voltages V_2 for the two circuits.
c) Sketch the ratios of amplitudes and the phases as a function of normalised frequency $x = \omega/\omega_0$, with $\omega_0 = 1/RC$ and $\omega_0 = R/L$ respectively for the two circuits.

18. An AC current $I(t) = I_0 \sin(\omega t)$ is flowing through the circuit below made from a series combination of capacitance C, inductance L and resistance R. The numerical values are: C = 10nF, L = 0.2mH, R = 100Ω, I_0= 0.1A and $\omega = 10^6 s^{-1}$.

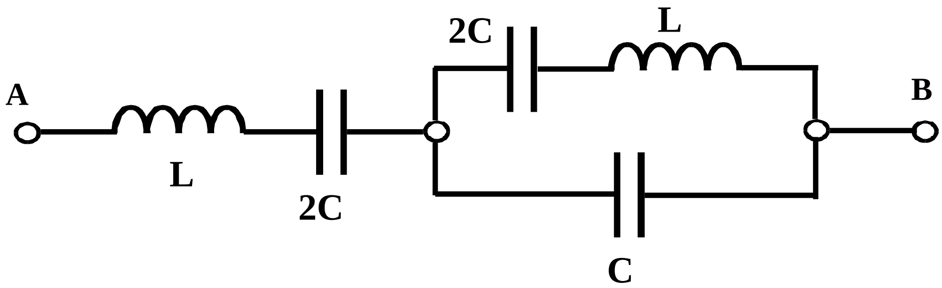


a) Find the amplitudes and phase angles of the voltages across the capacitor, inductor and resistor $V_C(t)$, $V_L(t)$ and $V_R(t)$ and of the total voltage V(t).
b) Find the total power dissipated by the circuit.
c) Find expressions for the energy contents $W_L(t)$ and $W_C(t)$ of the inductor and capacitor.
d) What are the maximum values for these energies?
e) Sketch $W_L(t)$ and $W_C(t)$.
f) For which value of L (provided all other values remain constant) would the sum of $W_L(t)$ and $W_C(t)$ be constant?

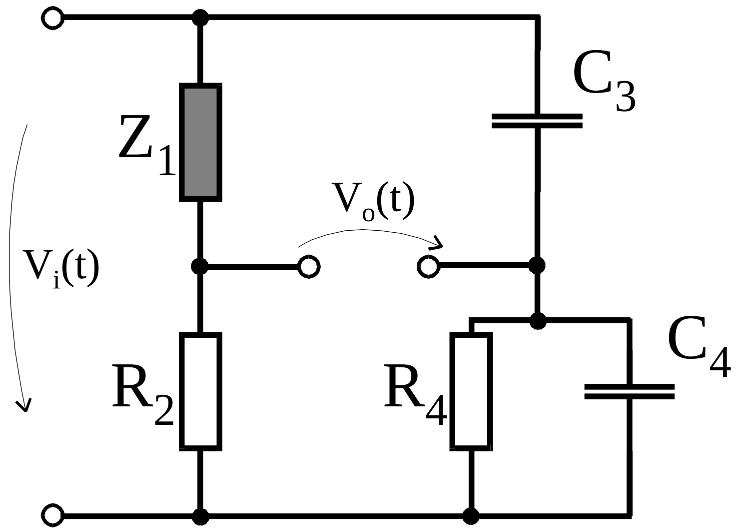
19. At what frequency does the network below have its minimum impedance ?
If the driving voltage is of the form $V=V_0\sin(\omega t)$ with $V_0=10V$, show that the voltage across the capacitor at the frequency of minimum impedance is 3.16 V.



20. Show that there are two frequencies, $\omega_1 = \sqrt{(5/(2LC))}$ and $\omega_2 = \sqrt{(1/(2LC))}$, for which the impedance between points A and B in the network below is zero.

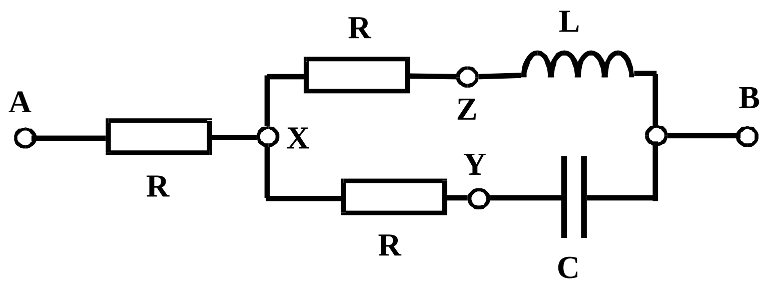


21. In the following bridge circuit R_2, C_3, R_4 and C_4 are fixed and Z_1 is variable.



a) Find the complex value of Z_1 for which the bridge is balanced.
b) Given that Z_1 is the parallel combination of a resistor R_1 and a capacitor C_1, for which values of R_1 and C_1 is the bridge balanced?
c) When Z_1 is a series combination of R_1 and C_1 what values must they have to balance the bridge?

22. A voltage $V_{AB}=V_0\cos(\omega t)$ is applied between points A and B in the circuit below. C and L have the values $(\omega R\sqrt{3})^{-1}$ and $(R\sqrt{3})/\omega$ respectively.



i) Show that the total impedance between A and B is 3R;
ii) Verify that voltages of equal amplitude are developed between the points X-A, the points X-Y and the points X-Z;
iii) Show that the phases of these three voltages relative to V_{AB} are 0, $+\sqrt{3}$ and $-\sqrt{3}$.

23. In the circuit below, show that the amplitude of the voltage V_{XY} between points X and Y is independent of R, and show that if $R=1/\omega C$, the phase of V_{XY} with respect to the applied voltage V is $\pi/2$.