Problem Class

The following problems will worked through to show an example solution. You will get maximimum benefit from this class if you have tried to solve these problems for yourself ***before*** coming to the class.

1. Explain what is meant by the *Fraunhofer condition* for diffraction. [4]

An aperture lies in the plane $z = 0$ and has amplitude transmission function $T(y)$ independent of x. It is illuminated by coherent monochromatic light of wavelength λ at normal incidence. Show that, in the Fraunhofer case, the diffracted intensity is proportional to $|A(\theta)|^2$, where

$$A(\theta) = \int_{-\infty}^{\infty} T(y) \exp[-\mathrm{i}(2\pi/\lambda) y \sin\theta] \, \mathrm{d}y.$$

[6]

An aperture consists of two long slits parallel to the x axis. The centres of the slits are separated by a distance d, and each slit has a width a. Write down $T(y)$ for this aperture and use it to calculate $A(\theta)$. Hence sketch the diffracted intensity as a function of $\sin\theta$. [8]

A two-slit aperture of this type has $d = 0{\cdot}1$ mm and $a \ll d$. It is illuminated at normal incidence with light of wavelength $\lambda = 650$ nm. A lens of focal length 90 mm and diameter 10 mm is used to form an image of the aperture on a screen situated 1 m behind the aperture. Calculate the two possible positions of the lens, and explain which one of these positions gives the sharper image of the aperture. [7]

2. Describe with the aid of a labelled diagram the important features of a transmission-grating spectrograph. [6]

A diffraction grating has a line spacing of d and a total of N lines. The grating is illuminated at normal incidence by light of wavelength λ. Show that the minima adjacent to a principal maximum in the diffracted intensity are separated in angle by

$$\Delta\theta = \frac{2\lambda}{Nd\cos\theta_m} \ ,$$

where θ_m is the angle of diffraction for the mth order. [6]

A transmission-grating spectrograph has a grating with 500 lines mm^{-1} and total width 50 mm. The spectrograph is used at normal incidence to examine a spectral line with a wavelength of 540 nm consisting of two components separated by 0·02 nm. Determine whether the two components can be resolved with the grating provided. [6]

The resolution is also affected by the size of the entrance slit of the spectrograph. Assuming that the entrance slit has width 0·01 mm and the collimator lens has focal length 0·5 m, estimate the contribution of the entrance slit to the resolution and hence discuss whether the two components of the spectral line can be resolved in practice. [7]

3. Describe how a Michelson interferometer may be used to measure the wavelength of monochromatic light. [5]

A Michelson interferometer is used to examine light from a vapour of atoms. The intensity of light at the centre of the circular interference pattern is monitored as a function of the mirror displacement x, and found to have the form:

$$I(x) = 3I_0 + [2I_0 \cos \alpha x + I_0 \cos \beta x] \exp(-\gamma^2 x^2)$$

where $\alpha = 1{\cdot}6405 \times 10^7\,\mathrm{m}^{-1}$, $\beta = 1{\cdot}6319 \times 10^7\,\mathrm{m}^{-1}$ and $\gamma \ll \alpha, \beta$. Show that the spectrum consists of two components and find

(a) their relative intensity and their mean wavelength;

(b) the minimum distance x required to resolve the two spectral lines. [10]

Make a rough sketch of the interferogram and suggest a physical origin for the term $\exp(-\gamma^2 x^2)$. [5]

How could light from a helium-neon laser which provides a standard wavelength known to one part in 10^8 be used to improve the accuracy of the wavelength measurement? [5]

4. Derive an expression for the intensity transmitted through a Fabry-Perot etalon with mirrors of reflectivity R, as a function of the mirror separation d, the wavelength λ and the angle of incidence θ. Define the *finesse* and *free-spectral range*. [10]

A short etalon with mirrors of reflectivity $R = 0{\cdot}75$ is used as an interference filter transmitting infrared radiation of wavelength $4{\cdot}3\,\mu$m, at normal incidence. The full-width at half maximum of the transmitted intensity is about $\Delta\lambda = 0{\cdot}2\,\mu$m. Any phase change of the light on reflection may be neglected.

(a) Calculate the spacing of the mirrors.

(b) Calculate the change in mirror spacing required to shift the centre wavelength to $4{\cdot}5\,\mu$m.

(c) The mirror spacing is fixed at the value which gives maximum transmission at $4{\cdot}3\,\mu$m for normal incidence and the filter is tilted so that the angle of incidence becomes $17{\cdot}5^\circ$. Calculate the wavelength of the light transmitted.

(d) Find a wavelength longer than $4{\cdot}3\,\mu$m for which the intensity transmitted at normal incidence is a maximum, and a wavelength shorter than $4{\cdot}3\,\mu$m which also gives a maximum. Comment on how the transmission of other wavelengths affects the usefulness of the filter. [15]

5. Explain what is meant by *polarized light* and describe the different types of polarization that are possible. Describe briefly the principle of operation of a polarizing device made from a birefringent material. [8]

A beam of light is elliptically polarized with the major axis vertical. The ratio of the major and minor axes of the ellipse is $a : b$. Explain how you would use a quarter-wave plate to obtain linearly polarized light, and determine the angle of the plane of polarization to the vertical. [6]

A beam of light consisting of a mixture of elliptically polarized and unpolarized light is passed through a linearly polarizing filter. The maximum of the transmitted light intensity is observed when the transmission axis of the filter is vertical, and is twice the minimum intensity. In a second experiment, the beam is passed through a quarter-wave plate with the fast axis vertical followed by the polarizing filter. The maximum is now observed when the transmission axis is at $33{\cdot}21^\circ$ to the vertical. Calculate the ratio of the intensities of the polarized to unpolarized components of the light. [8]

How could the handedness of the elliptically polarized component be determined? [3]