Second Year Physics: Optics Problems P.Ewart

1. Use both the analytical and the phasor methods to compare the Young's slits patterns for the two cases (i) the usual arrangement of narrow slits (ii) with both slits still narrow, but one giving twice the amplitude of the other in the plane of observation. Sketch the two patterns using the same axes for each.

2. The single most important result in physical optics is that a collimated beam of ljght of width w has an angular divergence $\sim \lambda/w$ or greater. Use this result to find the rough size of the smallest patch of light one can get on the moon when a beam of diameter ~20 mm from a ruby laser is pointed at it in lunar ranging experiments. Find the new minimum size when the laser beam is expanded so as to fill the aperture of a Newtonian telescope having a mirror diameter of 3 m. [The wavelength of the light from a ruby laser may be taken to be 700 nm and the distance from the earth to the moon to be 390,000 km]

3. A plane transmission grating consists of a large number N of narrow slits separated by equal distances d. Show that the intensity of radiation, $I(\theta)$, diffracted at an angle θ by the grating when it is illuminated by monochromatic light of wavenumber $\overline{\nu}$ at normal incidence is given by

$$I(\theta) = \frac{I(0)}{N^2}\frac{\sin^2(Nud/2)}{\sin^2(ud/2)} \qquad \text{where } u = 2\pi\overline{\nu}\sin\theta.$$

4. The centremost $N/2$ slits of a diffraction grating are obscured by an opaque strip. Show that the angular size of a principal maximum (i.e., the separation of the minima either side of it) is 2/3 that displayed by the unobscured grating. [This can be done in several different ways, the simplest being a phasor diagram.]

5. Explain what is meant by Fraunhofer diffraction. [Your answer should contain no reference to parallel light or observation at infinity.] Why is it so important in practice?

Plane waves of monochromatic light of wavelength λ fall on a slit of width a. Estimate the minimum distance from the slit of the plane of observation if the observed diffraction pattern is to fulfil the Fraunhofer condition.

Derive an expression for the angular distribution of the intensity in the Fraunhofer pattern, and sketch it.

6. The width of a slit, roughly 100 microns, is to be measured by studying its Fraunhofer diffraction pattern. Describe how you would carry out this measurement, including in your account reasoned order of magnitude estimates. Mention also all of the important experimental components, and how they would be used.

As part of the driving test (when I took it, anyway) one is required to read a car number plate at a stated distance. How far would this have to be before Fraunhofer diffraction might be expected to cause difficulties?

7. The grating of question 3 is removed, so that where the slits were is now an open aperture with the width of the original grating. Show that at small angles the form of the intensity distribution remains unchanged. Why are the two patterns so similar? Estimate, with reasons, the angle at which they begin to be significantly different.

[Hint: compare the phasor diagrams for N slits and for a rectangular aperture of width Nd for the same angle of observation.]

8. A pair of slits, each of width a, and separated centre-to-centre by a distance d, is illuminated normally by a plane monochromatic wave. Light transmitted by the slits is observed in the focal plane of a lens of focal length f. Draw sketch graphs showing the variation of the observed intensity in the focal plane of the lens for the cases (i) $d \gg a$ (ii) $d = 3a$ (iii) $d = 2a$. In each case your sketch graph should show quantitatively the scales being used on the co-ordinate axes.

Explain what happens to the general features of these patterns as more slits are added (keeping the same values of d and a in each case).

9. A telescope in a satellite is directed towards (a) a star (b) an extended nebula of uniform brightness. In each case explain how the illumination (flux falling on unit area) of the image in the primary focal plane will depend on the focal length and the diameter of the telescope mirror.

An earth-based telescope with a mirror of diameter 2.4 m can record the image of a faint star in 1 hour. A telescope placed in space (above the atmosphere) has a mirror of the same focal length as that of the earth-based instrument, but its diameter is 1.2 m. Atmospheric turbulence is assumed to place a limit of 0.25 seconds of arc on the angular resolution obtainable by earth-bound telescopes, and the mean wavelength of the radiation detected is 550 nm. Estimate the time required by the telescope placed in space to record the same star.

[Note that telescopes, in their usual configuration do not form an image directly but require a further lens e.g. the eye or a camera lens to form the image. In practice, the brightness of the final image will depend on the overall focal length and aperture of the system. For the purposes of this question consider only the brightness of the image formed by the objective.]

Estimate the approximate linear dimensions of the smallest feature on the planet Mars which could be resolved by each of the above telescopes when Mars is at a distance of 9×10^7 km.

10. A laser produces a beam of coherent light of wavelength λ with plane wave-fronts travelling along z. The amplitude of light transmitted by a transparency in the x, y plane is independent of y and varies with x as $A[1 +$

$\cos(2\pi x/d)]$ where A is a constant and $d > \lambda$. Show that three beams emerge from the transparency, and find their angles to the z-axis.

An ordinary transmission diffraction grating illuminated normally by plane monochromatic light of wavelength 465 nm is examined under a microscope whose objective subtends an angle of 0.5 radians at the grating. What is the smallest grating spacing that can be seen in the image?

11. A monochromatic plane wave of wavelength λ is incident normally on a screen which transmits a fraction $\phi(x)$ of the amplitude incident upon it, where x is measured across the screen and $\phi(x)$ is given by the Gaussian function

$$\phi(x) = \exp\left(\frac{-x^2}{2d^2}\right)$$

where d is a constant. Sketch this function. A lens of focal length f is placed behind the screen. Derive an expression for the intensity distribution in the focal plane of the lens.

[The point being that you get another Gaussian. This is because the amplitude distribution in the Fraunhofer pattern is the Fourier Transform of the transmission function of the screen, and the FT of a Gaussian is another Gaussian. Taking the modulus squared still keeps the Gaussian form. But you don't need to use FT's explicitly to do the question.]

Explain briefly why the use of a screen of this type can be helpful in practice. Suppose the screen to be moved to a position beyond the lens, halfway between the lens and its focal plane. What would the intensity distribution then be in the focal plane? Would the pattern still be correctly described as Fraunhofer?

Finally, the lens is removed altogether. and the pattern observed on a wall. How far from the screen must the wall be for the pattern to fulfil the Fraunhofer condition?

$$\left[\int_{-\infty}^{\infty} \exp(-\alpha x^2 + i\beta x)dx = \left(\frac{\pi}{\alpha}\right)^{1/2} \exp\left(\frac{-\beta^2}{4\alpha}\right)\right]$$

12. Give a brief account of the principal features of a Micheison interferometer used with visual observation and illuminated by an extended monochromatic source. Explain what configurations of the mirrors will give (i) circular fringes (ii) straight, equally spaced fringes. State where the fringes are localized and why. Why in case (ii) does the mirror spacing have to be small?

The instrument is set up to give circular fringes and illuminated with light in the spectral range 800 -900 nm from an atomic caesium (^{133}Cs) discharge source. The intensity $I(x)$ at the centre of the interference pattern is recorded as a function of the distance x of the moving mirror from that corresponding to zero path difference. It is found to have the form

$$I(x) = 3I_0 + 3I_0 \cos K_1 x \cos K_2 x - I_0 \sin K_1 x \sin K_2 x$$

for $0 \leq x \leq 5$ mm, where $K_1 = 1.44 \times 10^7 \mathrm{m}^{-1}$, $K_2 = 3.48 \times 10^5 \mathrm{m}^{-1}$, and I_0 is a constant.

Show that this can be written as the sum of the patterns due to two monochromatic spectral components. Hence determine:

(a) the mean wavenumber, $\overline{\nu}$, of the two caesium lines in this wavelength range

(b) their wavenumber separation, $\Delta\overline{\nu}$

(c) their relative intensities.

When the interference pattern is recorded over a larger range of path difference it is found that the periodic terms in the pattern are in fact multiplied by a function

$$f(x) \propto \exp(-K_3^2 x^2)$$

where $K_3 = 5.02 \mathrm{~m}^{-1}$. Make a rough (but reasonably realistic) sketch of the interference pattern as a function of x over a range large enough to show the effect of this term.

[It comes about because of Doppler broadening of the spectral lines. When you know enough about this effect, come back to this question and work out the Doppler width of the lines and hence the temperature of the source. If you want to try a simplified version of this experiment, it is on the Optics and Atomic Physics Practical course.]

13. Give a diagram of an apparatus, and explain how you would use it, to determine the wavelength of a spectral line from a given source, known to be in the region of 500 nm.

14. Describe, with the aid of a labelled diagram including all essential components, how a Fabry-Perot etalon can be used to measure the separation of components in one of the spectral lines emitted by a discharge lamp.

How would you choose the plate separation for a given investigation? [One then has also to choose the plate reflectivity, to give the required resolving power.]

A certain spectral line is known to consist of two equally intense components with a wavenumber separation $\Delta\overline{\nu}$ less than 20 m^{-1}. The Fabry-Perot fringes produced by this line are photographed, using a plate separation of 25 mm. The diameters of the smallest rings are found to be, in mm :

$$1.82,\ 3.30,\ 4.84,\ 5.57,\ 6.60,\ 7.15.$$

Explain why this experiment does not allow $\Delta\overline{\nu}$ to be determined uniquely, but gives two possible values. What are they? Suggest a further experiment which could be carried out to resolve the ambiguity.

15. Explain what is meant by the *instrumental width* associated with a spectroscopic device. Why is the instrumental width such an important property?

The rest of this question takes you through a "sledgehammer" calculation of the resolving power of a grating using the instrumental width.

(a) A grating with N slits of spacing d is illuminated normally with monochromatic light of wavelength λ, and the p^{th} order principal maximum is observed at an angle θ_p The intensity at angle θ is given by

$$I(\delta) = \frac{\sin^2\left(\frac{N\delta}{2}\right)}{\sin^2\left(\frac{\delta}{2}\right)} \qquad \text{where } \delta = \frac{2\pi}{\lambda} d \sin\theta$$

Show from this equation that adjacent minima fall at angles $\theta_p \pm \delta\theta_p$ where

$$\delta\theta_p = \frac{\lambda}{N d \cos\theta_p}$$

(b) Show that the dispersion $\frac{d\theta}{d\lambda}$ of the grating in the p^{th} order of interference is

$$\frac{d\theta}{d\lambda} = \frac{p}{d\cos\theta}.$$

(c) Hence show that the instrumental width of the grating (in terms of wavelength) is

$$\Delta\lambda_{INST} = \frac{\lambda}{Np}$$

(d) Deduce that the resolving power of the grating is Np.

Does the resolving power of a grating change if it is used in a chamber containing a gas of high refractive index?

16. Prove that the transmission function of a Fabry-Perot interferometer for monochromatic light of wavelength λ incident at an angle θ to the normal is

$$I(\phi) = I_0 \left[1 + \left(\frac{4F^2}{\pi^2}\right)\sin^2\left(\frac{\phi}{2}\right)\right]^{-1}$$

where $I(0)$ is the incident intensity, $\phi = (2\pi/\lambda)2nt\cos\theta$, t is the plate separation, and n is the refractive index of the gas between the plates. Obtain an expression for F in terms of the intensity reflectivity of the plates.

Find the instrumental width and resolving power of the etalon, as follows.

(a) Show that the values of ϕ at which the intensity has dropped to 50% of maximum near the p^{th} order of interference are

$$\phi = 2\pi\left(p \pm \frac{1}{2F}\right)$$

(b) Show that two spectral lines for which the p^{th} maximum of one falls on the $(p+1)^{th}$ maximum of the other are separated by $\delta\overline{\nu}$ given by

$$\delta\overline{\nu} = \frac{1}{2nt}$$

(c) From (a) and (b), show that the instrumental width $\Delta\lambda_{INST}$ (FWHM, full width at half maximum intensity) of the etalon is given by

$$\Delta\lambda_{INST} = \frac{1}{F} \times \frac{1}{2nt}$$

(d) Hence show that the resolving power is Fp.

17. A transmission grating spectrograph, working at normal incidence, employs a grating with 500 lines mm^{-1}. Calculate the minimum width the grating must have if it is to resolve two equally intense spectral lines with wavelengths spaced 0.04 nm apart at 600 nm. The collimating lens of the spectrograph has a focal length of 500 mm; estimate the maximum slit width one could use without significantly affecting the resolution of the lines.

How far would it be necessary to scan a Michelson Fourier Transform Spectrometer to resolve these lines?

Would a Fabry-Perot interferometer be a good choice of instrument to measure their separation?

18. (a) Why is reflection at Brewster's angle from a dielectric surface not a good all-purpose method of producing polarized light? Explain how polarized light can be produced using some form of crystal polarizer.

(b) Demonstrate analytically that unpolarized light can be converted to circularly polarized light by passing the radiation through a linear polarizer followed by a quarter-wave plate at a suitable orientation.

(c) A beam of initially unpolarized light passes through this system, and is then reflected back normally through it from a plane mirror. Describe the state of polarization of the beam in each part of its path through the system.

19. Show that any elliptically polarized light can be converted to plane polarized light using a suitably oriented quarter-wave plate. Is the converse also true? Show also that plane polarized light remains plane polarized after passing through a half-wave plate, but with the direction of the electric vector altered.

How would you use a half-wave plate to change this direction by 40^{o}?

20. (a) A beam of, initially unpolarized, light is incident normally on a piece of polarizing sheet (polaroid) **A**, the transmitted light then falling on a second similar sheet **B**. **A** and **B** are set with their axes at right angles. A third polaroid sheet **C** is placed between the first two with its axis at an angle ϕ to that of sheet **A**. What fraction of the incident energy passes through the system?

(b) In a Young's double slit experiment, both slits are covered by polarizing sheets, one with its axis horizontal, the other with its axis vertical. Explain why no fringes are seen in the plane of observation.

(c) Given a third polarizing sheet and a half-wave mica plate, explain where you would place them, and in what orientation, so that fringes would appear .

Could the pattern be produced using only the half-wave plate?