OPTICS

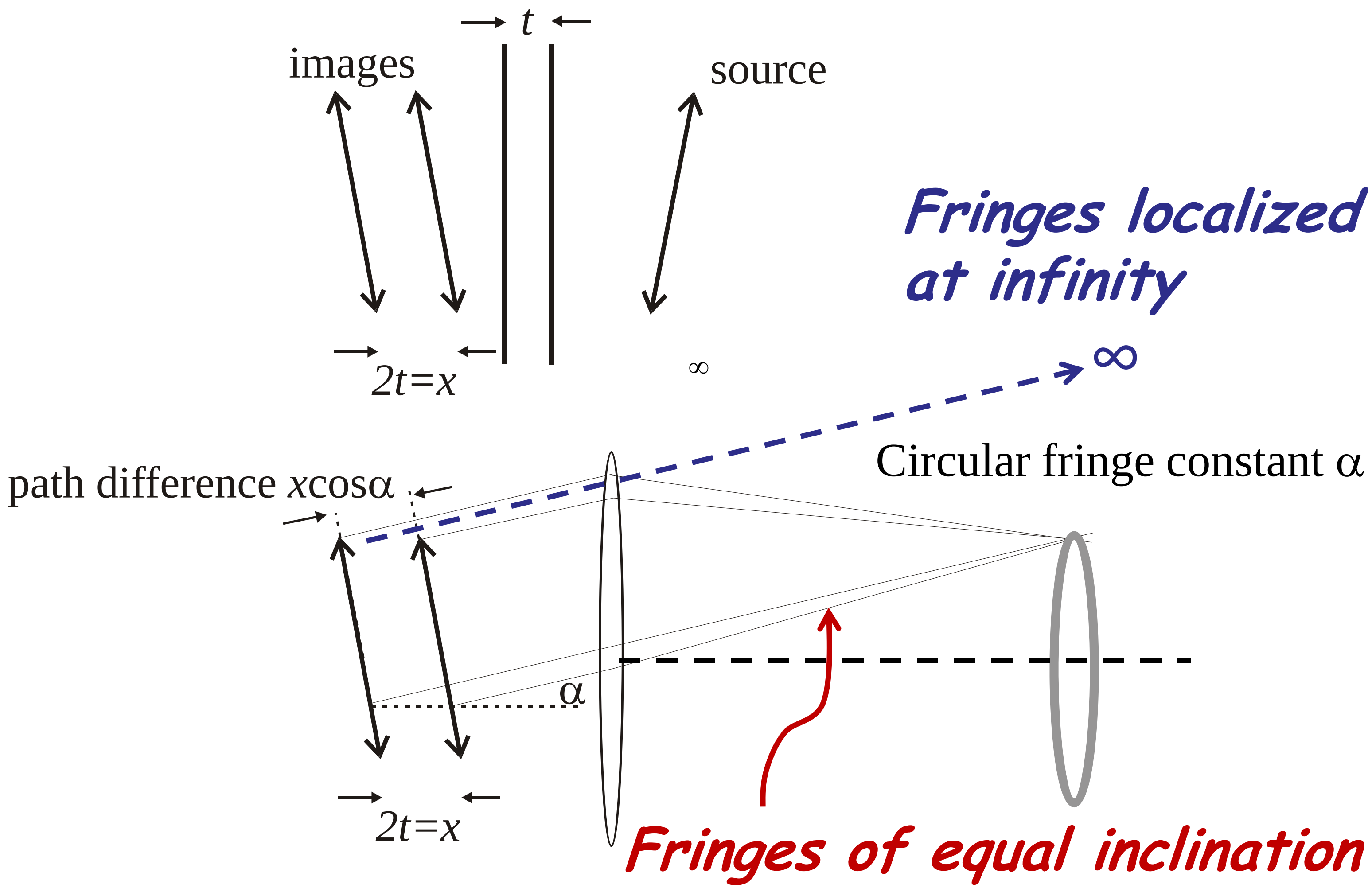
The science of light

P. Ewart

The story so far

- Geometrical optics: image formation
- Physical optics: interference → diffraction
- Phasor methods: physics of interference
- Fourier methods: Fraunhofer diffraction = Fourier Transform
  Convolution Theorem
- Diffraction theory of imaging
- Fringe localization: interferometers

Parallel reflecting surfaces   Extended source



Summary: fringe type and localisation

	Wedged	Parallel
Point Source	Non-localised Equal thickness	Non-localised Equal inclination
Extended Source	Localised in plane of Wedge Equal thickness	Localised at infinity Equal inclination

Optical Instruments for Spectroscopy

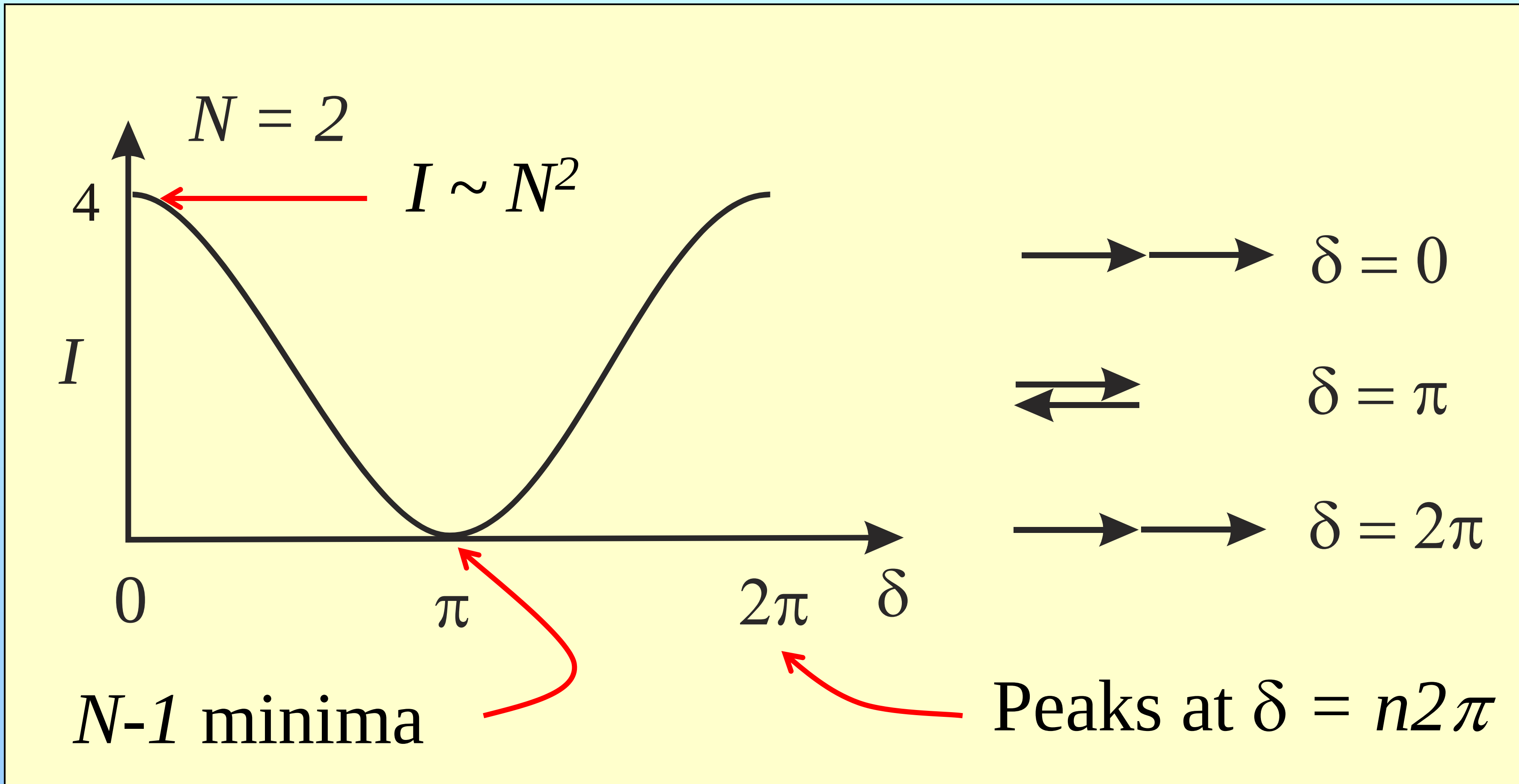
Some definitions:

- **Dispersion:** $d\theta/d\lambda$, angular separation of wavelengths
- **Resolving Power:** $\lambda/\Delta\lambda$, dimensionless figure of merit
- **Free Spectral Range:**
  extent of spectrum covered by interference pattern before overlap with fringes of same λ and different order
- **Instrument width:**
  width of pattern formed by instrument with monochromatic light.
- **Etendu:** or throughput –
  a measure of how efficiently the instrument uses available light.

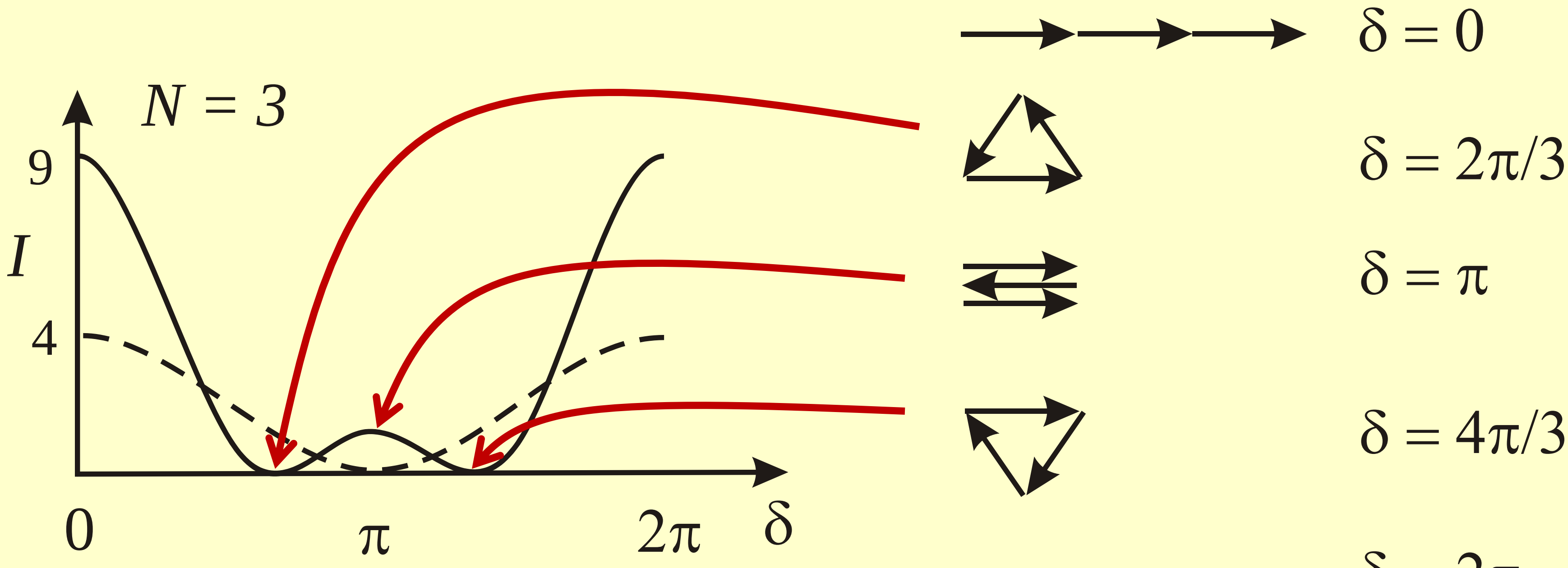
The Diffraction Grating Spectrometer

- Fringe formation – *phasors*
- Effects of groove size – *Fourier methods*
- Angular dispersion
- Resolving power
- Free Spectral Range
- Practical matters, blazing and slit widths

N-slit grating

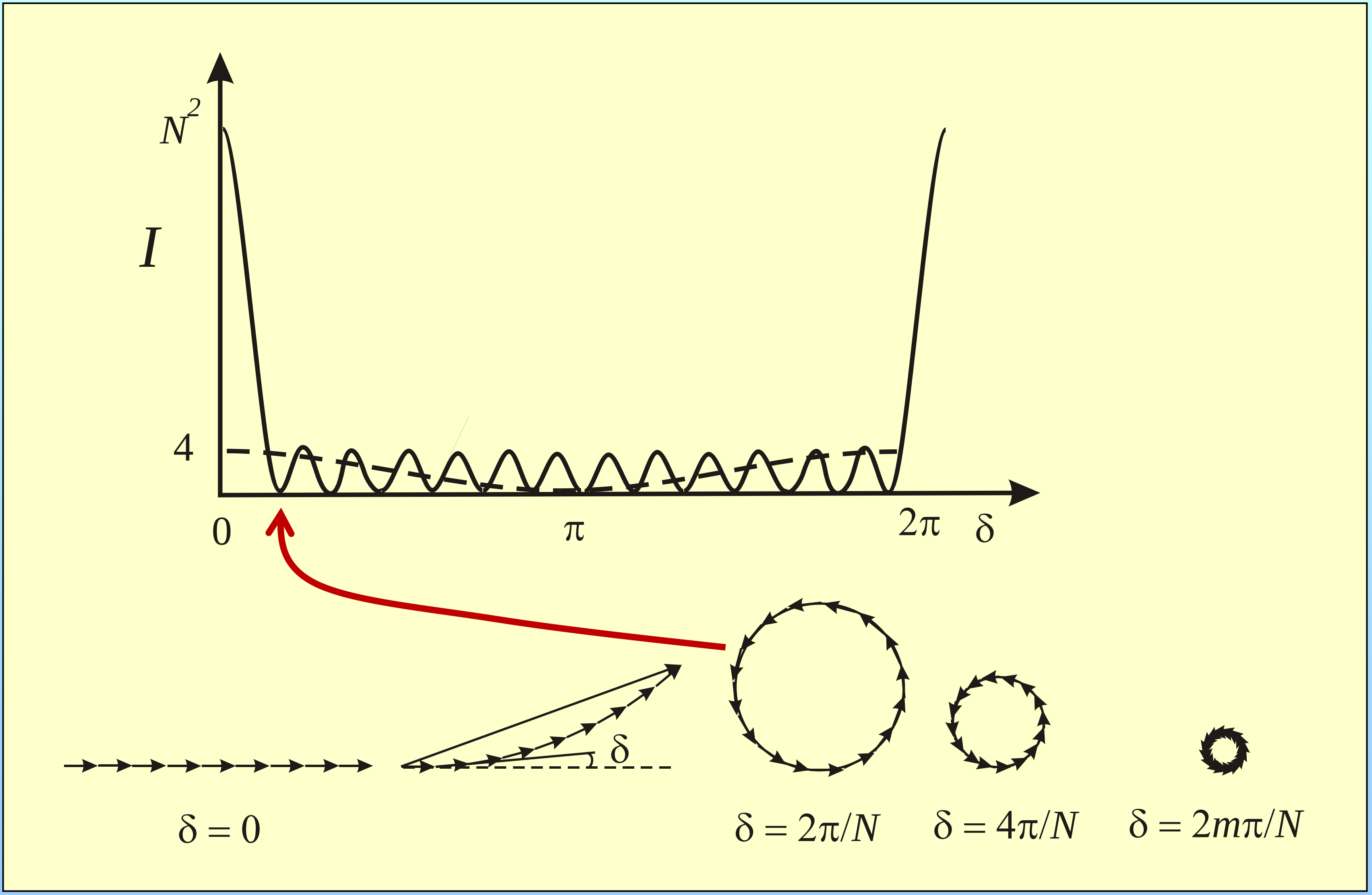


N-slit grating



Peaks at $n2\pi$, N-1 minima, $I \sim N^2$ 1^{st} minimum at $2\pi/N$

N-slit grating



Optical Instruments for Spectroscopy

Some definitions:

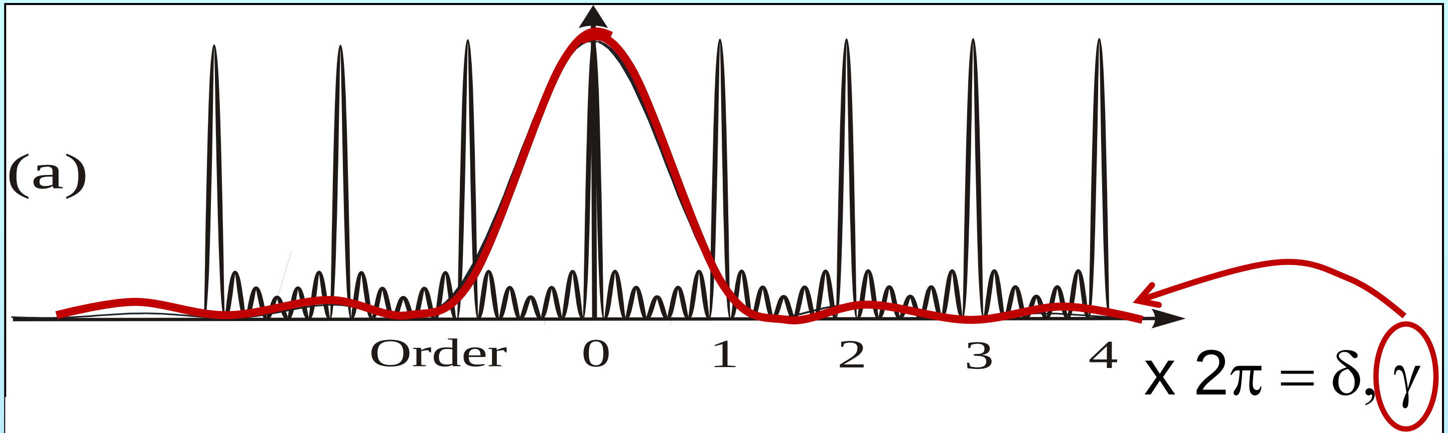
- Dispersion: $d\theta/d\lambda$, angular separation of wavelengths
- Resolving Power: $\lambda/\Delta\lambda$, dimensionless figure of merit
- Free Spectral Range:

  extent of spectrum covered by interference pattern before overlap with fringes of same λ and different order
- Instrument width:

  width of pattern formed by instrument with monochromatic light.
- Etendu: or throughput –

  a measure of how efficiently the instrument uses available light.

N-slit diffraction grating

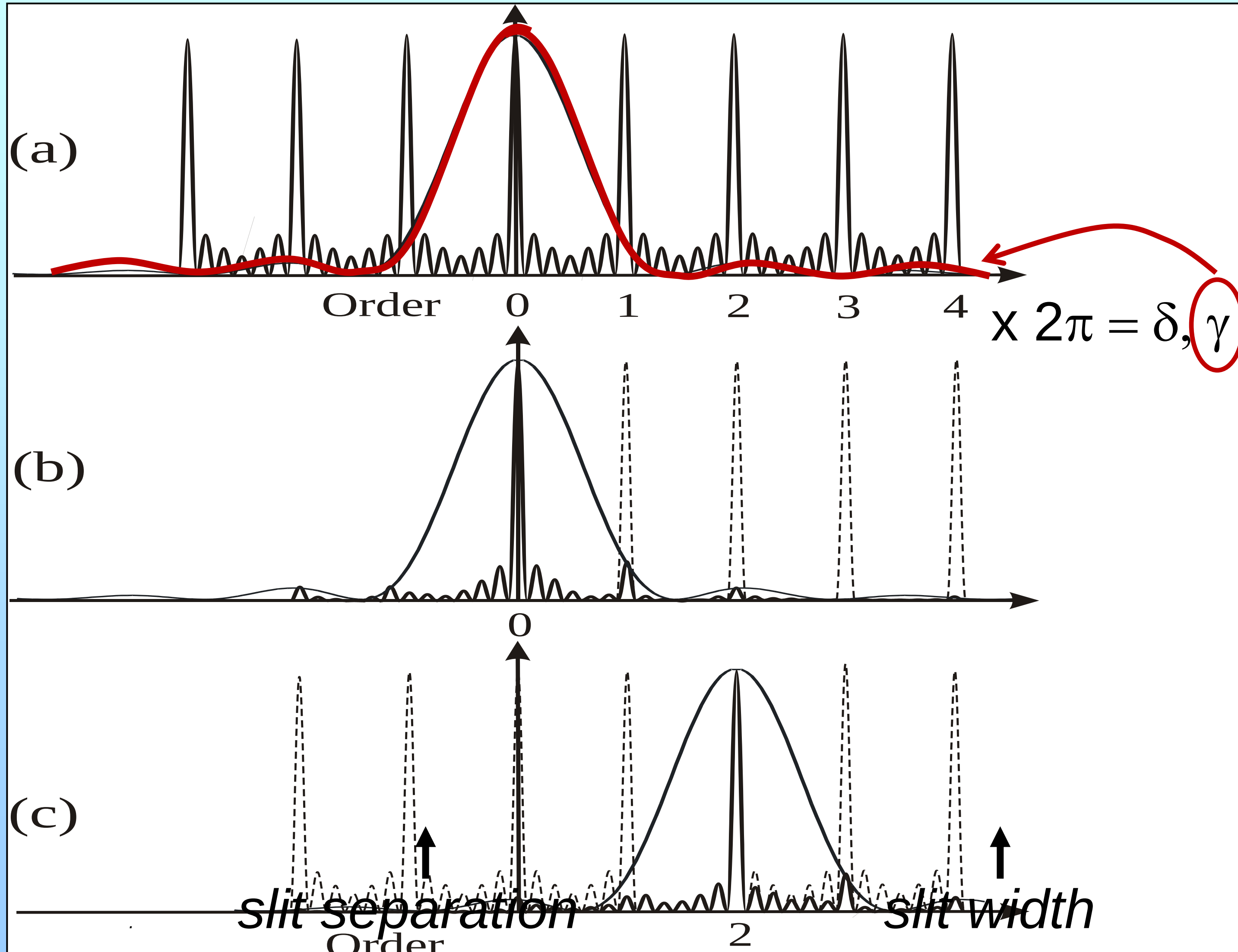


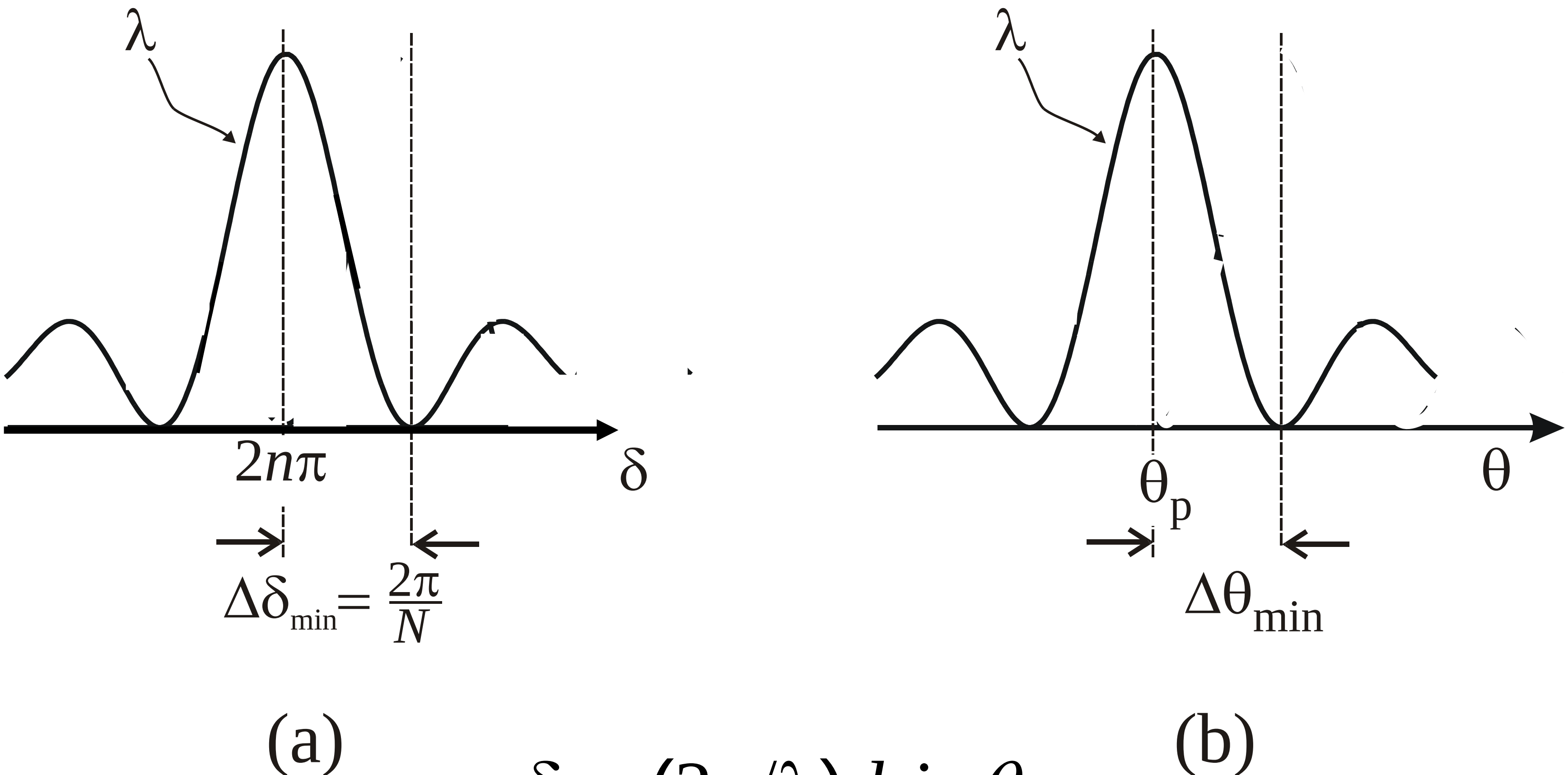
$$I(\theta) = I(0)\ \frac{\sin^2\{N\delta/2\}}{\sin^2\{\delta/2\}}\ \frac{\sin^2\{\gamma/2\}}{\{\gamma/2\}^2}$$

$$\delta = (2\pi/\lambda)\, d \sin\theta \qquad \gamma = (2\pi/\lambda)\, a \sin\theta$$

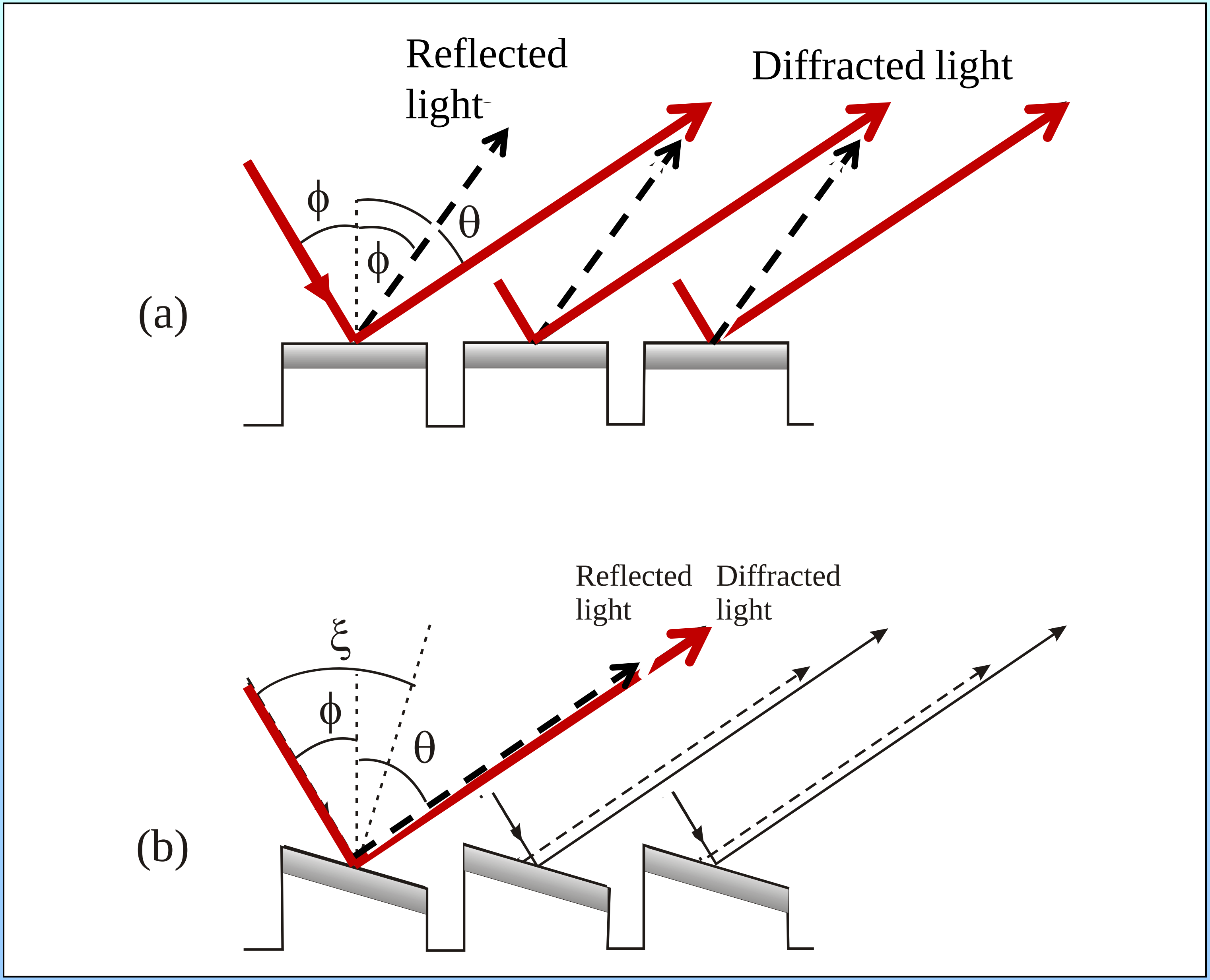
slit separation *slit width*

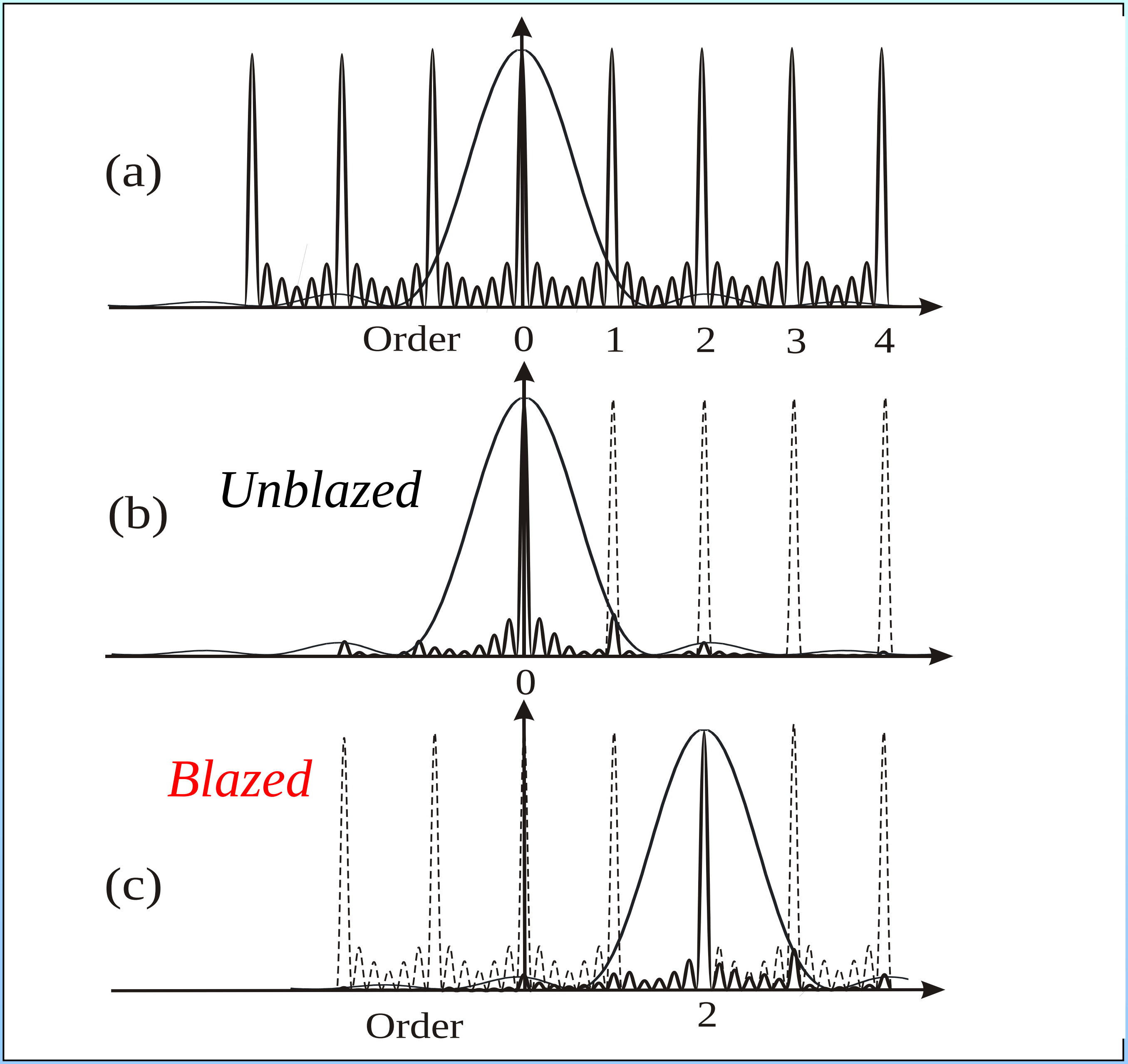
N-slit diffraction grating

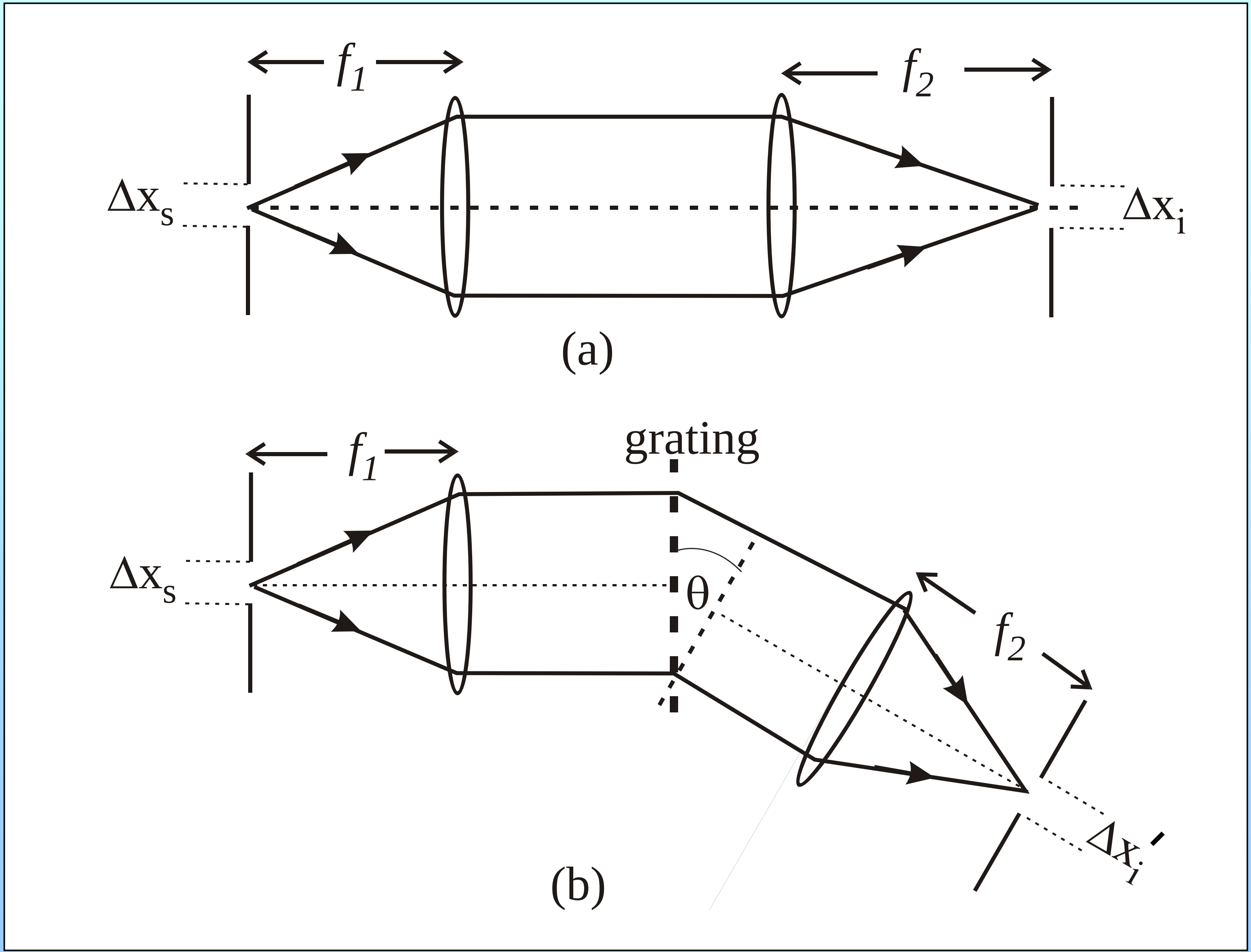




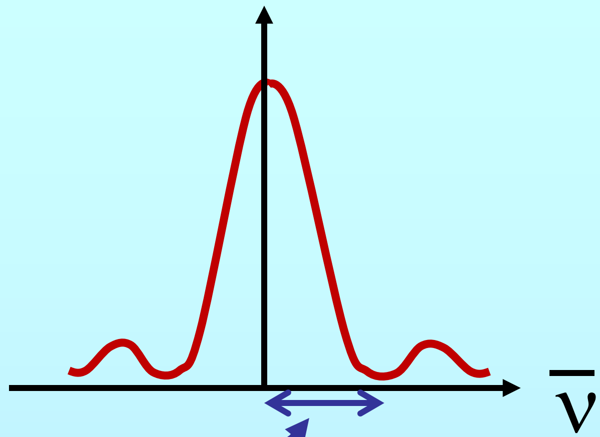
$$\delta = (2\pi/\lambda)d\sin\theta$$


Reflected light
Diffracted light
ϕ
θ
ϕ
(a)
Reflected light
Diffracted light
ξ
ϕ
θ
(b)


(a)
Order
0
1
2
3
4
Unblazed
(b)
0
Blazed
(c)
Order
2



f1
f2
Δxs
Δxi
(a)
grating
f1
Δxs
θ
f2
Δxi
(b)



Instrument function

Instrument width

(Grating spectrometer): $$\Delta\bar{\nu}_{Inst} = \frac{1}{2W}$$

$$\textbf{Instrument width} = \frac{1}{\textbf{Maximum path difference}}$$

Resolving Power

$$= \lambda/\Delta\lambda_{Inst}$$

$$= \bar{\nu}/\Delta\bar{\nu}_{Inst}$$

$$= 2W/\lambda$$

Maximum path difference in units of wavelength

Optical Instruments for Spectroscopy

- **Interference by division of wavefront:**

  The Diffraction Grating spectrograph

- **Interference by division of amplitude:**

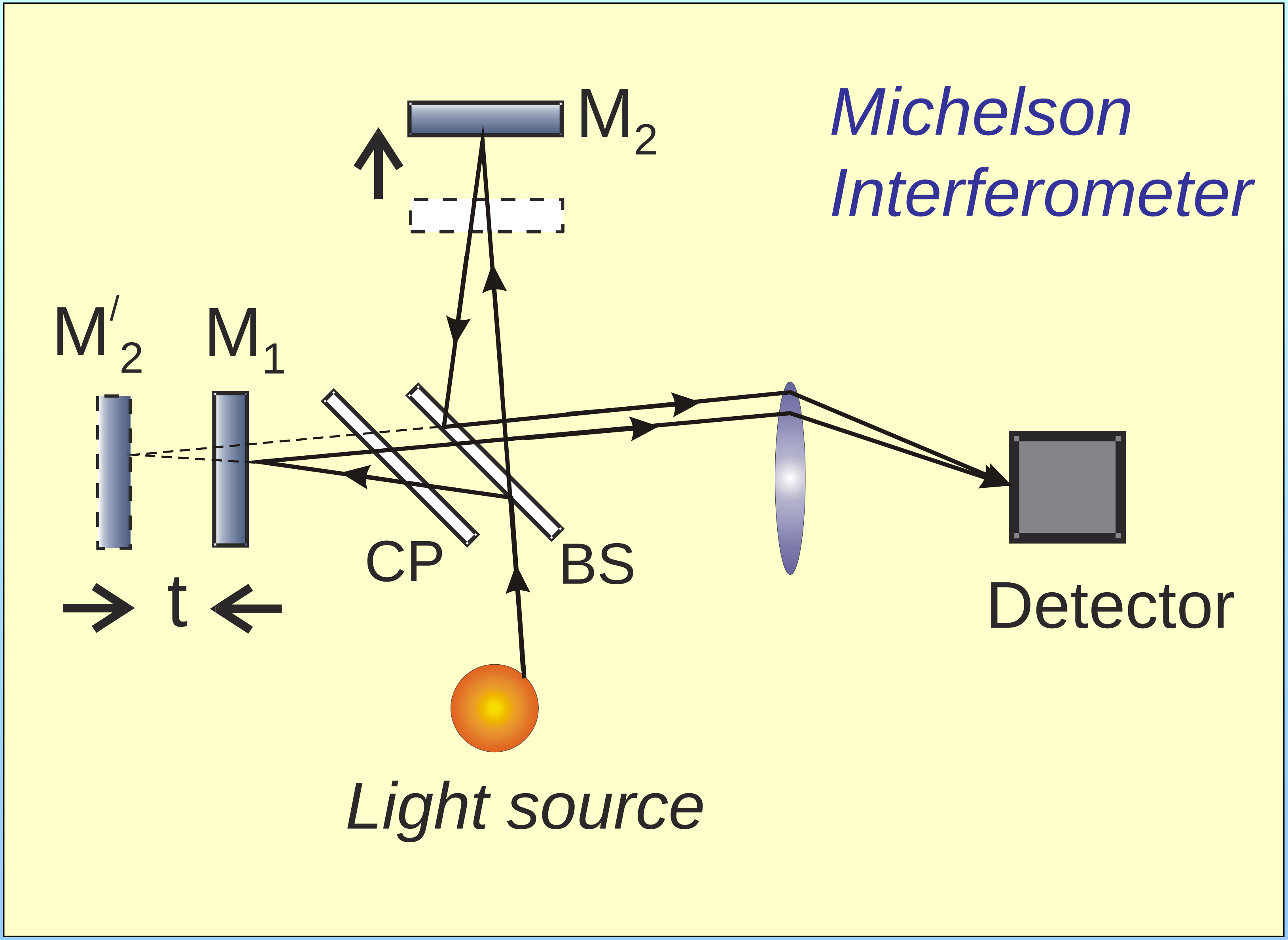
  2-beams -

  The Michelson Interferometer

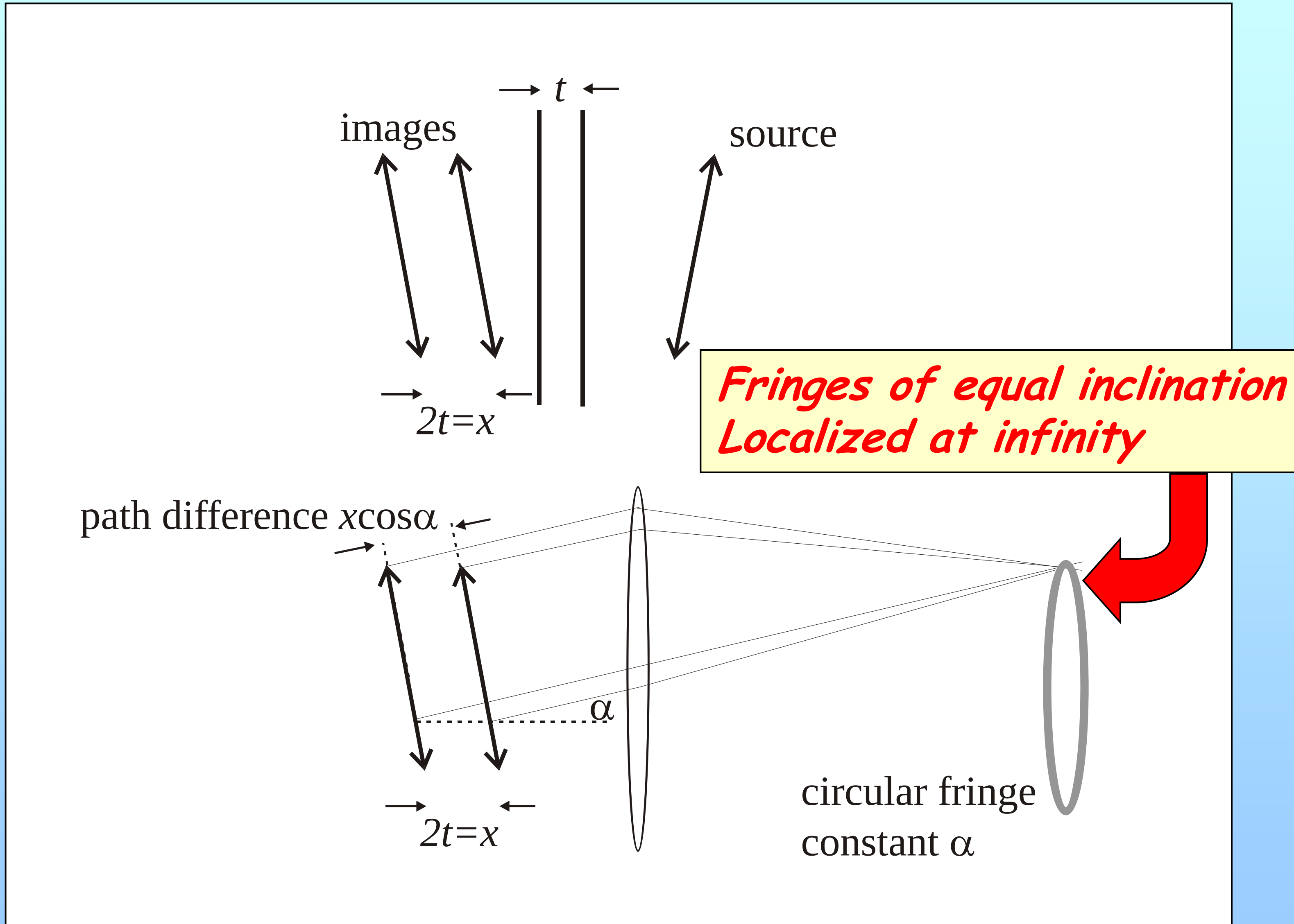
Albert Abraham Michelson
1852 –1931

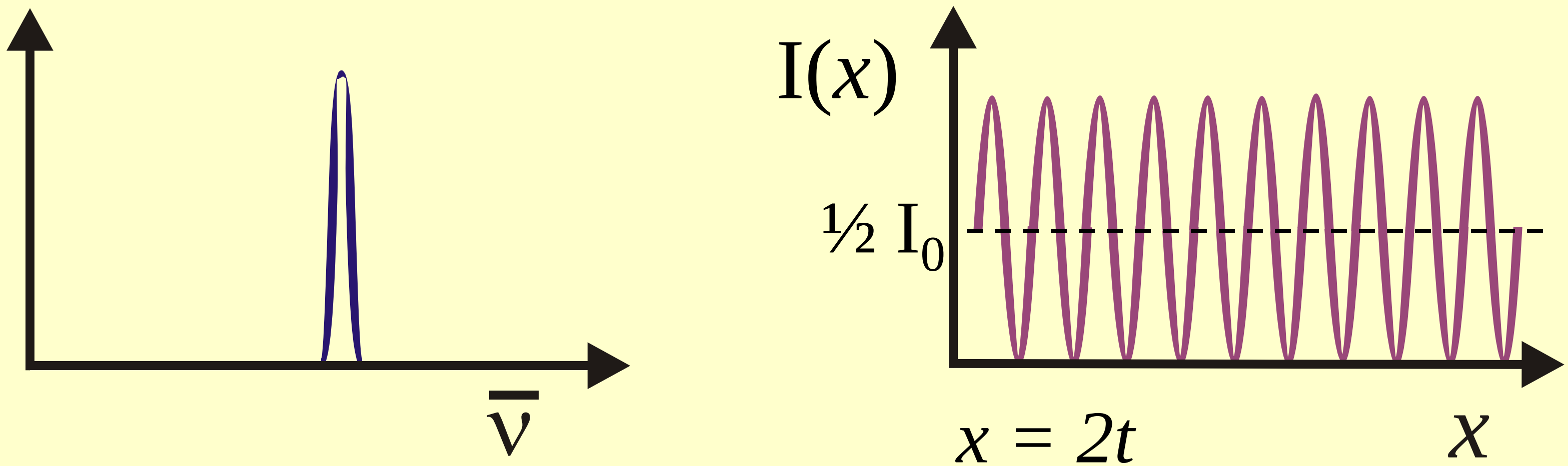
Michelson interferometer

- Fringe properties – interferogram
- Resolving power
- Instrument width



Michelson
Interferometer
M2
M'2
M1
CP
BS
t
Detector
Light source

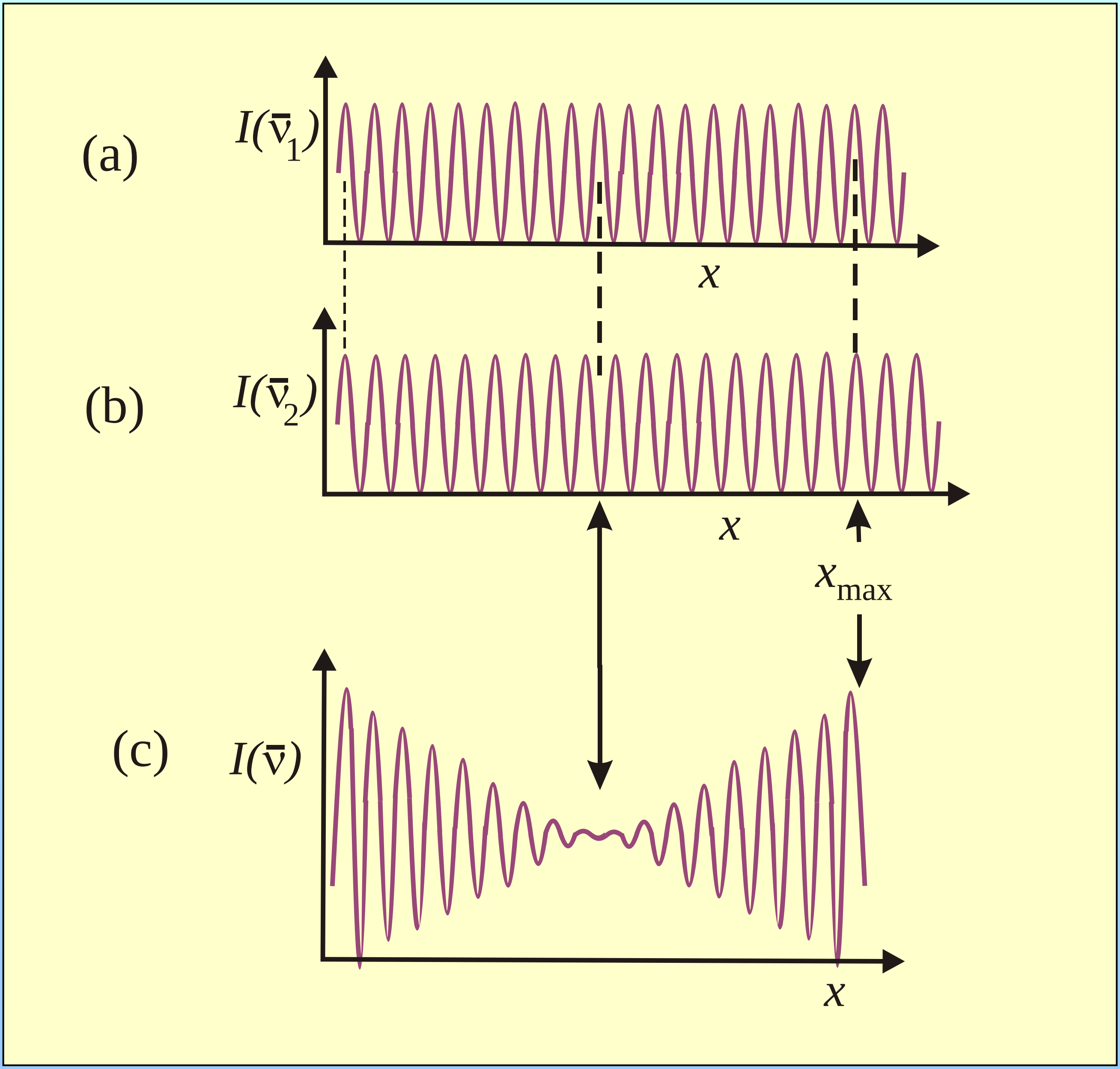

t
images
source
2t=x
Fringes of equal inclination
Localized at infinity
path difference xcosα
α
2t=x
circular fringe
constant α



Input spectrum

Detector signal
Interferogram

$$I(x) = \tfrac{1}{2} I_0 \left[1 + \cos 2\pi\bar{\nu}x \right]$$


(a)
I(v̄1)
x
(b)
I(v̄2)
x
xmax
(c)
I(v̄)
x

Michelson Interferometer

$$\Delta\bar{\nu}_{Inst} = 1/x_{\max}$$

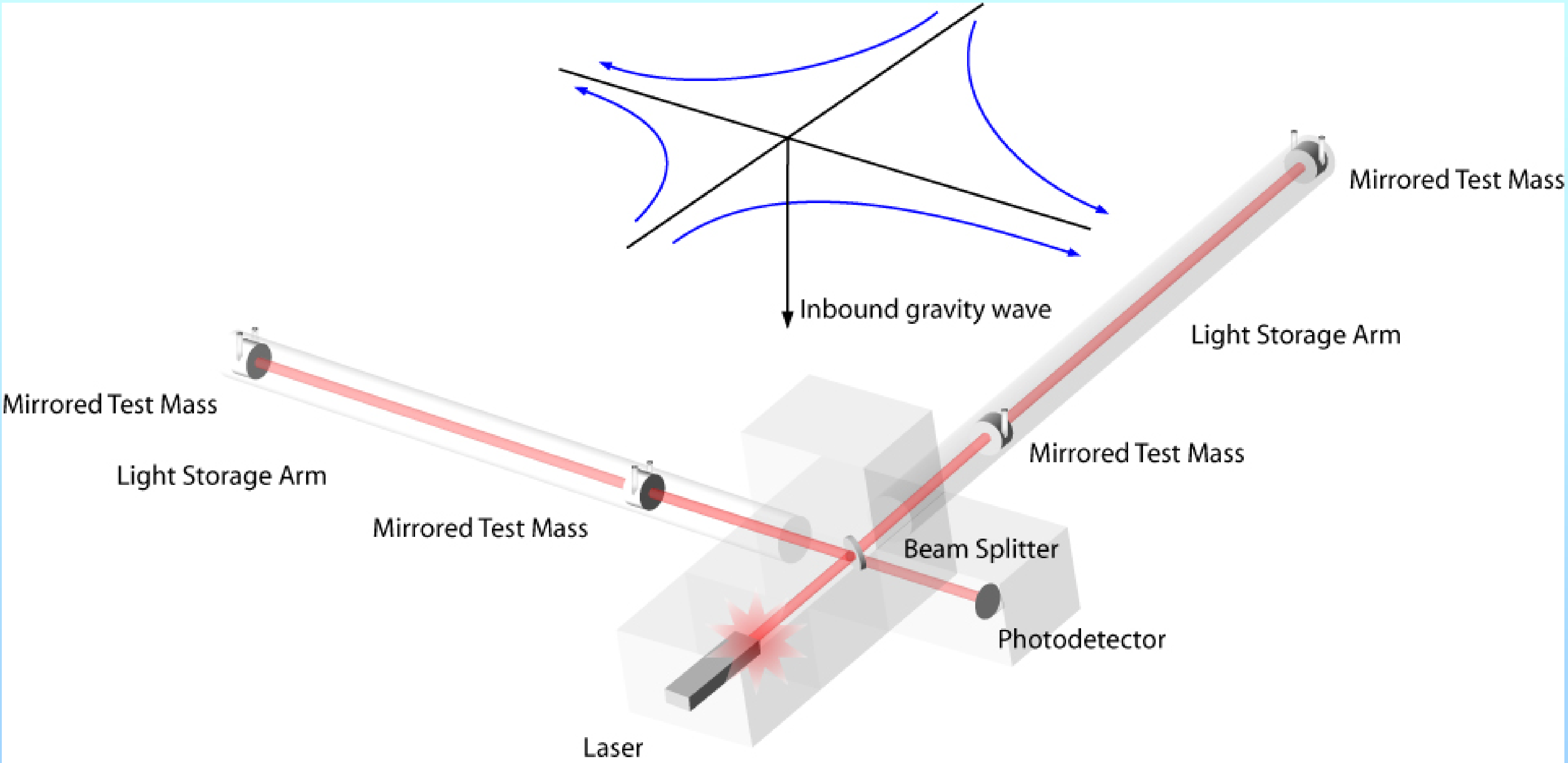
Instrument width = $\dfrac{1}{\text{Maximum path difference}}$

Size of instrument

Resolving Power = Maximum path difference in units of wavelength

LIGO, Laser Interferometric Gravitational-Wave Observatory

LIGO, Laser Interferometric Gravitational-Wave Observatory



Vacuum ~ 10^{-12} atmosphere

Precision ~ 10^{-18} m

Lecture 10

Michelson interferometer

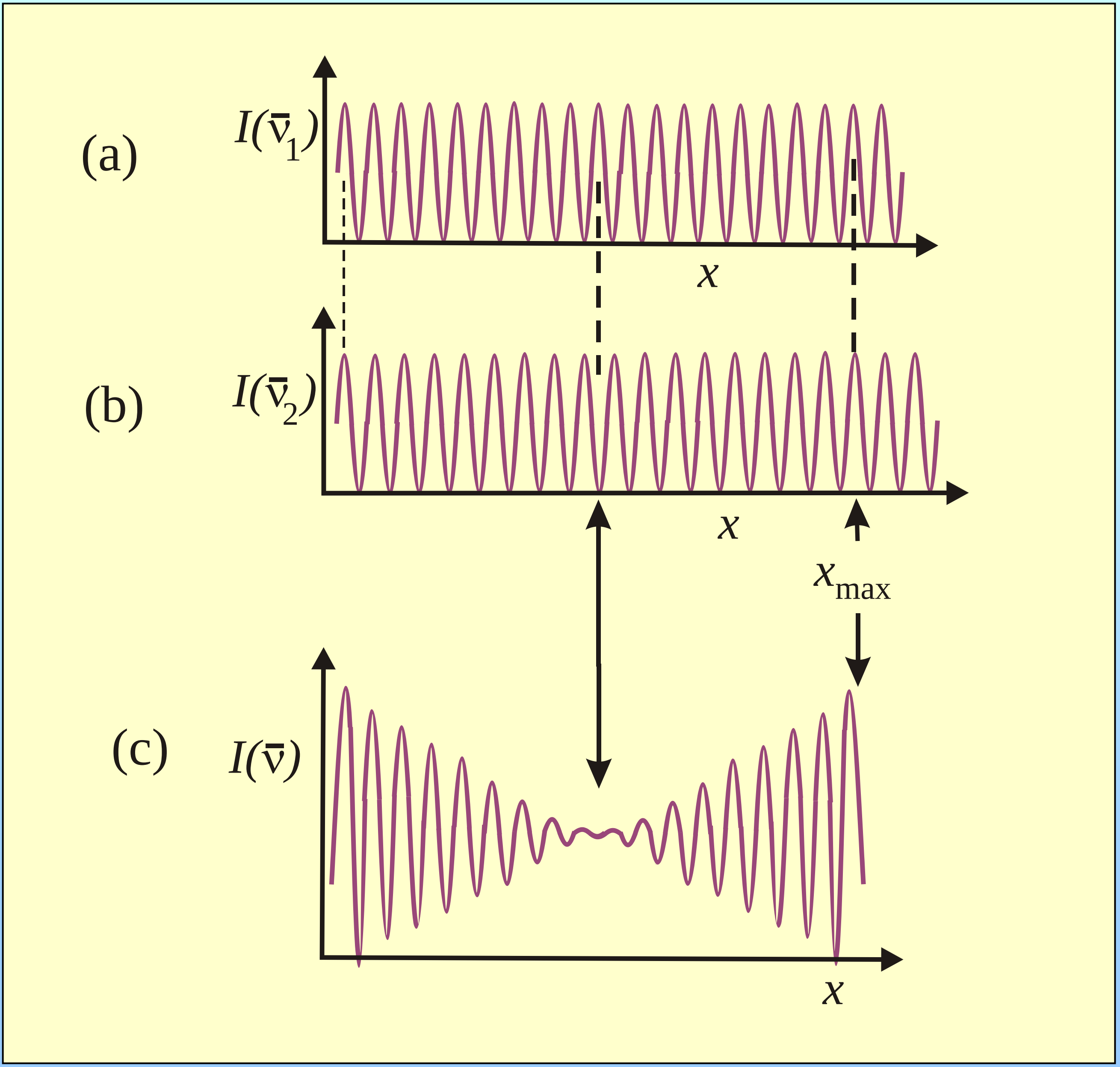
- Path difference: $x = p\lambda$

$\therefore$ measure λ by reference to known $\lambda_{\text{calibration}}$

- Instrument width: $\Delta\bar{\nu}_{Inst} = 1/x_{\max}$

- Fourier transform interferometer
- Fringe visibility and relative intensities
- Fringe visibility and coherence

Albert Abraham Michelson
1852 –1931



(a)
$I(\bar{\nu}_1)$
x
(b)
$I(\bar{\nu}_2)$
x
x_{max}
(c)
$I(\bar{\nu})$
x

Coherence

Longitudinal coherence

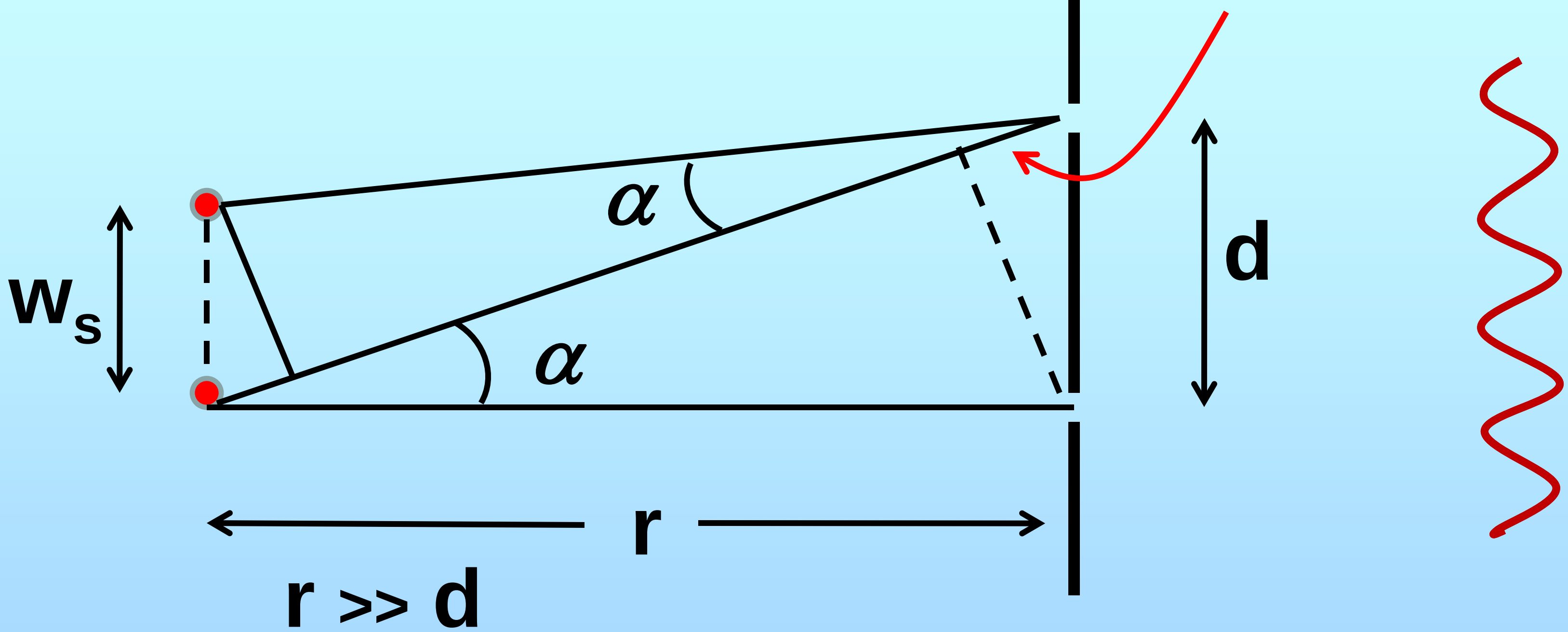
Coherence length: $\ell_c \sim 1/\Delta\bar{\nu}$

Transverse coherence

Coherence area: size of source or wavefront with fixed relative phase

Transverse coherence

$$d \sin \alpha \leq \lambda$$

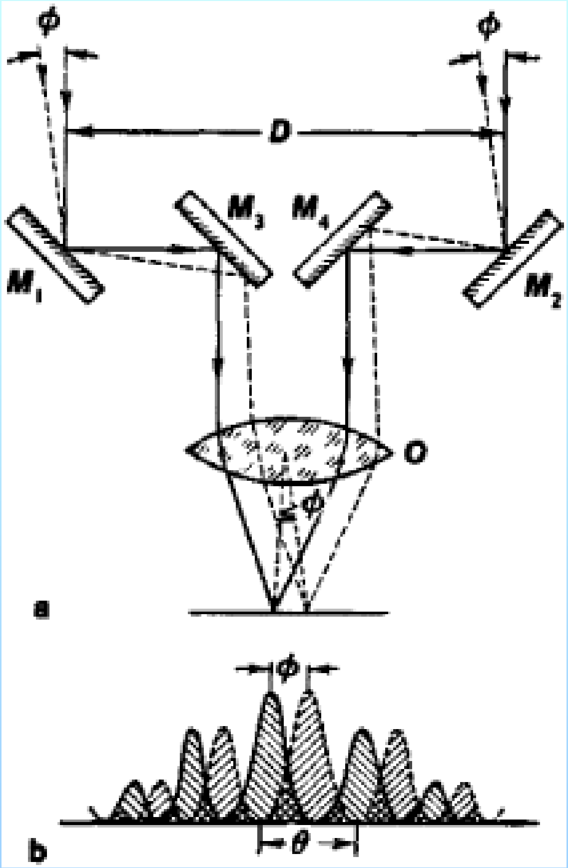


Interference Fringes

$$\alpha \leq \lambda/d$$

d defines **coherence area**

Michelson Stellar Interferometer



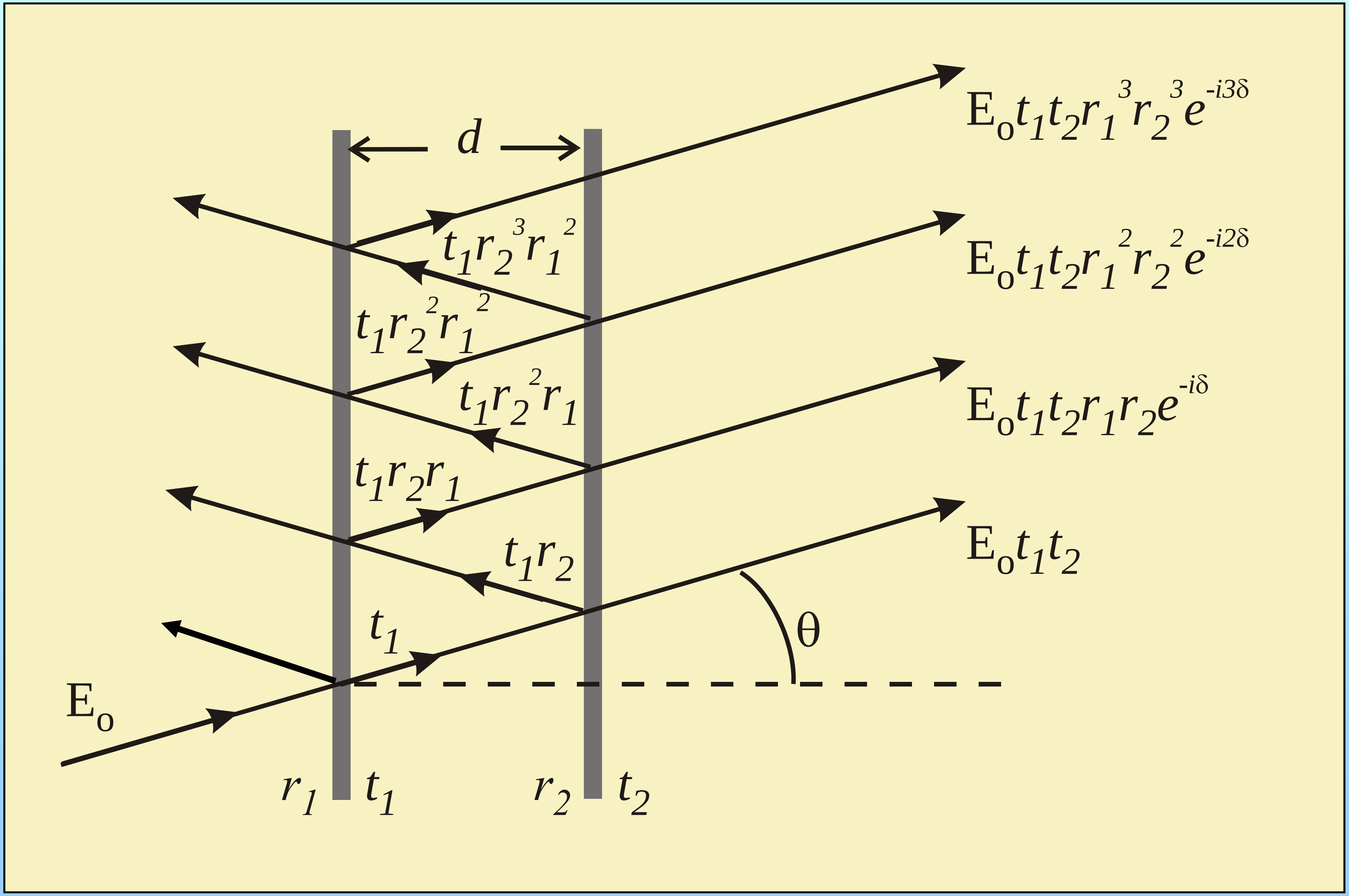
Measures angular size of stars

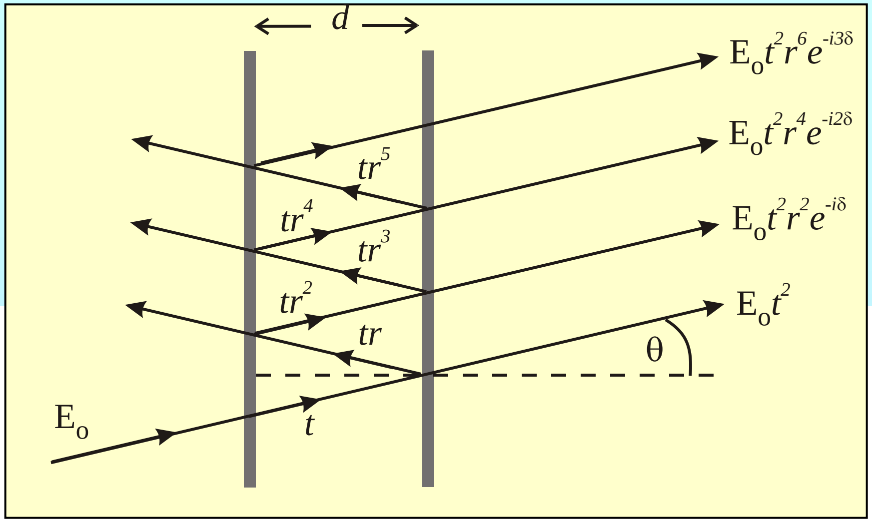
Division of wavefront

	fringes
Young's slits (*2-beam*)	$\cos^2(\delta/2)$
Diffraction grating (*N-beam*)	$\dfrac{\sin^2(N\delta/2)}{\sin^2(\delta/2)}$
Sharper fringes	

Division of amplitude

	fringes
Michelson (*2-beam*)	$\cos^2(\delta/2)$
Fabry-Perot (*N-beam*)	?


d
$E_o t_1 t_2 r_1^3 r_2^3 e^{-i3\delta}$
$t_1 r_2^3 r_1^2$
$E_o t_1 t_2 r_1^2 r_2^2 e^{-i2\delta}$
$t_1 r_2^2 r_1^2$
$t_1 r_2^2 r_1$
$E_o t_1 t_2 r_1 r_2 e^{-i\delta}$
$t_1 r_2 r_1$
$t_1 r_2$
$E_o t_1 t_2$
t_1
θ
E_o
r_1 t_1 r_2 t_2



$$\delta = \frac{2\pi}{\lambda} 2d\cos\theta$$

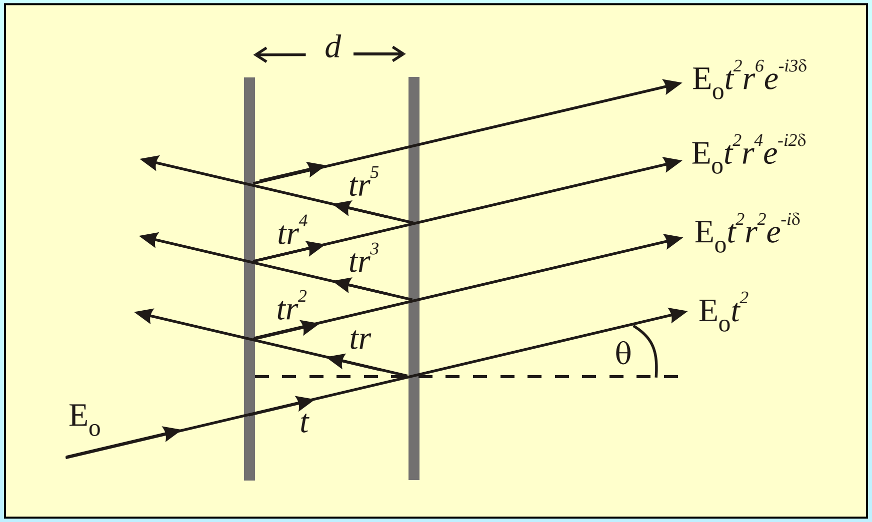
$$E_t = E_0 t^2 e^{i\omega t} + E_0 t^2 r^2 e^{i(\omega t - \delta)} + E_0 t^2 r^4 e^{i(\omega t - 2\delta)} + \ldots etc.$$

Geometric Progression in $r^2 e^{-i\delta}$

$$E_t = E_0 t^2 e^{i\omega t}\left[\frac{1}{1 - r^2 e^{-i\delta}}\right]$$

Transmitted Intensity:

$$I_t = E_t E_t^* = E_0^2 t^4 \left[\frac{1}{1 + r^4 - 2r^2\cos\delta}\right]$$



writing $E_0^2 = I_0$, $r^2 = R$, $t^2 = T$, and $\cos\delta = (1 - 2\sin^2\delta/2)$:

$$I_t = I_0 \frac{T^2}{(1-R^2)}\left[\frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2\delta/2}\right] \qquad (8.2)$$

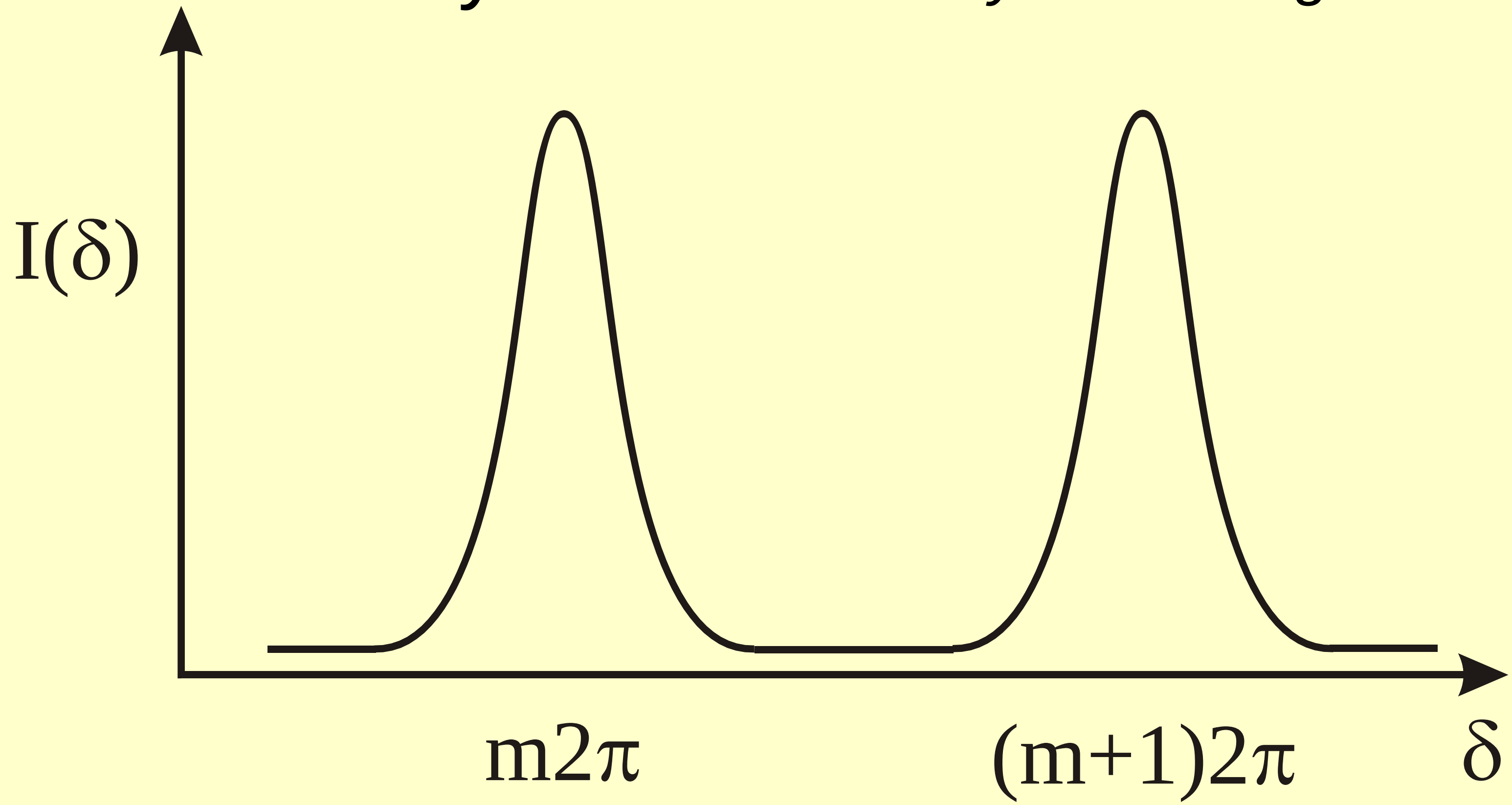
No absorption $T = (1 - R)$ then defining

$$\frac{4R}{(1-R)^2} = \Phi \qquad (8.3)$$

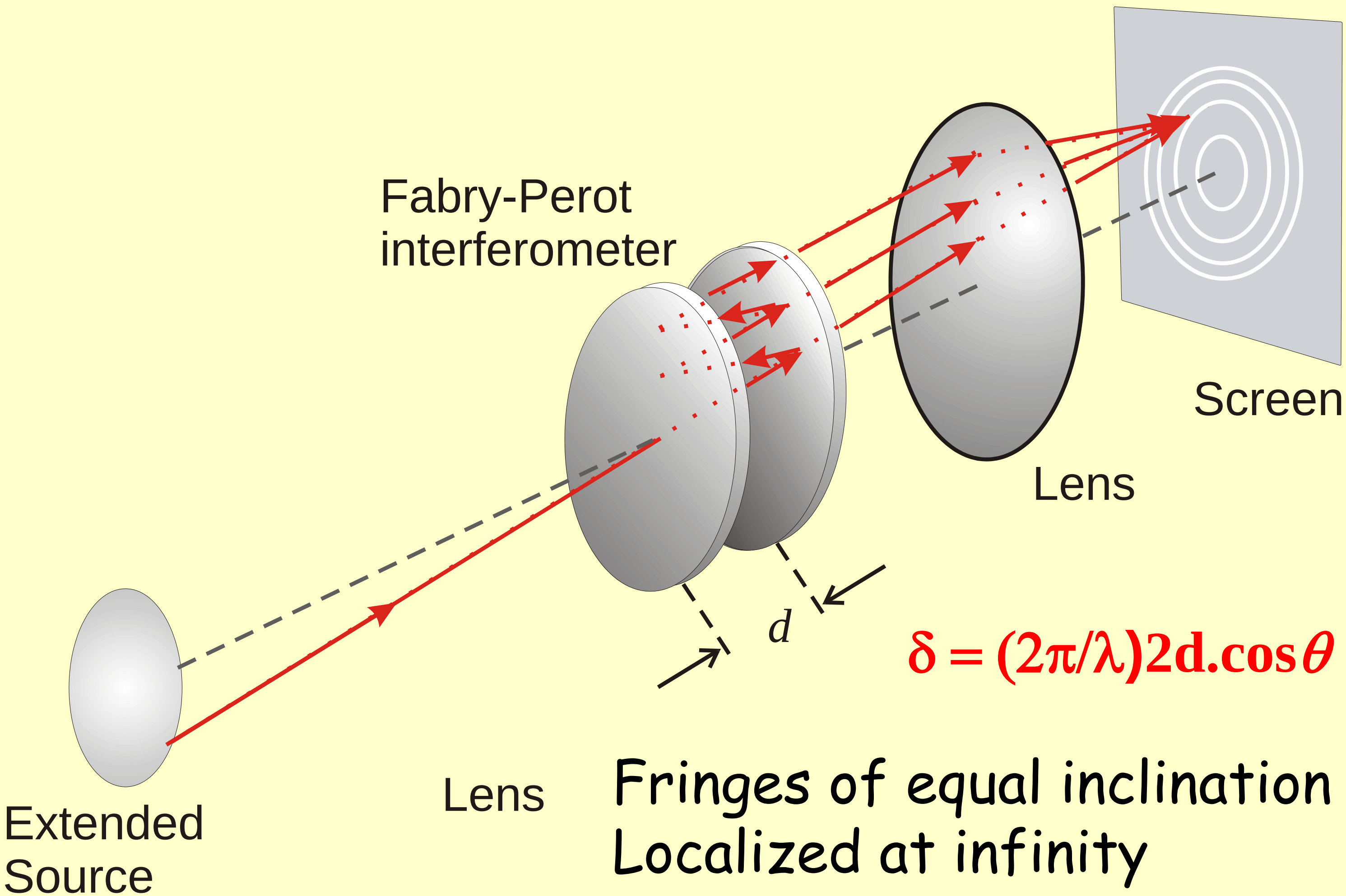
Airy Function

$$I_t = I_0\left[\frac{1}{1 + \Phi\sin^2\delta/2}\right] \qquad (8.4)$$

The Airy function: *Fabry-Perot fringes*



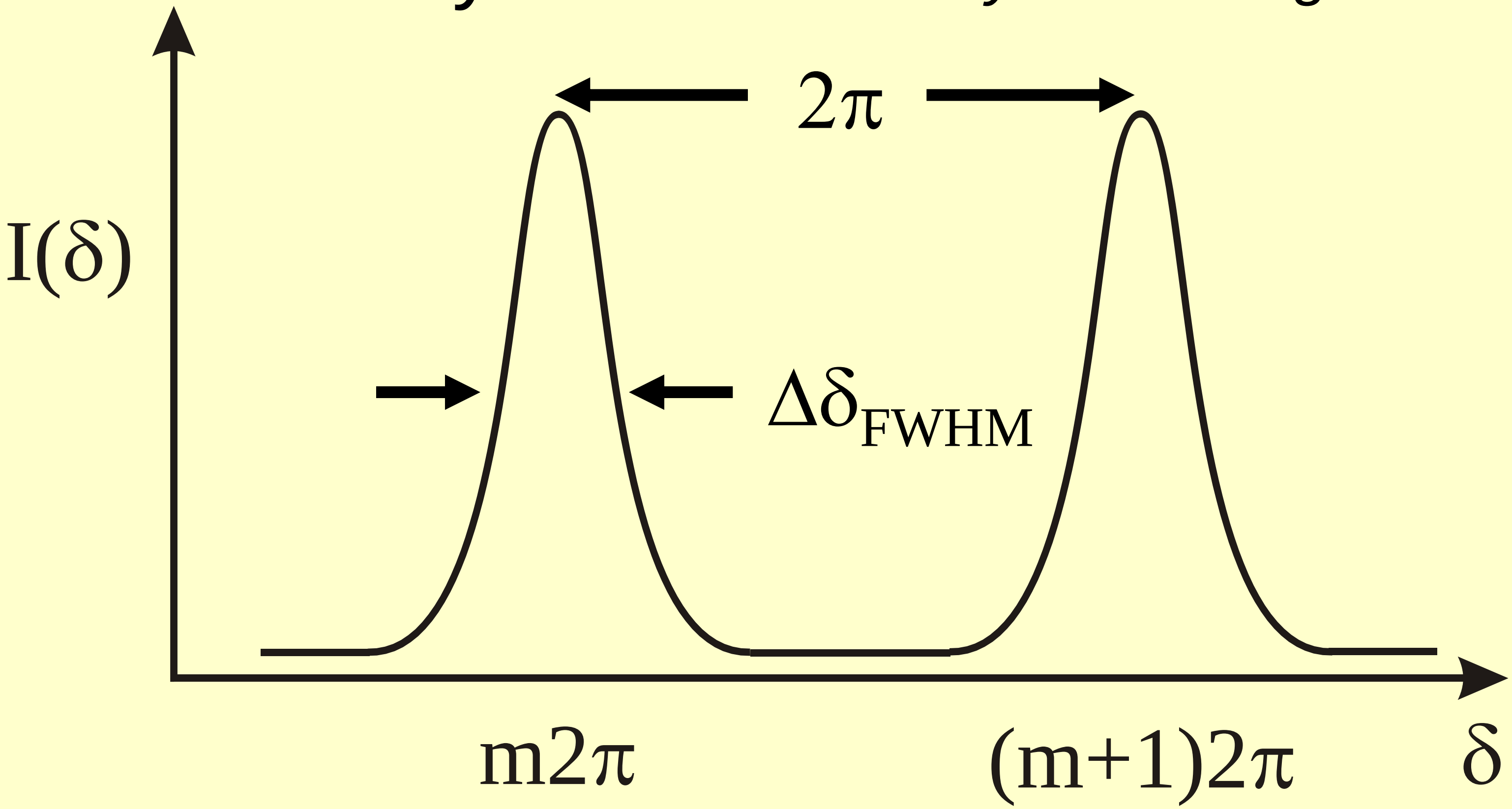
$\delta = (2\pi/\lambda)2d.\cos\theta$



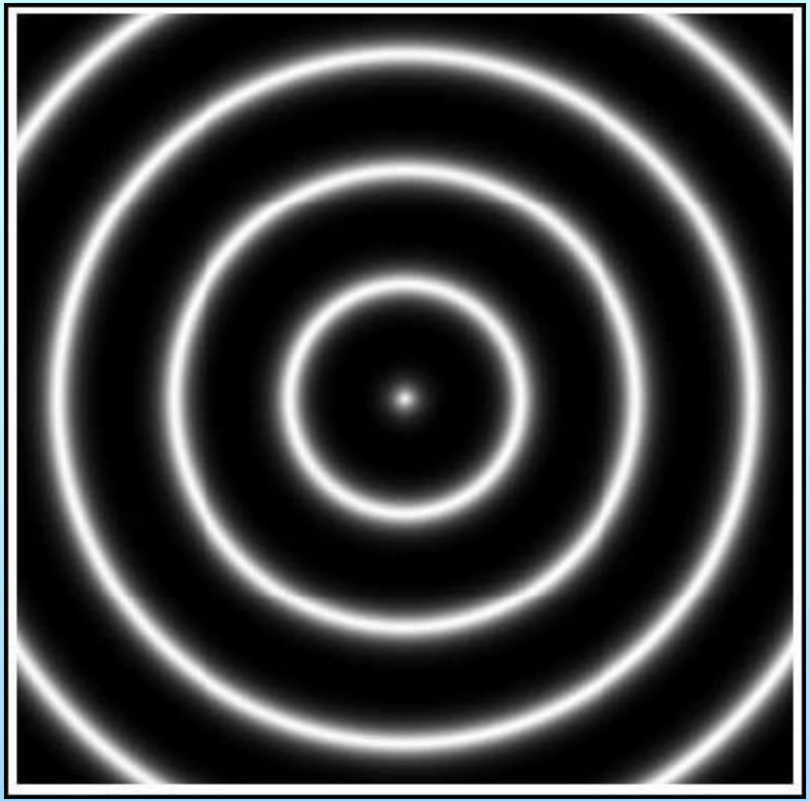
$$\delta = (2\pi/\lambda)2d.\cos\theta$$

Fringes of equal inclination
Localized at infinity

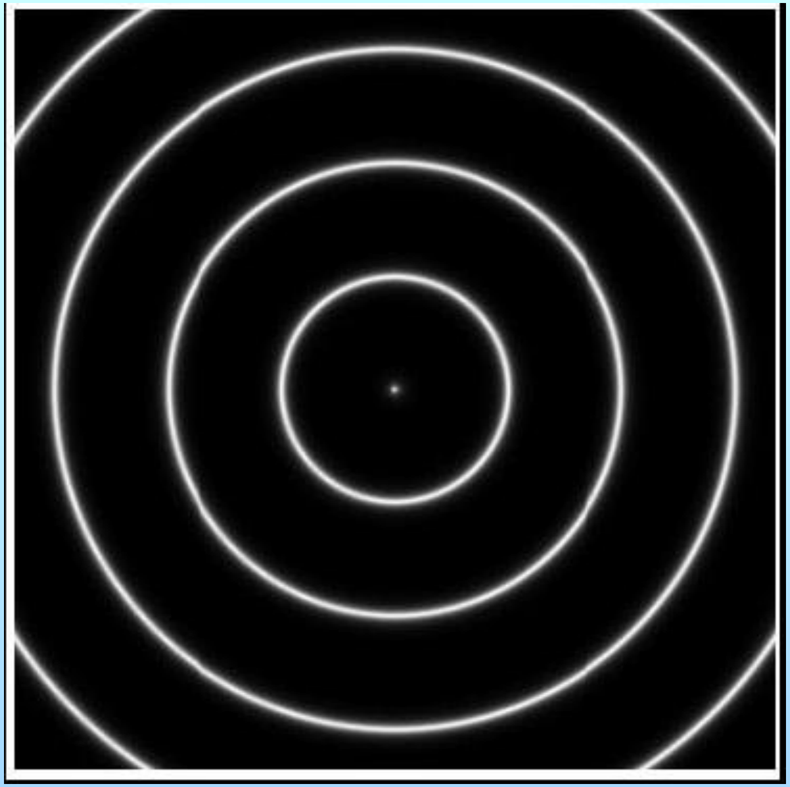
The Airy function: *Fabry-Perot fringes*



Finesse = $2\pi/\Delta\delta_{FWHM}$



Finesse = 10



Finesse = 100

Fabry-Perot Interferometer

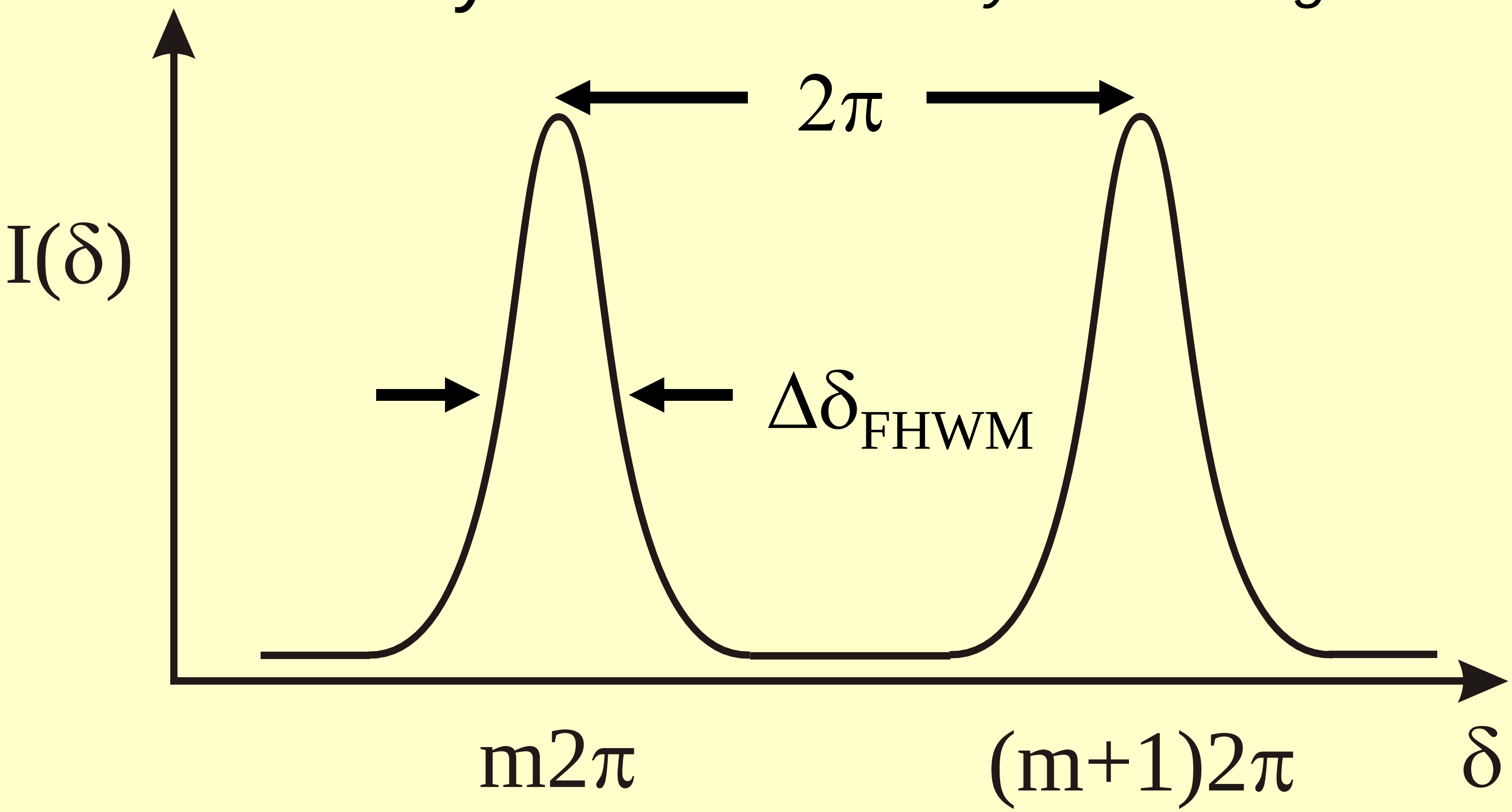
Multiple beam interference:

Fringe sharpness set by Finesse

$$F = \frac{\pi\sqrt{R}}{(1-R)}$$

- Instrument width $\Delta\nu_{Inst}$
- Free Spectral Range FSR
- Resolving power
- Designing a Fabry-Perot

The Airy function: *Fabry-Perot fringes*



Finesse, $F = 2\pi/\Delta\delta_{FHWM}$ $\quad \Delta\delta_{Inst} = 2\pi/F$

Fabry-Perot Interferometer: *Instrument width*

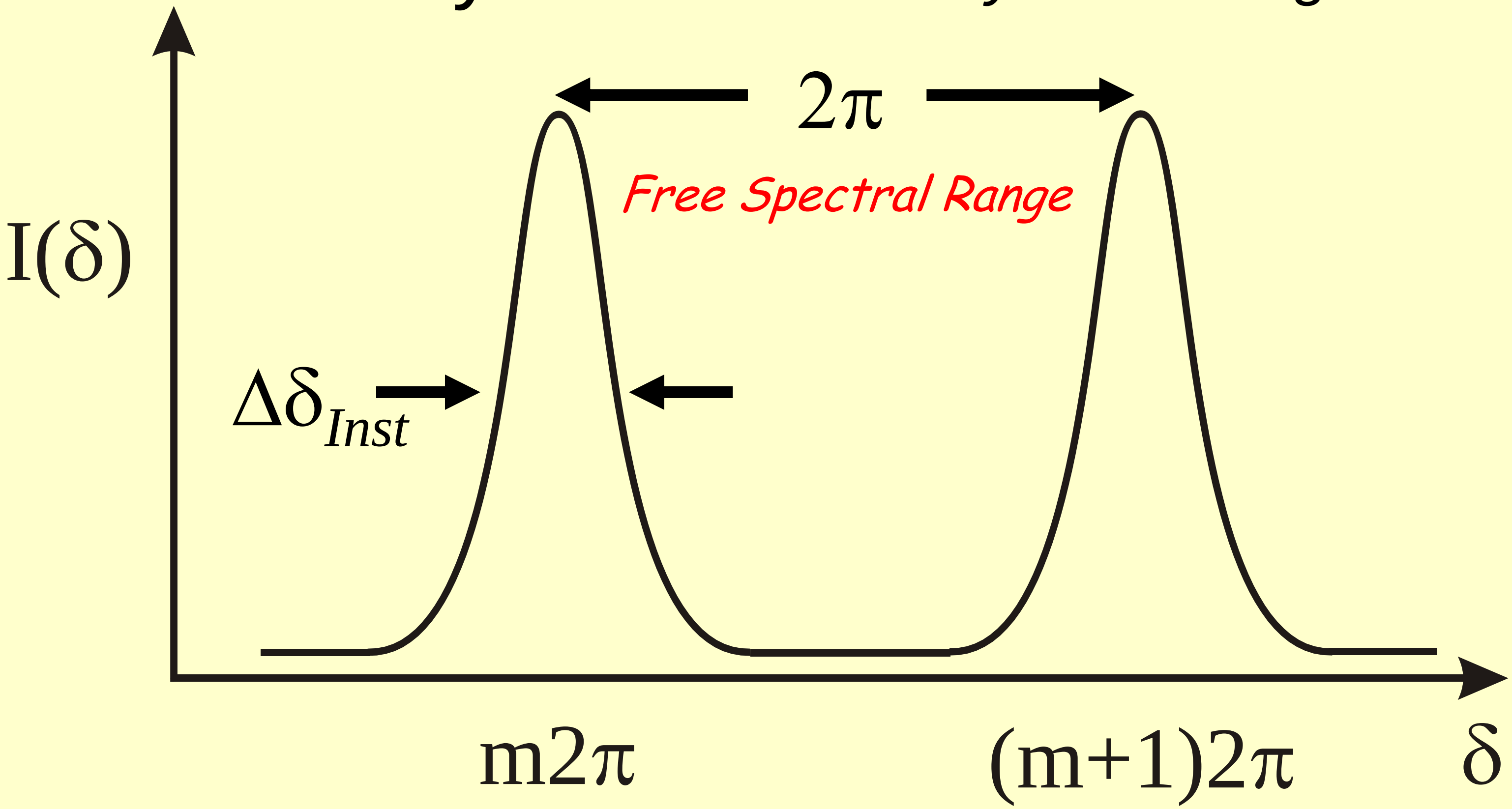
$$\Delta\bar{v}_{Inst} = \frac{1}{2d.F}$$

$$= 1/x_{\max}$$

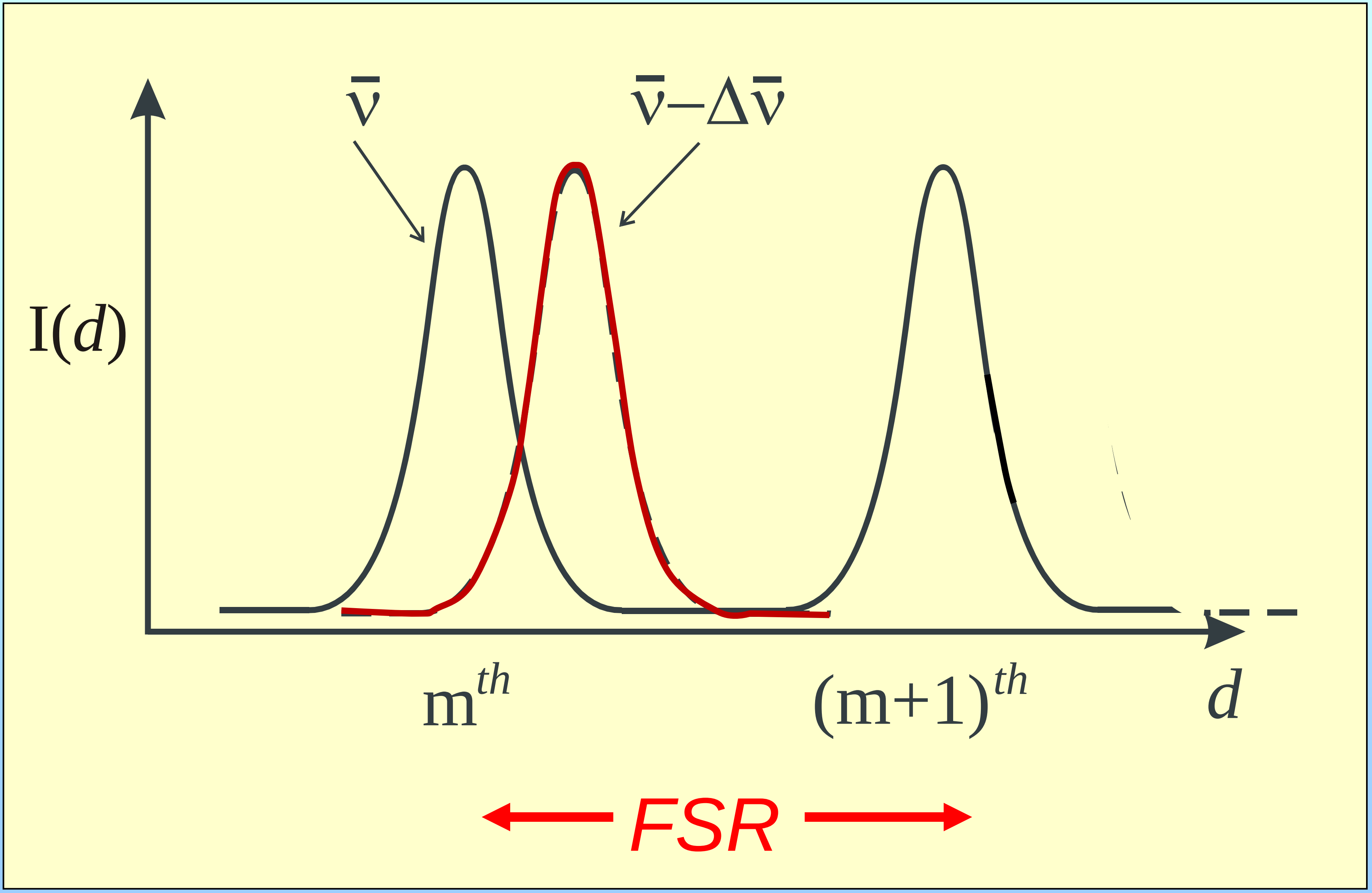
effective

Instrument width = $\dfrac{\mathbf{1}}{\textbf{Maximum path difference}}$

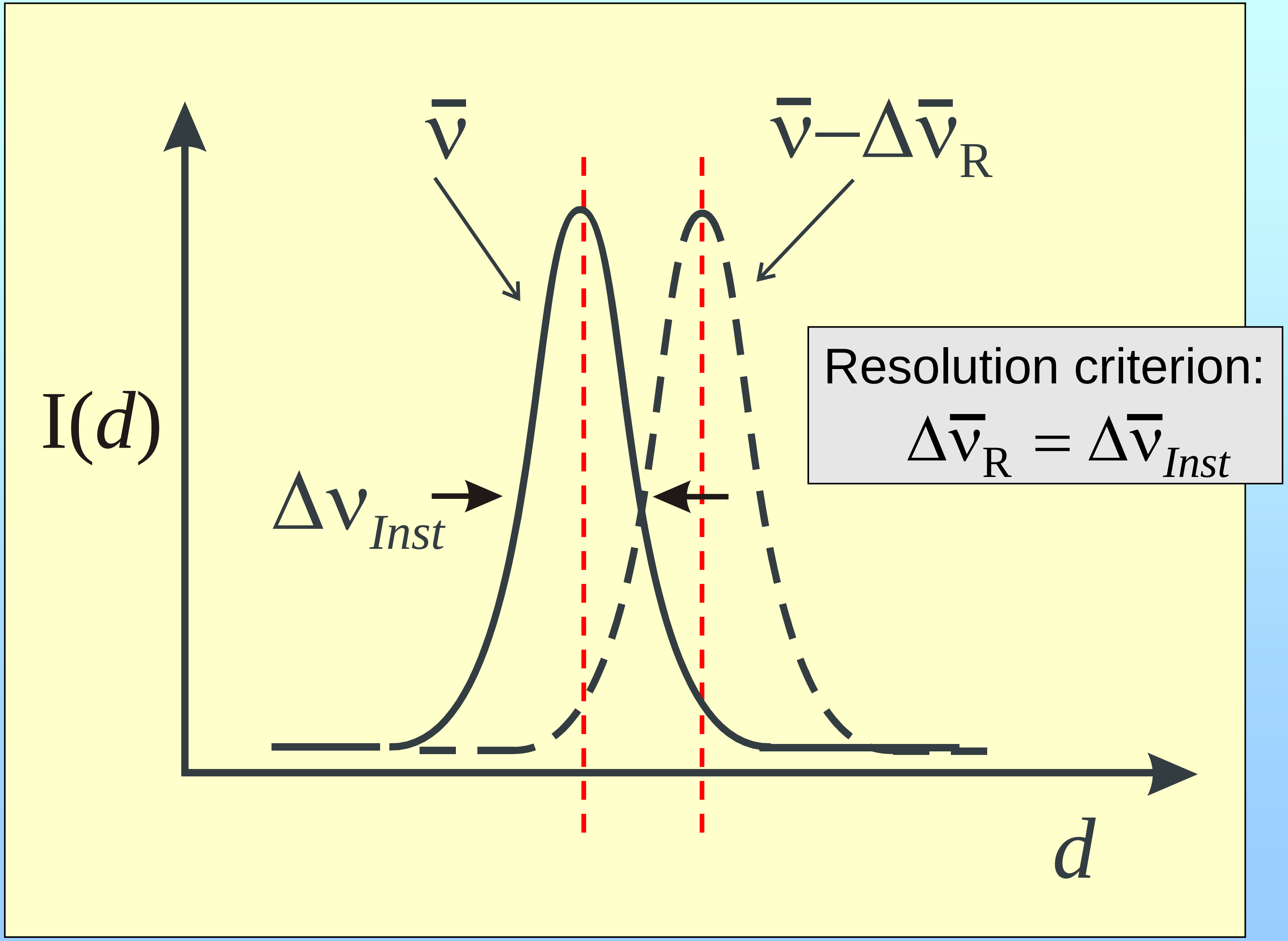
The Airy function: *Fabry-Perot fringes*



Finesse = $2\pi/\Delta\delta$

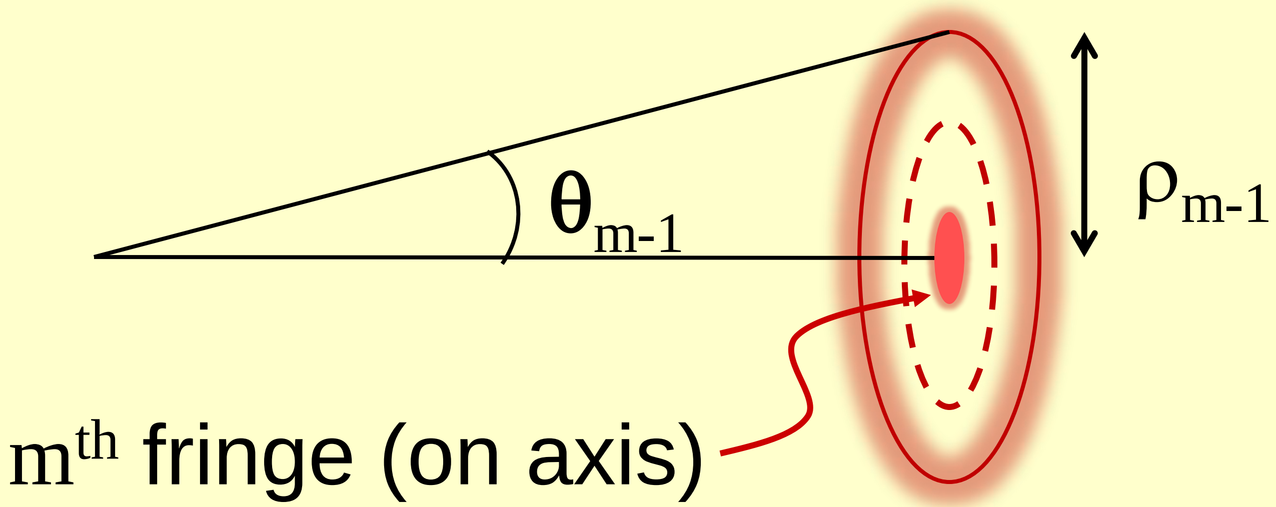


$\bar{\nu}$
$\bar{\nu}-\Delta\bar{\nu}$
I(*d*)
mth
(m+1)th
d
FSR



$\bar{\nu}$
$\bar{\nu}-\Delta\bar{\nu}_R$
I(d)
$\Delta\nu_{Inst}$
d
Resolution criterion:
$\Delta\bar{\nu}_R = \Delta\bar{\nu}_{Inst}$

Centre spot scanning



Aperture size to admit only m^{th} fringe

Typically aperture ~ $\dfrac{\rho_{m-1}}{10}$

Maxwell's equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

E.M. Wave equations

$$\nabla^2 E = \varepsilon_o \varepsilon_r \mu_o \mu_r \frac{\partial^2 E}{\partial t^2}$$

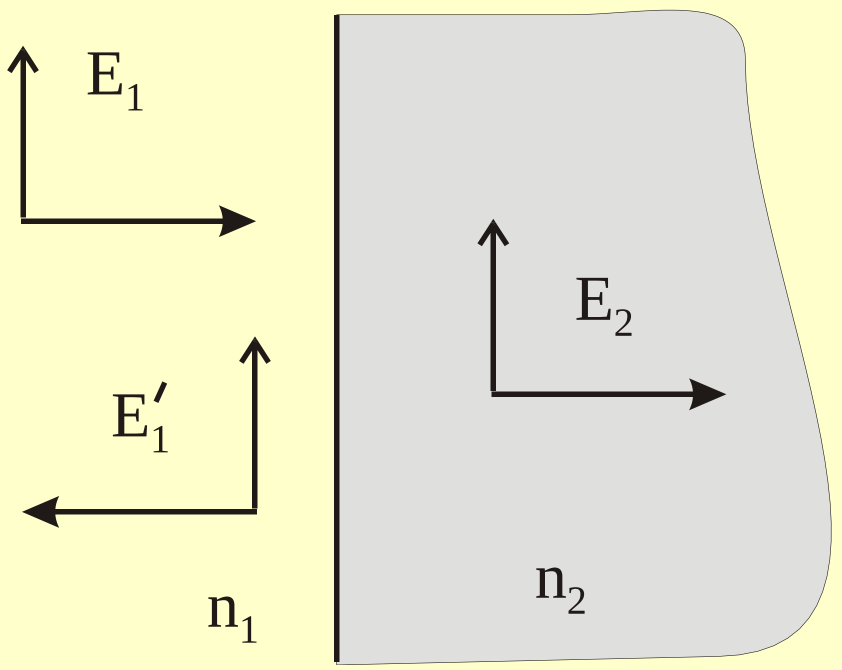
$$\nabla^2 H = \varepsilon_o \varepsilon_r \mu_o \mu_r \frac{\partial^2 H}{\partial t^2}$$

$$E = E_o e^{i(\omega t - k.r)}$$

$$E = \sqrt{\frac{\mu_o \mu_r}{\varepsilon_o \varepsilon_r}} H = \frac{1}{n} \sqrt{\frac{\mu_o}{\varepsilon_o}} H$$

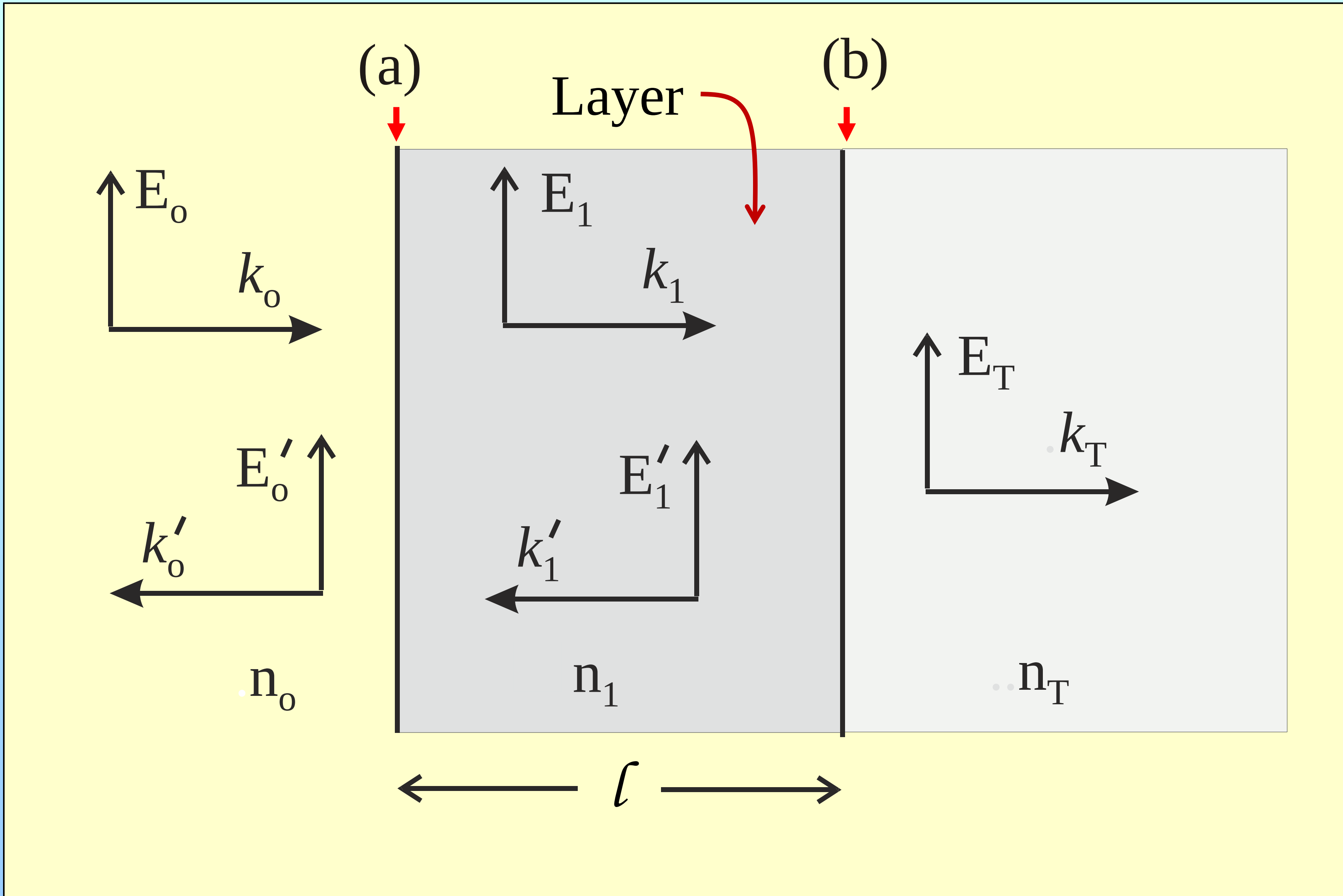
$$n = \textit{refractive index}$$

$$H = \text{constant} \times n \times E$$



Boundary conditions:

Tangential components of E and H are continuous



(a)
Layer
(b)
E_o
k_o
E₁
k₁
E_T
k_T
E_o′
k_o′
E₁′
k₁′
n_o
n₁
n_T
ℓ

$$E_O + E_O' = \left[\cos k_1 \ell - i\left(\frac{n_T}{n_1}\right)\sin k_1 \ell\right]E_T \qquad (9.7)$$

$$n_O(E_O - E_O') = \left[-i\sin k_1 \ell + n_T \cos k_1 \ell\right]E_T \qquad (9.8)$$

$$A = \cos k_1 \ell;\quad B = -i\left(\frac{1}{n_1}\right)\sin k_1 \ell;\quad C = -in_1 \sin k_1 \ell;\quad D = \cos k_1 \ell$$

$$\frac{E_O'}{E_O} = r = \frac{An_O + Bn_On_T - C - Dn_T}{An_O + Bn_On_T + C + Dn_T} \qquad (9.10)$$

$$\frac{E_T}{E_O} = t = \frac{2n_O}{An_O + Bn_On_T + C + Dn_T} \qquad (9.11)$$

$$\frac{E_O'}{E_O} = r = \frac{An_O + Bn_O n_T - C - Dn_T}{An_O + Bn_O n_T + C + Dn_T}$$

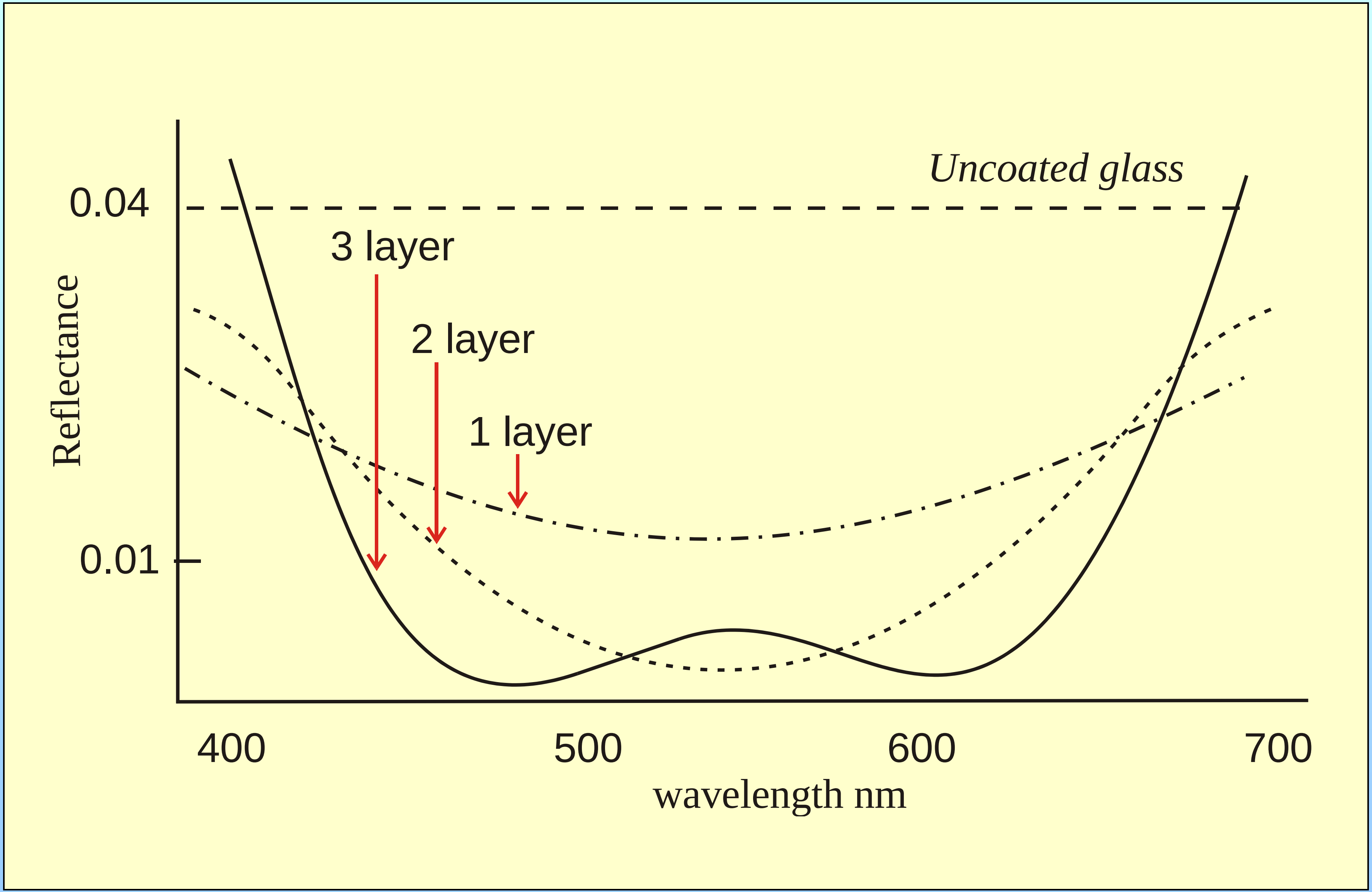
$\ell = \lambda/4$, **Quarter Wave Layer:**

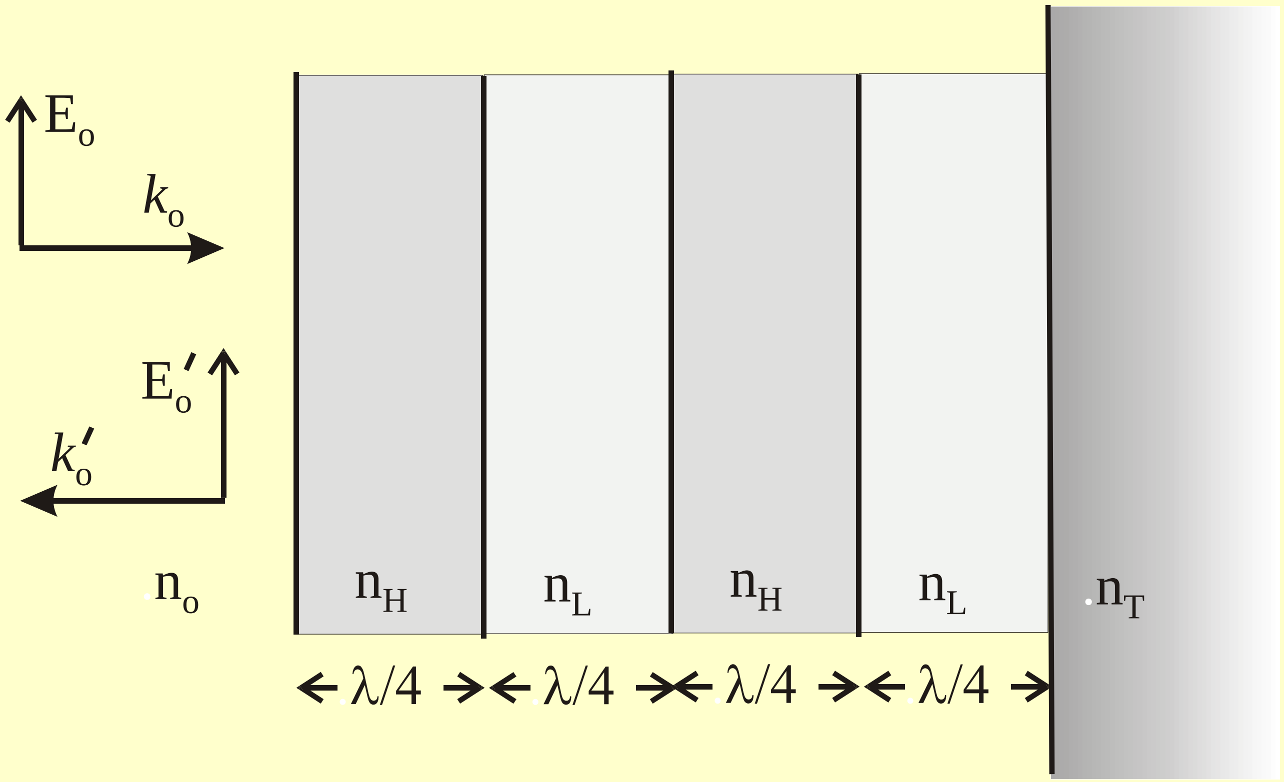
$A=0, \; B=-i/n_1, \; C=-in_1, \; D=0$

$$\mathrm{R} = |r|^2 = \left|\frac{n_O n_T - n_1^2}{n_O n_T + n_1^2}\right|^2$$

Anti-reflection coating, $\mathbf{n_1^2 \sim n_O n_T}$, $\mathrm{R} \rightarrow 0$

e.g. blooming on lenses


Uncoated glass
0.04
3 layer
2 layer
1 layer
Reflectance
0.01
400
500
600
700
wavelength nm



Multi-layer stack

$$E_O + E_O' = \left[\cos k_1 \ell - i\left(\frac{n_T}{n_1}\right)\sin k_1 \ell\right] E_T \qquad (9.7)$$

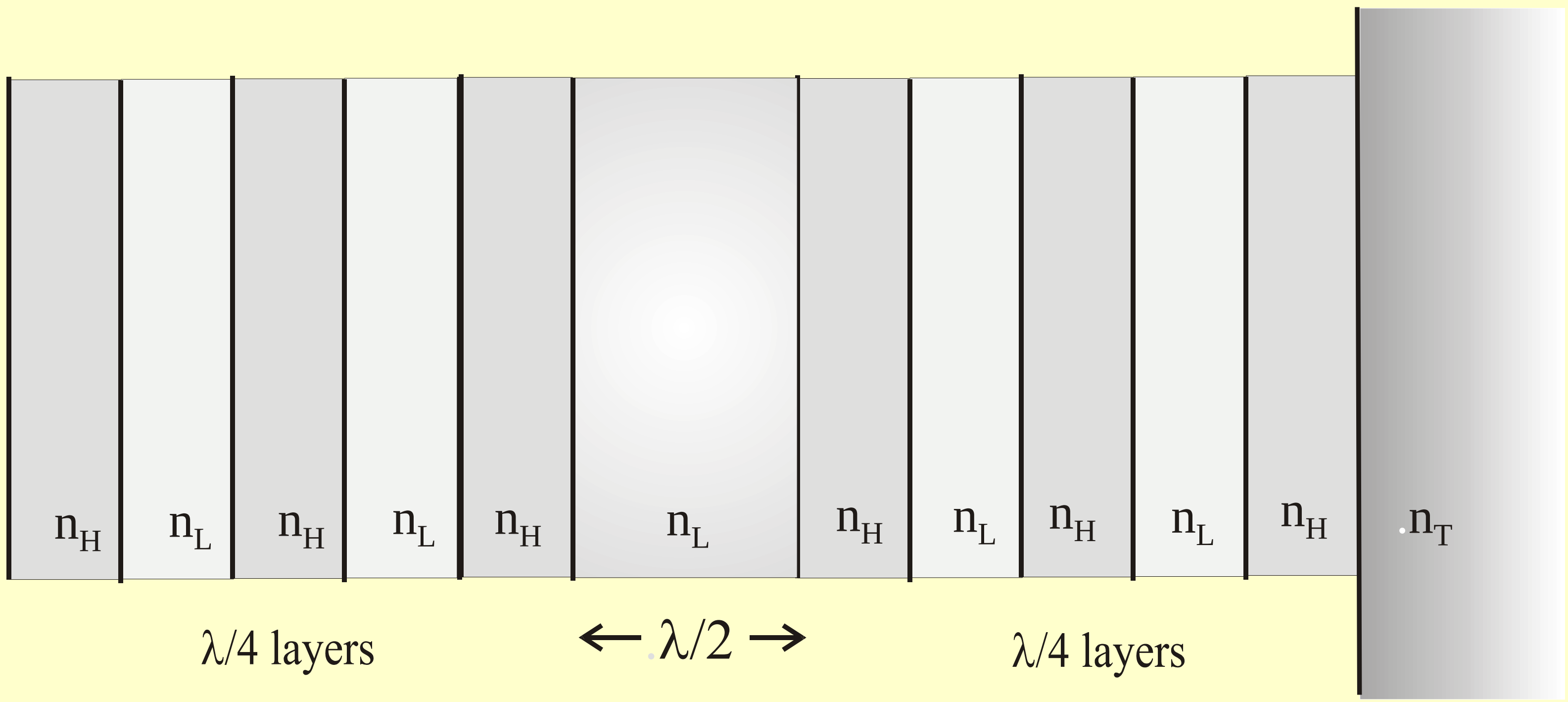
$$n_O(E_O - E_O') = \left[-i \sin k_1 \ell + n_T \cos k_1 \ell\right] E_T \qquad (9.8)$$

$$A = \cos k_1 \ell \quad B = -i\left(\frac{1}{n_1}\right)\sin k_1 \ell \quad C = -i n_1 \sin k_1 \ell \quad D = \cos k_1 \ell$$

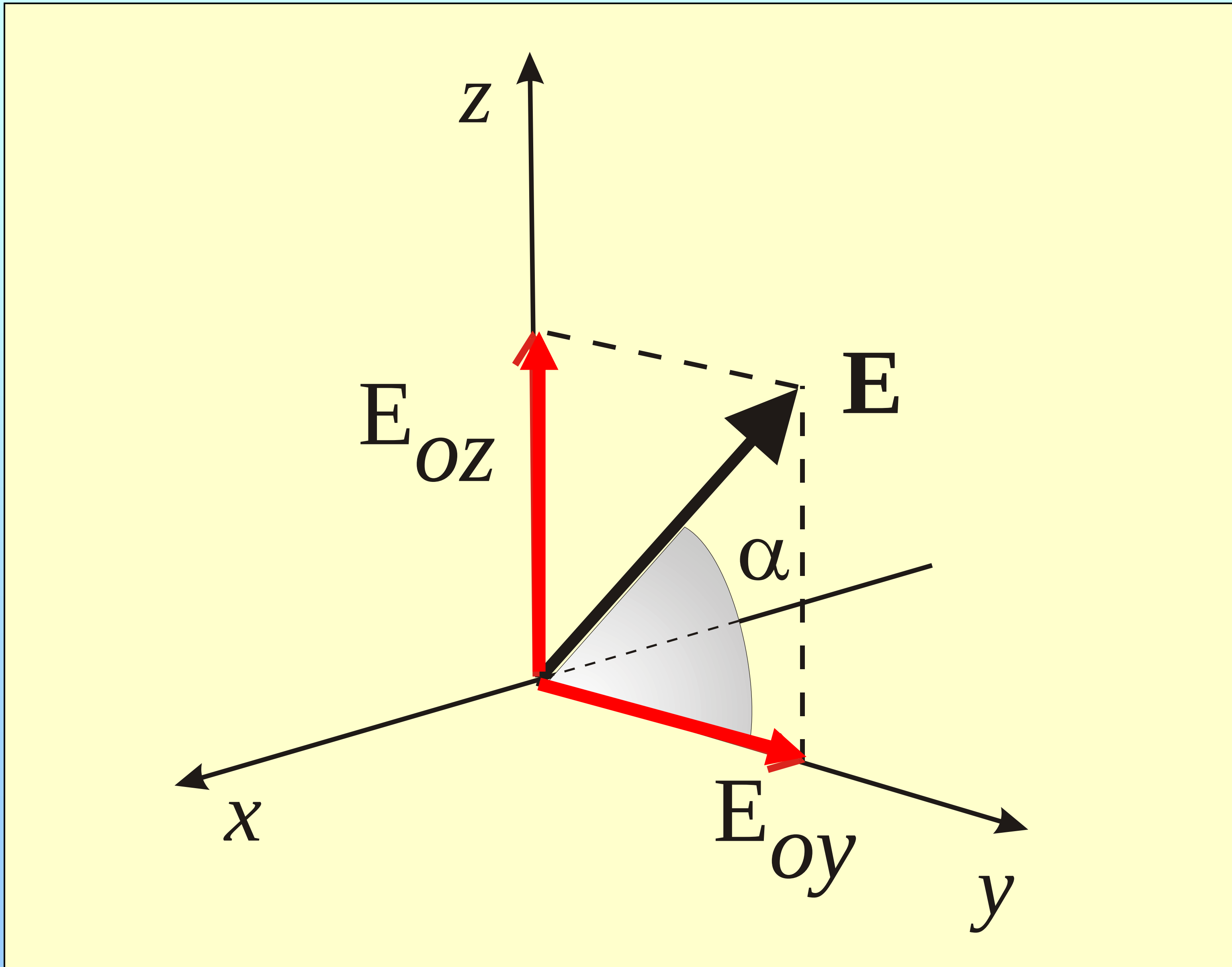
$$1 + r = (A + B n_T)t$$

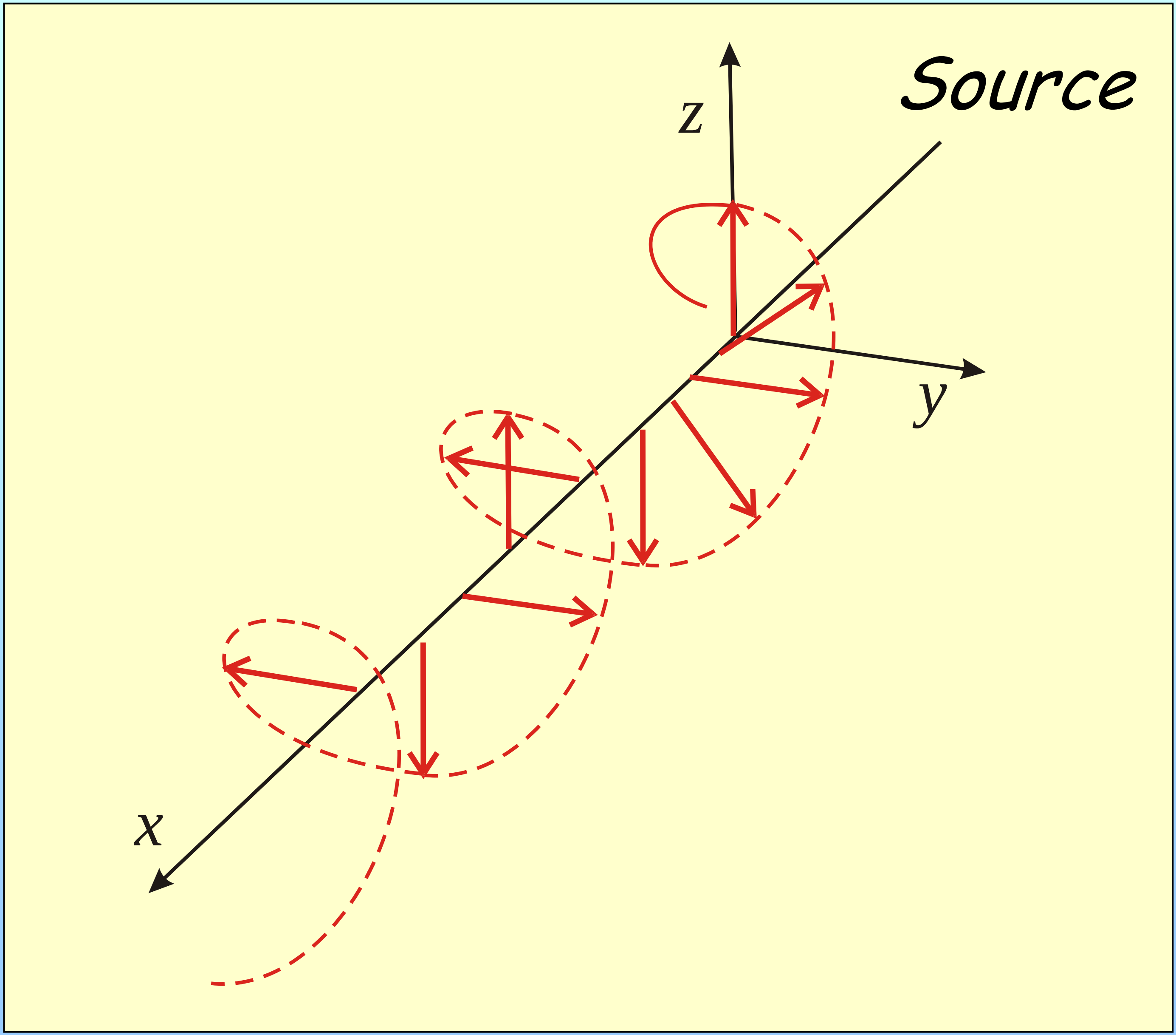
$$n_O(1 - r) = (C + D n_T)t$$

$$\begin{pmatrix} 1 \\ n_O \end{pmatrix} + \begin{pmatrix} 1 \\ -n_O \end{pmatrix} r = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} 1 \\ n_T \end{pmatrix} t$$



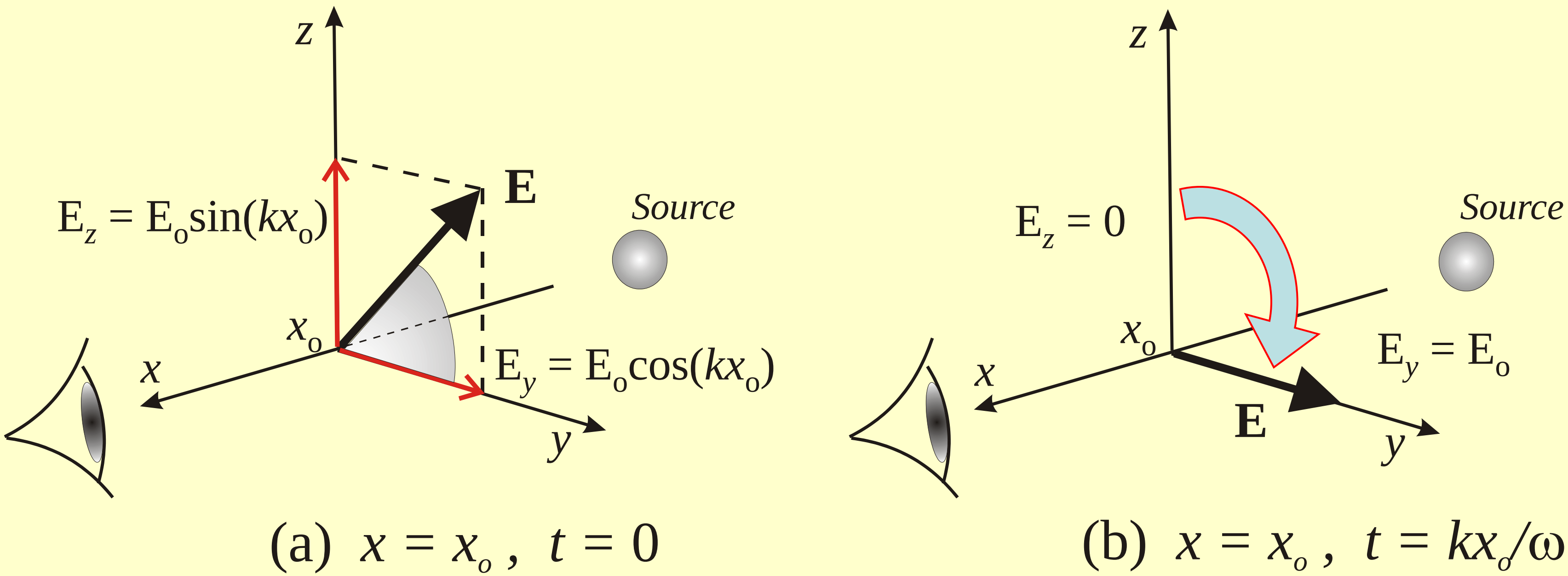
Interference filter; composed of multi-layer stacks

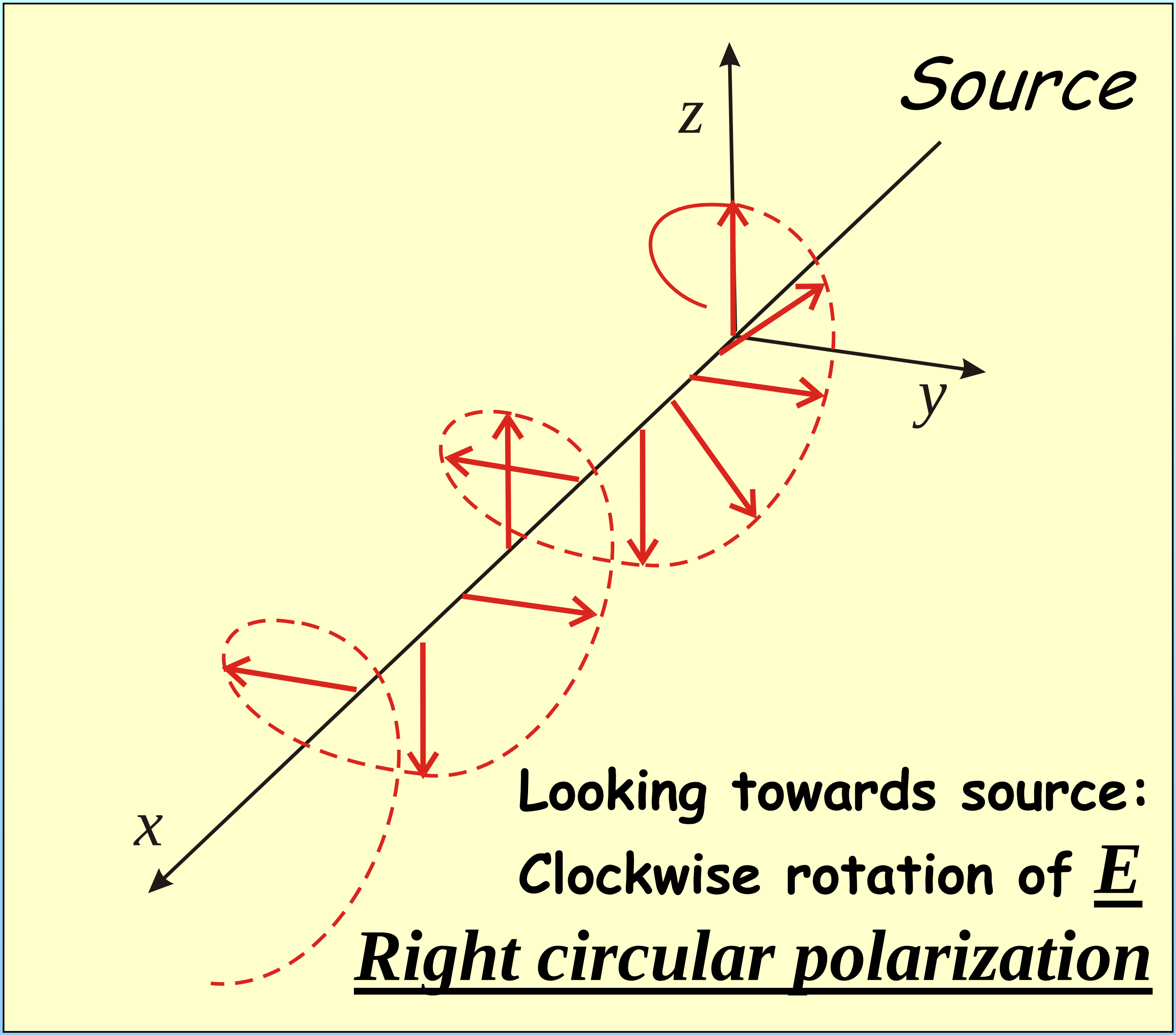

z
E
E_{oz}
α
x
E_{oy}
y

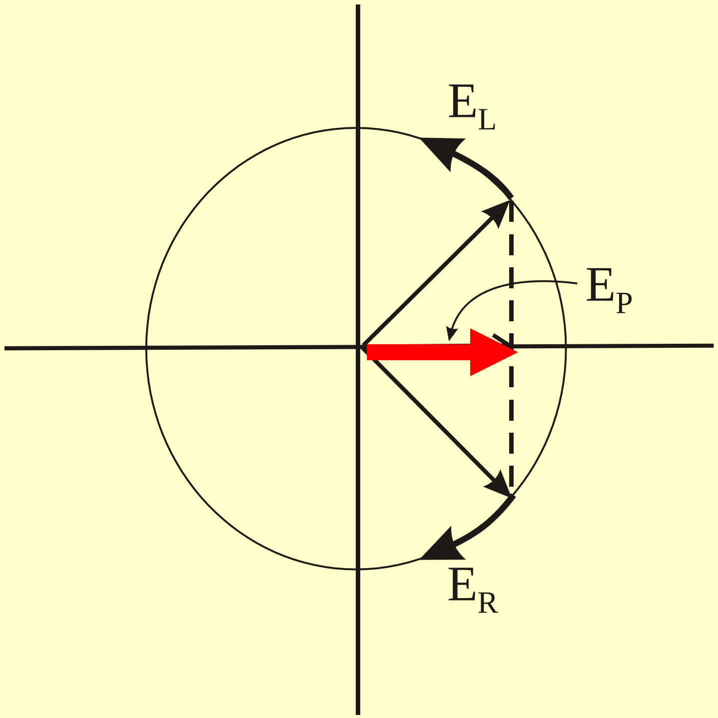

Source
z
y
x

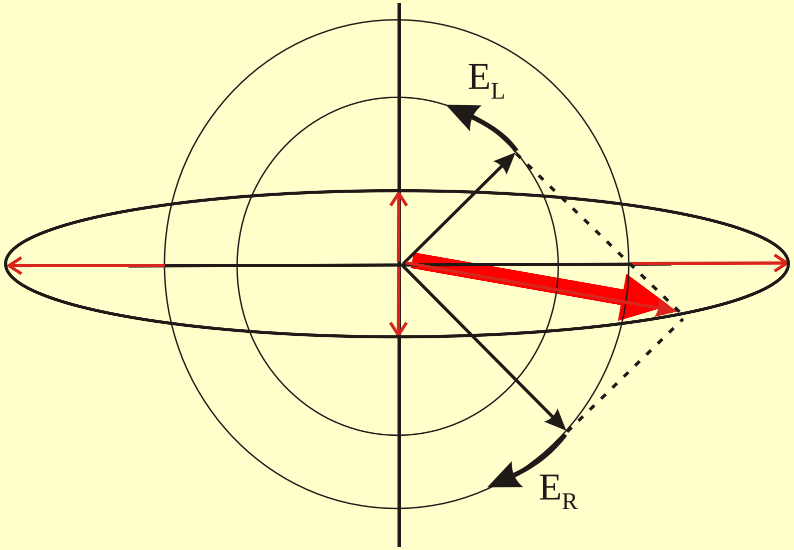
Circularly polarized light: $E_z = E_o \sin(kx_o - \omega t)$; $E_y = E_o \cos(kx_o - \omega t)$

$\delta = -\pi$



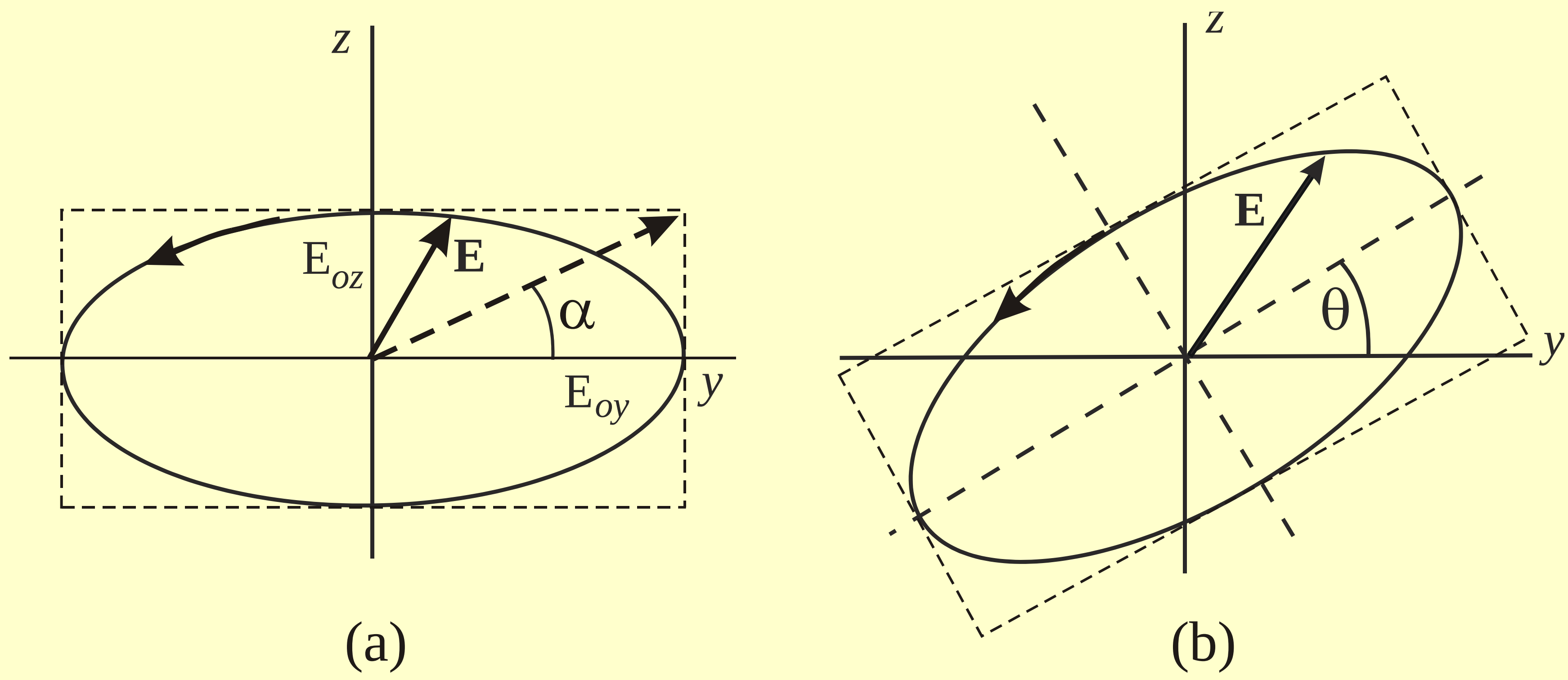

Source
z
y
x
Looking towards source:
Clockwise rotation of E
Right circular polarization





Superposition of equal
R & L Circular polarization
= Plane polarized light

Superposition of unequal
R & L Circular polarization
= Elliptically polarized light



z
E
E_{oz}
α
E_{oy}
y
(a)
z
E
θ
y
(b)

Polarization states defined by:

$$E_y = E_{oy} \cos(kx - \omega t) \qquad E_z = E_{oz} \cos(kx - \omega t - \delta)$$

• $\delta = 0$	Linearly Polarized
• $\delta \neq 0$	Elliptically Polarized

Polarization states defined by:

$E_y = E_{oy} \cos(kx - \omega t)$ $E_z = E_{oz} \cos(kx - \omega t - \delta)$

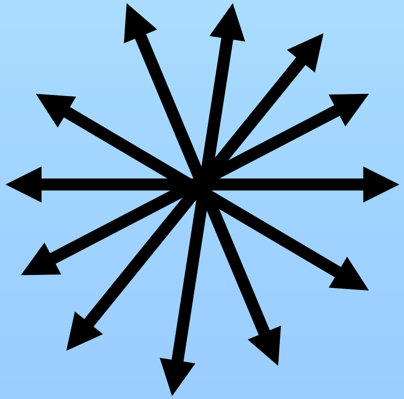
• $\delta = 0$		Linear
• $\delta = \pm \pi/2$	$E_{oy} = E_{oz}$	Left/Right Circular
• $\delta = \pm \pi/2$	$E_{oy} \neq E_{oz}$	Left/Right Elliptical axes along *y,z*
• $\delta \neq \pm \pi/2$	$E_{oy} \neq E_{oz}$	Left/Right Elliptical axes at angle to *y,z*

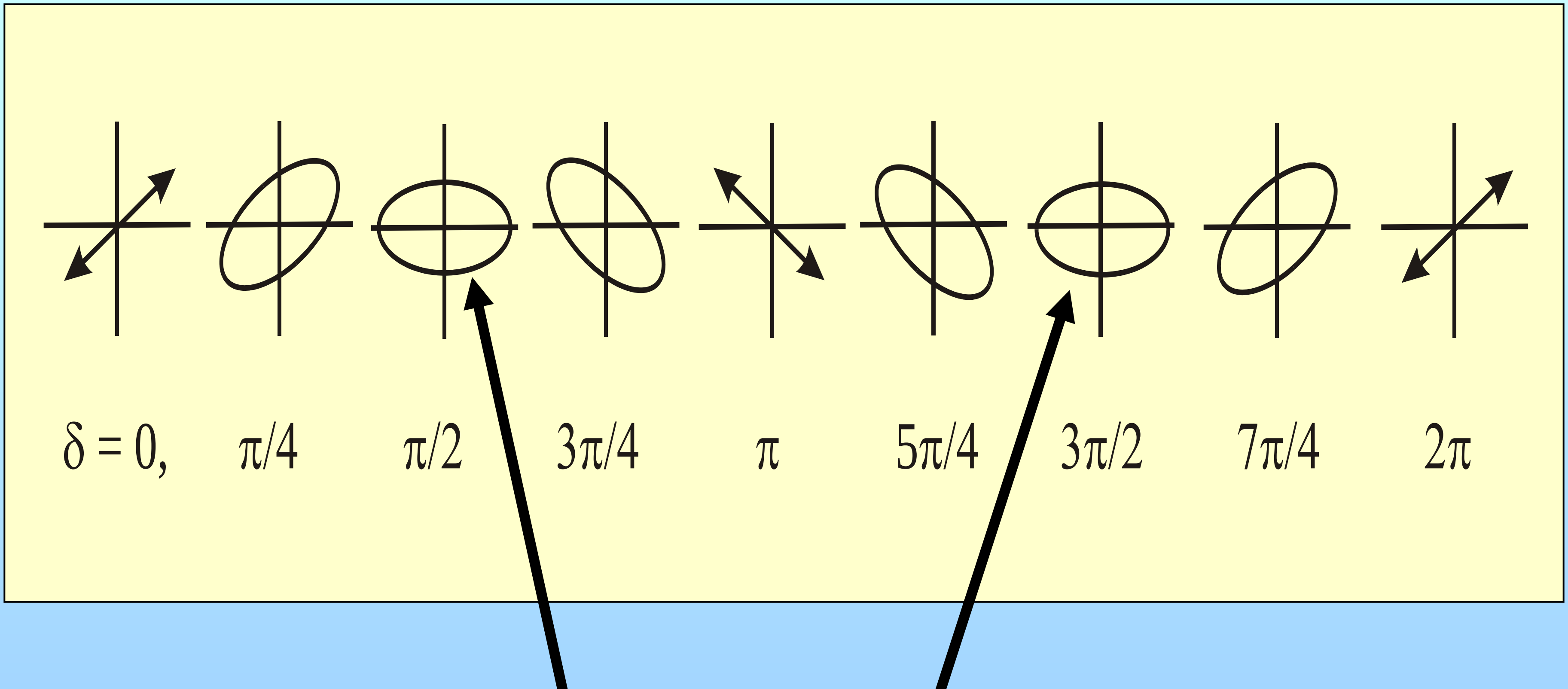
Unpolarized light defined by:

$$E_y = E_{oy}\cos(kx - \omega t) \qquad E_z = E_{oz}\cos(kx - \omega t - \delta(t))$$

- $\delta = \delta(t)$ Phase varies randomly in time

unpolarized light



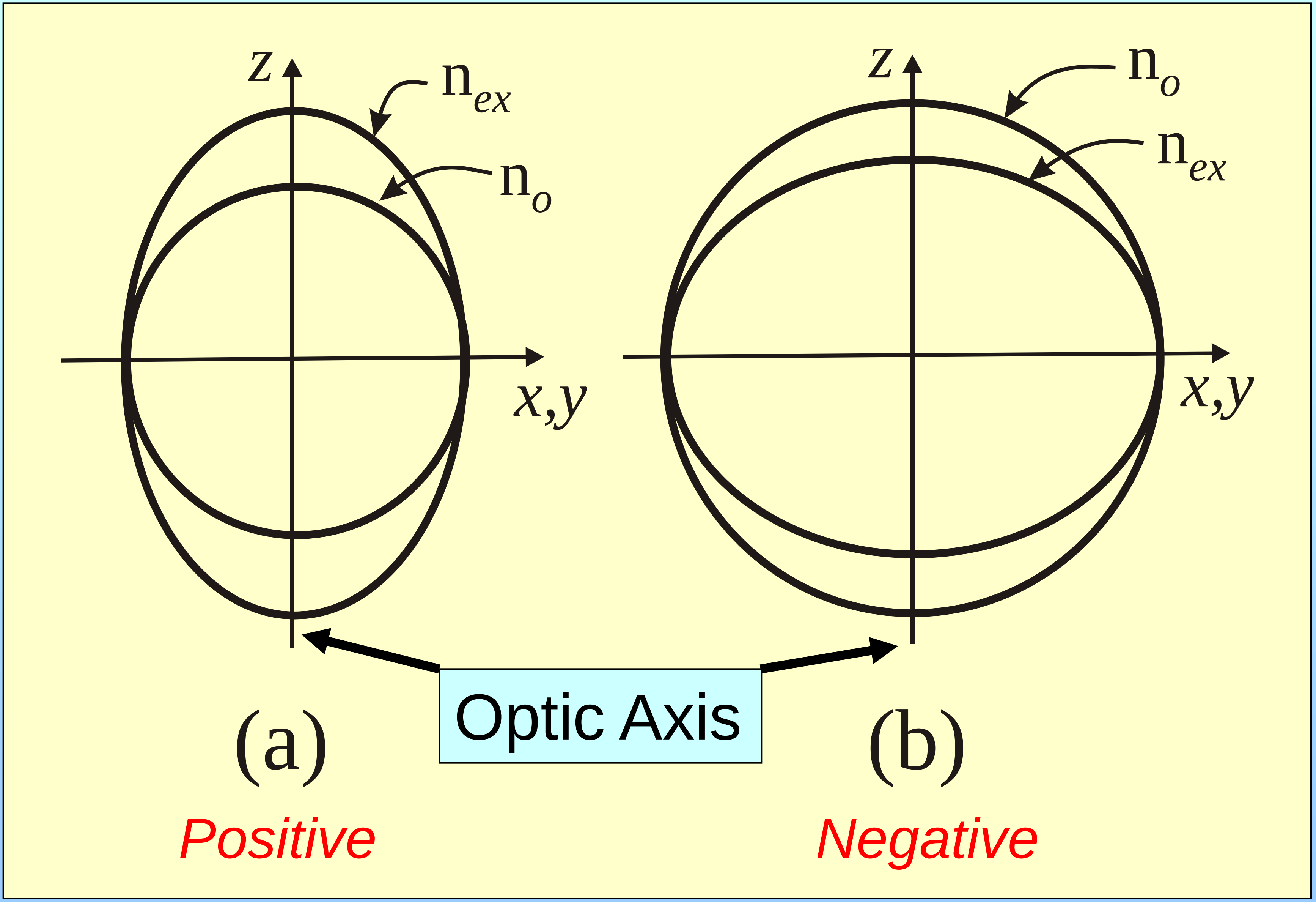


Note: if $E_{oy} = E_{oz}$
these ellipses become circles

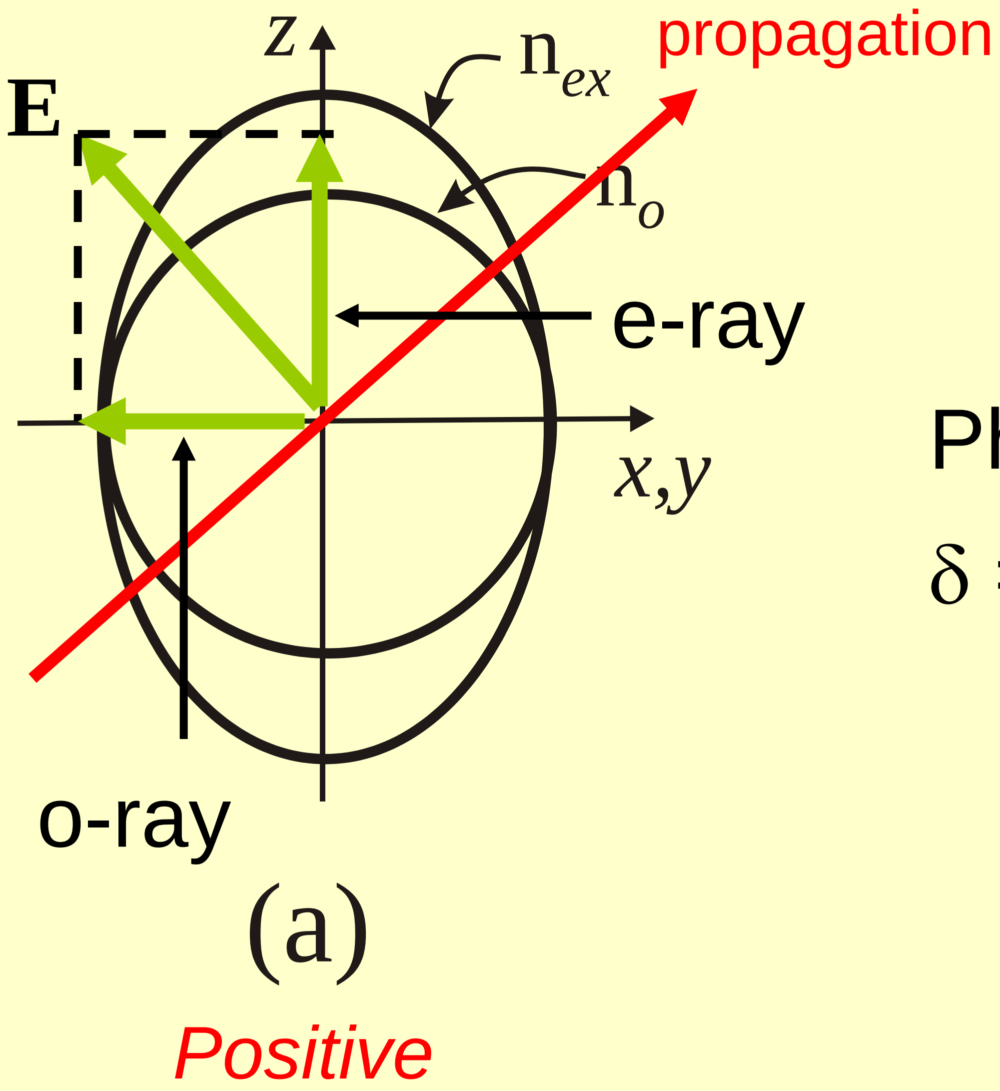
Polarized Light

- Production
- Analysis
- Manipulation

Uniaxial Crystals



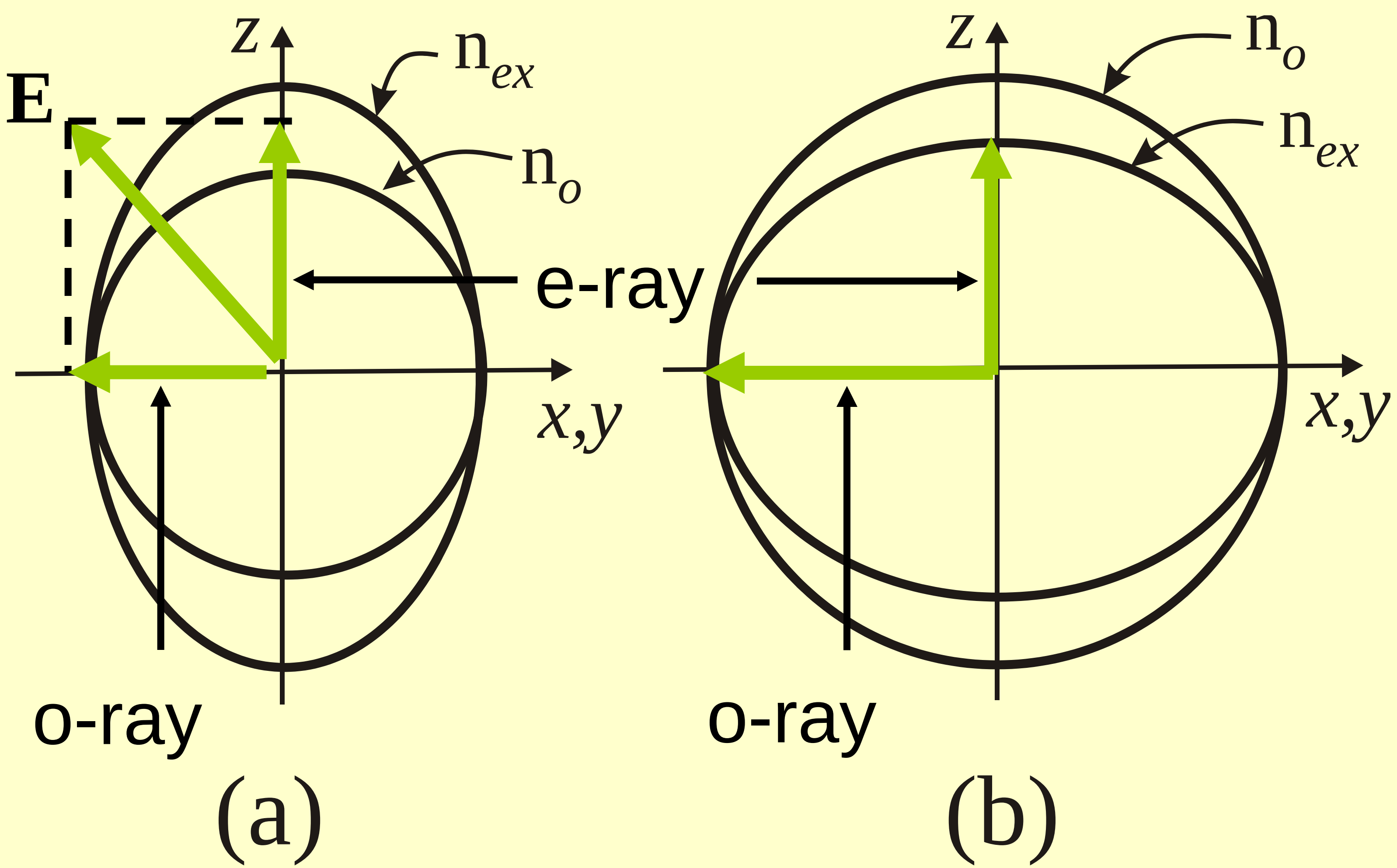
Uniaxial Crystals



Phase shift:

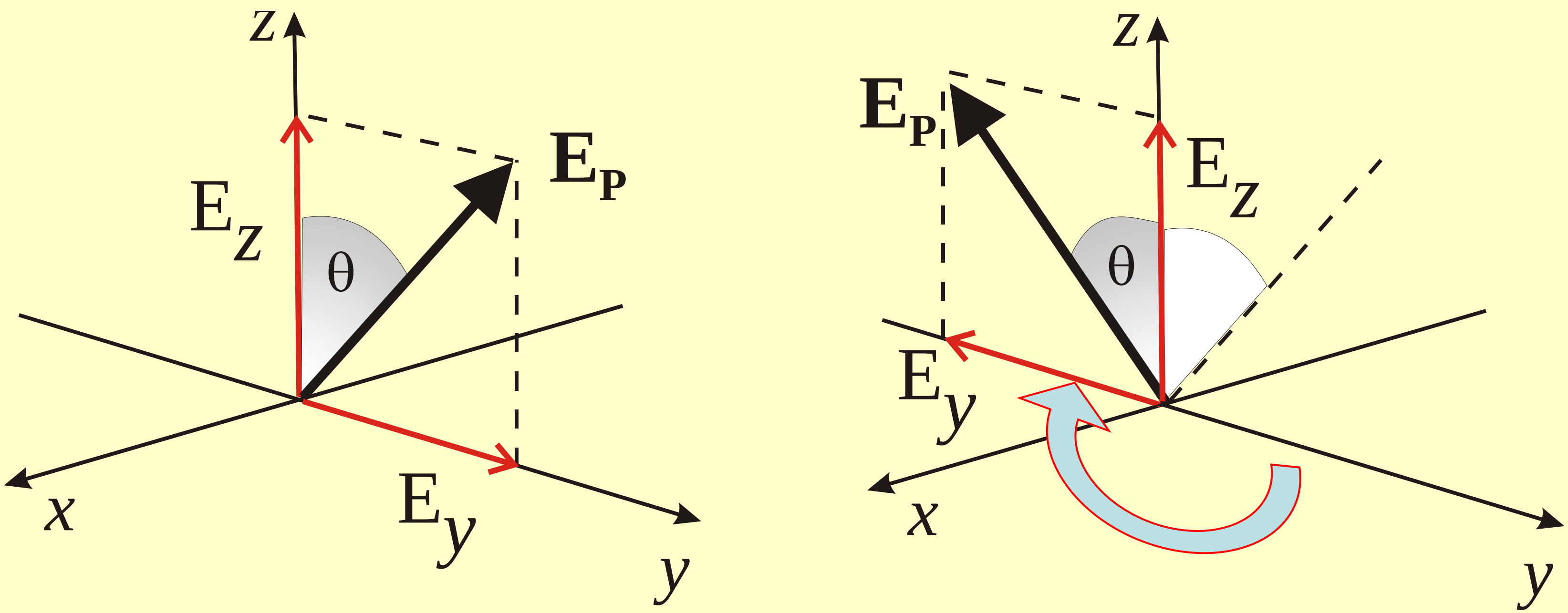
$$\delta = (2\pi/\lambda)[n_o - n_e]\ell$$

Uniaxial Crystals

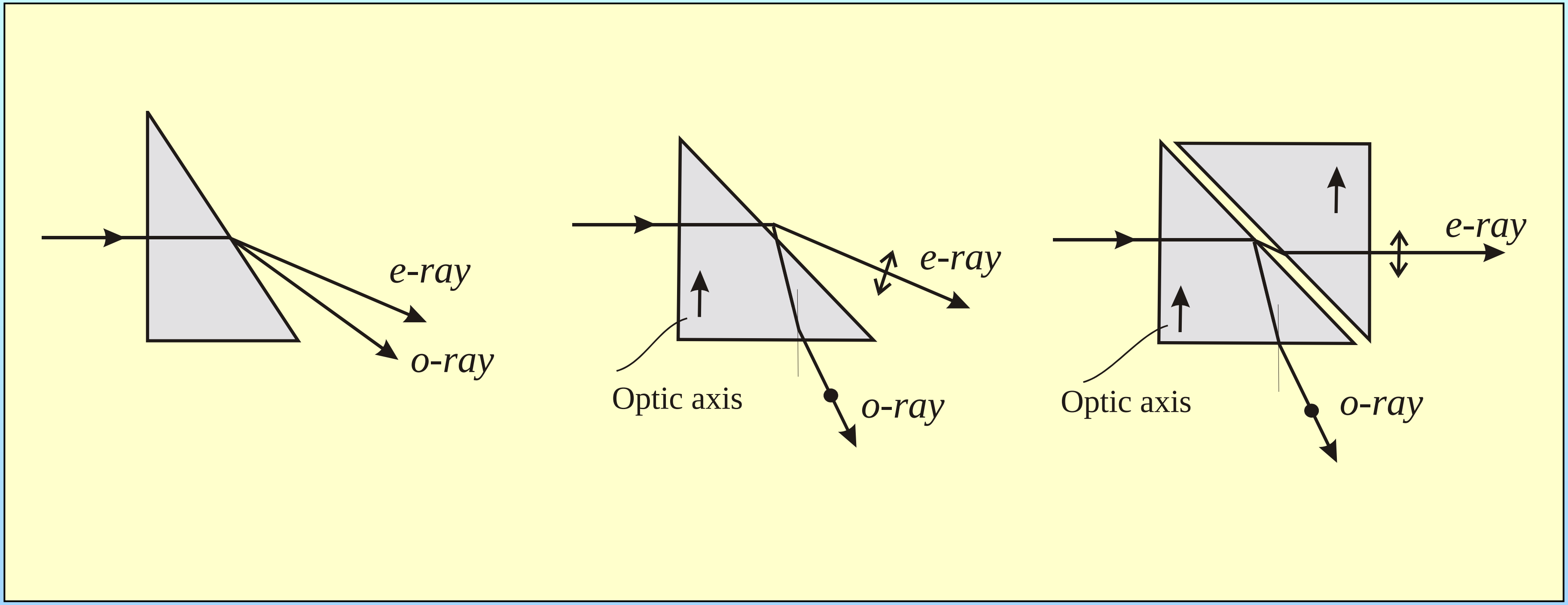


Positive *Negative*

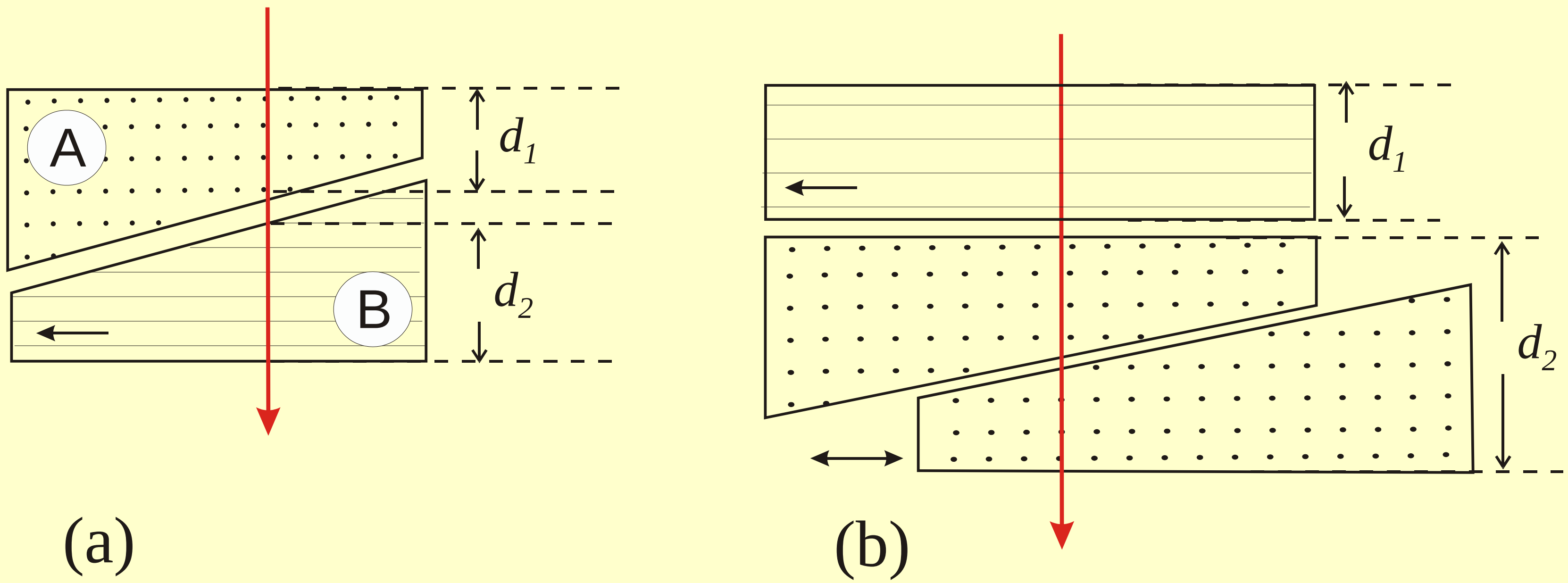
Linear Input	δ change	Output
$\theta = 45^{O}$, $E_{oy}=E_{oz}$	$\pm\pi/2$, $\lambda/4$	Left/Right Circular
$\theta \neq 45^{O}$, $E_{oy}\neq E_{oz}$	$\pm\pi/2$, $\lambda/4$	L/R Elliptical
$\theta \neq 45^{O}$, $E_{oy}\neq E_{oz}$	$\delta \neq \pi/2, \lambda/4$	Elliptical tilted axis
$\theta \neq 45^{O}$, $E_{oy}\neq E_{oz}$	$\delta = \pi$, $\lambda/2$	Linear at 2θ to input



Half-wave plate: introduces π-phase shift



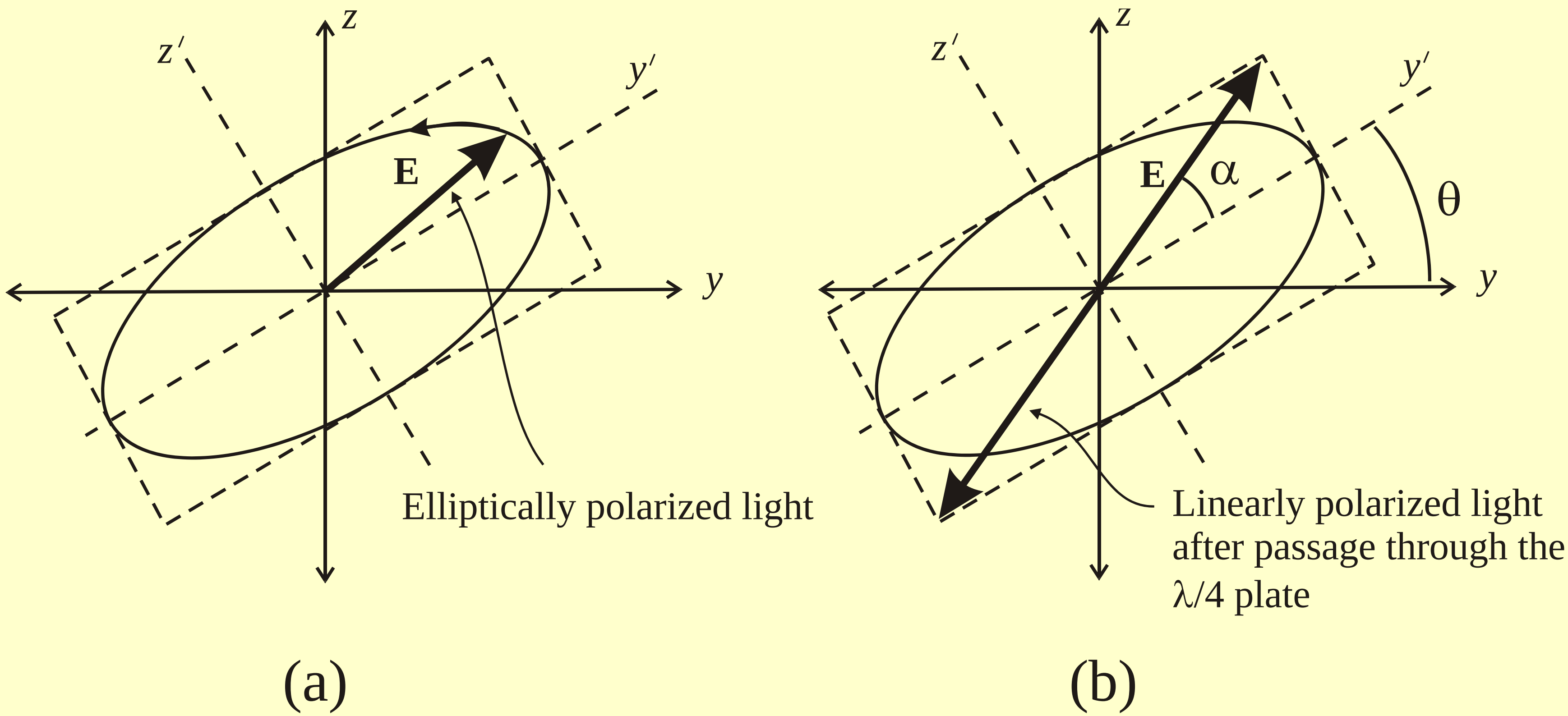
Prism Polarizers



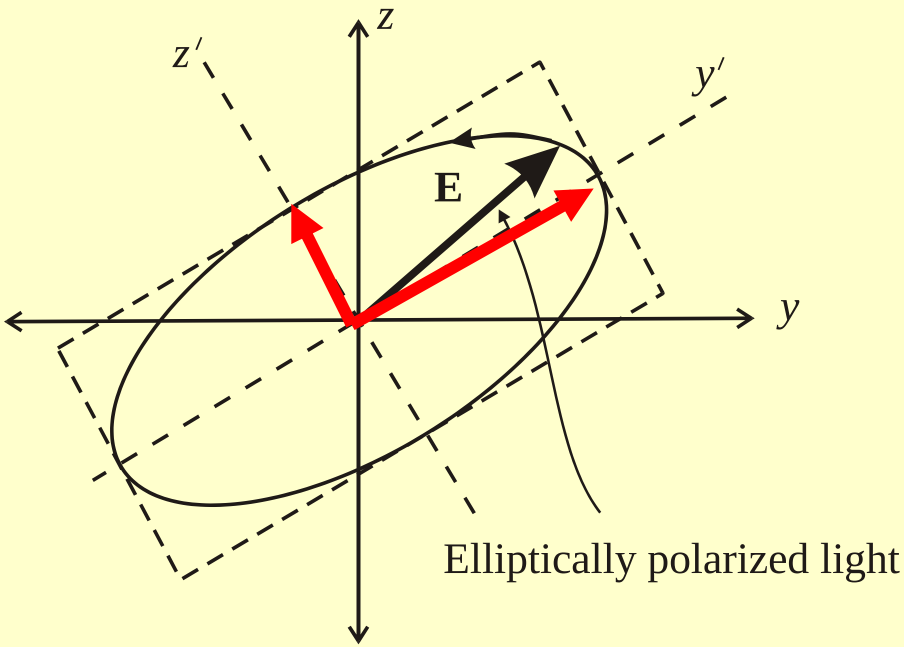
Babinet

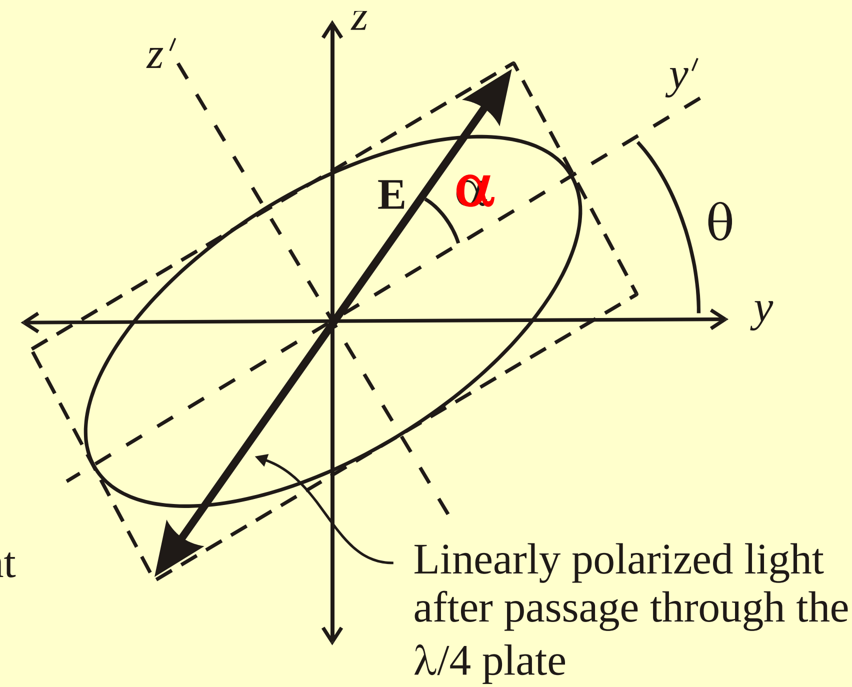
Babinet-Soleil

Analysis of polarized light



Analysis of polarized light **Find E_{oz}:E_{oy} and δ**



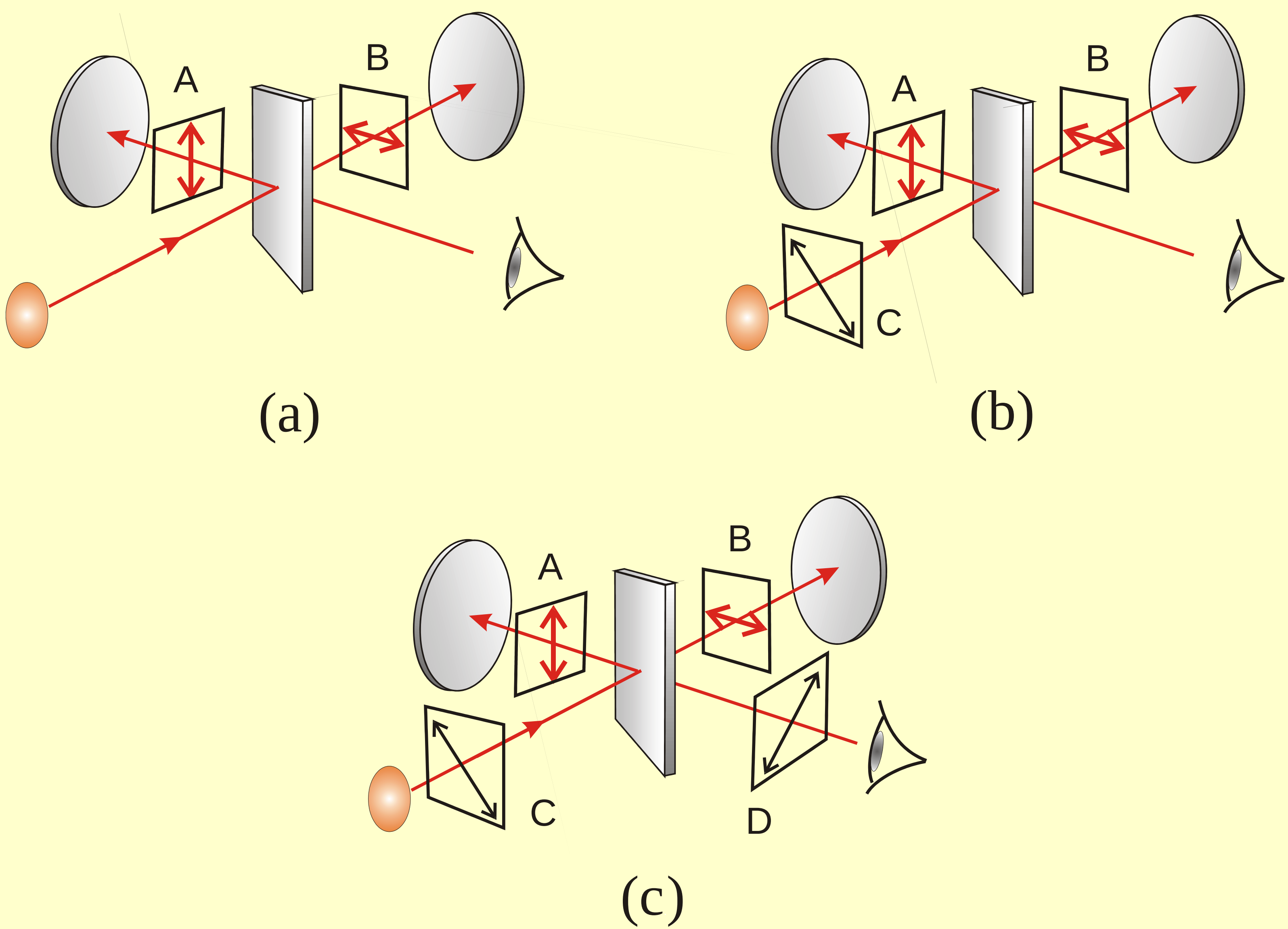


Find ratio of major:minor axes by rotating linear polarizer

$$E_{oz}/E_{oy} = \tan\alpha$$

Find angle of Linear polarization using λ/4 plate and linear polarizer

$$(\theta+\alpha)\rightarrow\theta,\ \tan 2\theta\rightarrow\cos\delta$$


A
B
(a)
A
B
C
(b)
A
B
C
D
(c)

The End