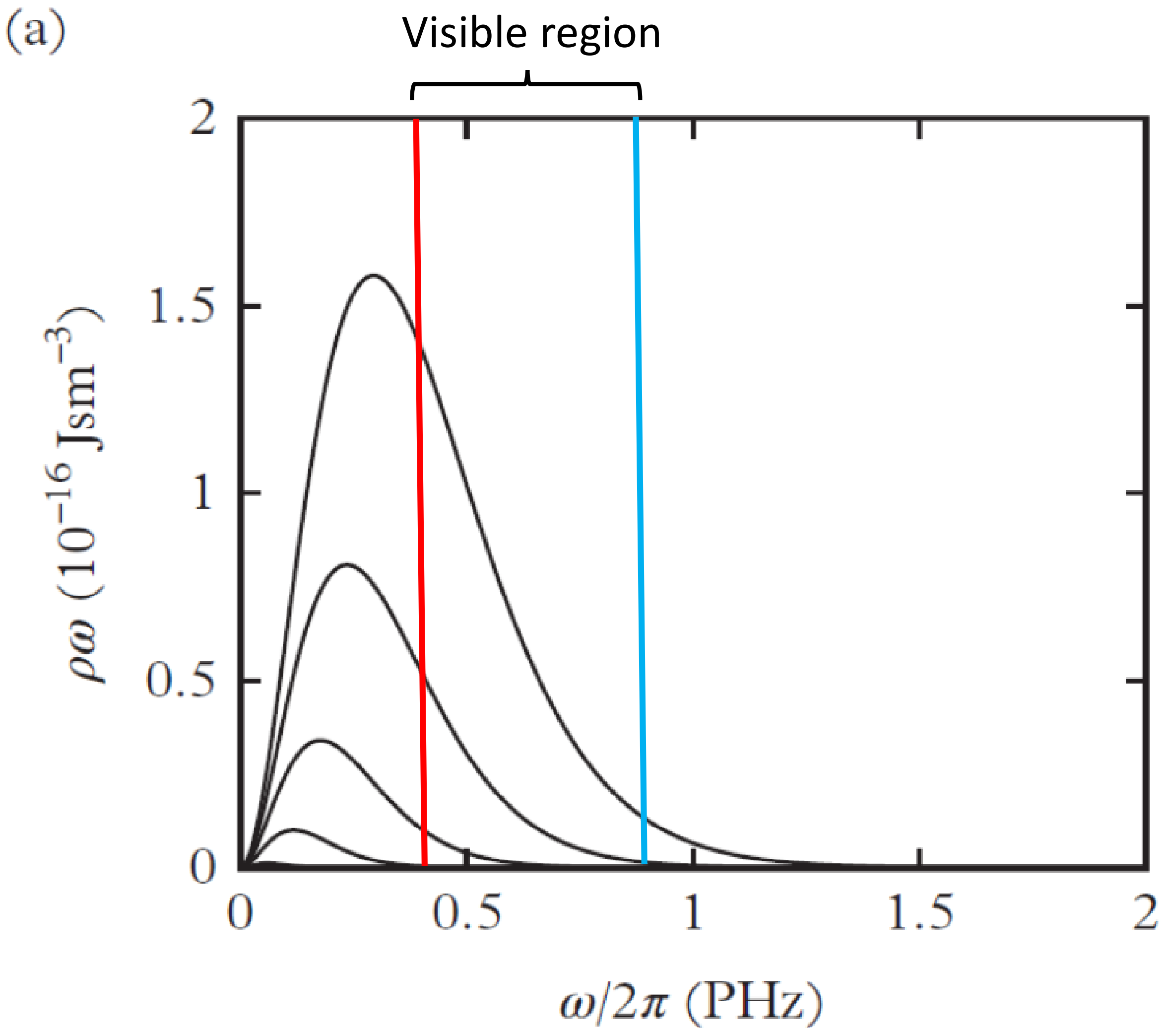
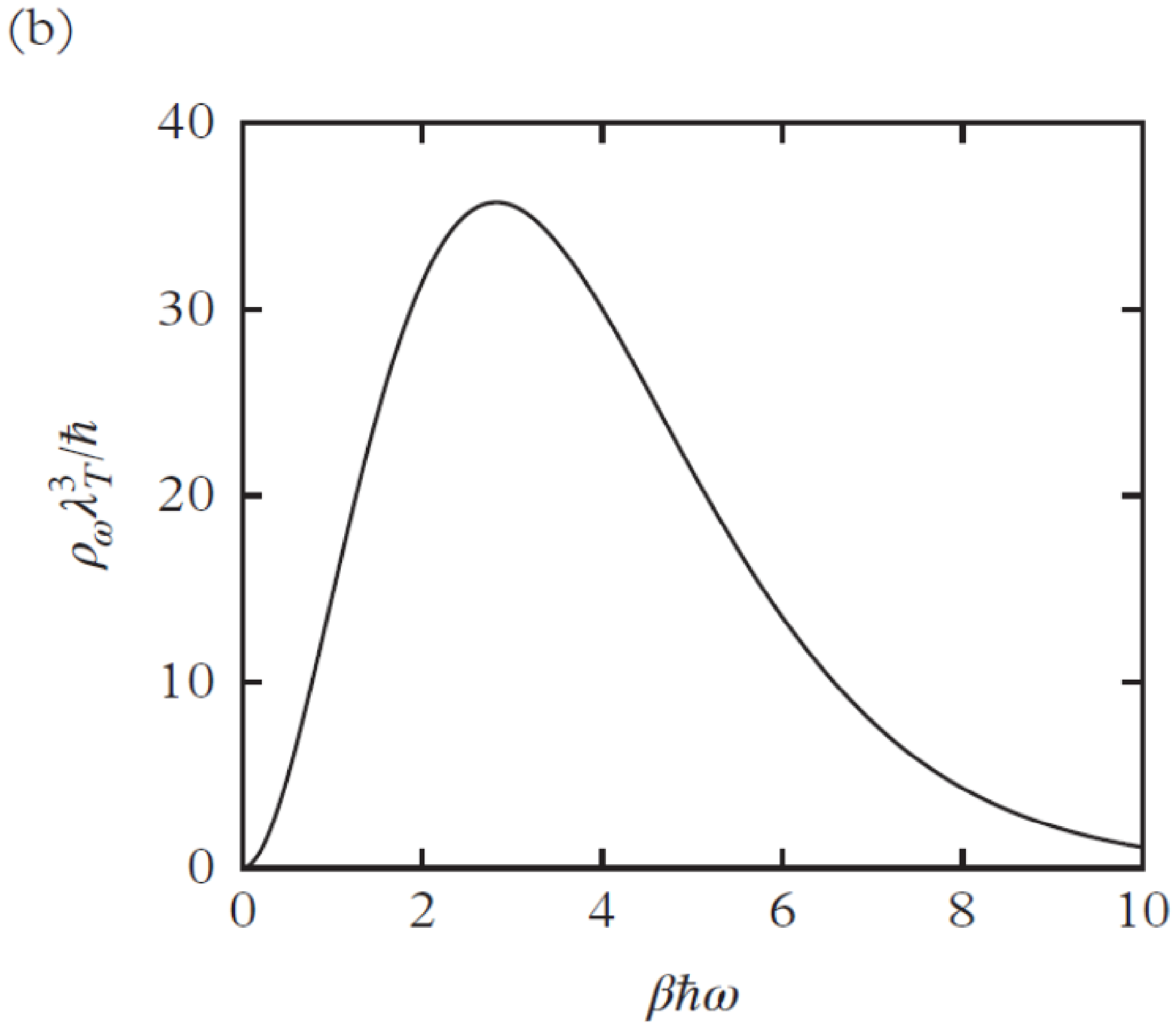
Thermodynamics lecture 9.

1. Wien’s laws:
   1. Wien’s displacement law
   2. Wien’s distribution law
2. Statistical mechanics of cavity radiation,
   Model 1: distinguishable modes (plane waves)
3. Energy, partition function, etc.
4. Model 2: indistinguishable photons
   (conceptually harder but equally important)



Spectral energy density at
1000, 2000, 3000, 4000, 5000 K

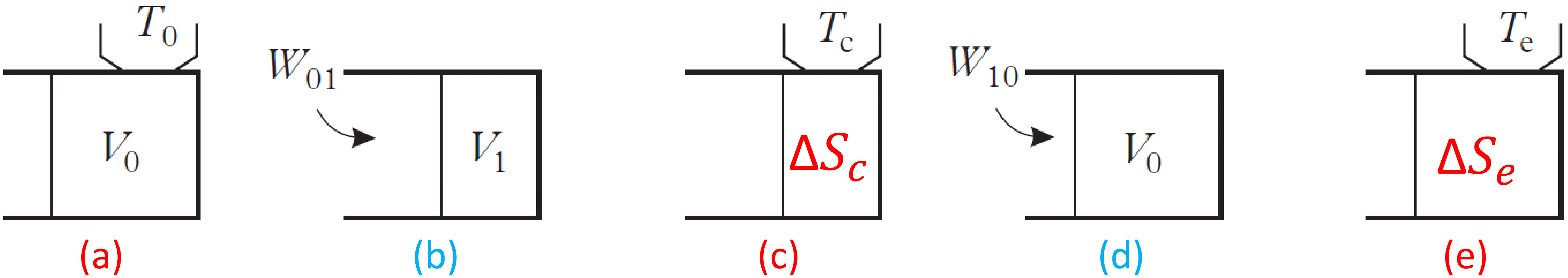


It's the same function each time,
scaled by λ_T^3

$$\lambda_T = 2\pi\hbar c/k_B T.$$

Slow expansion of cavity radiation in a reflecting cavity: does it remain in thermal equilibrium state?

Wien's argument to show the answer is *yes*:

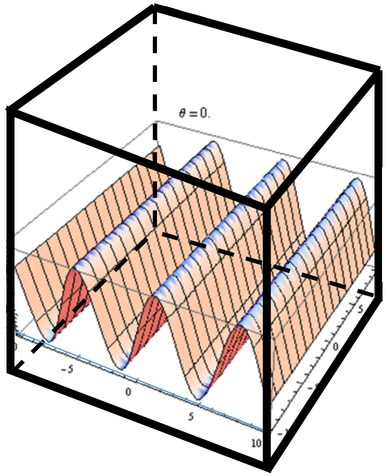


Stages (a), (c) and (e) :
Come to equil (at some temperature or other) with no net ΔU

Stages (b) and (d):
Adiabatic change of volume (so work is done and U changes and changes back)

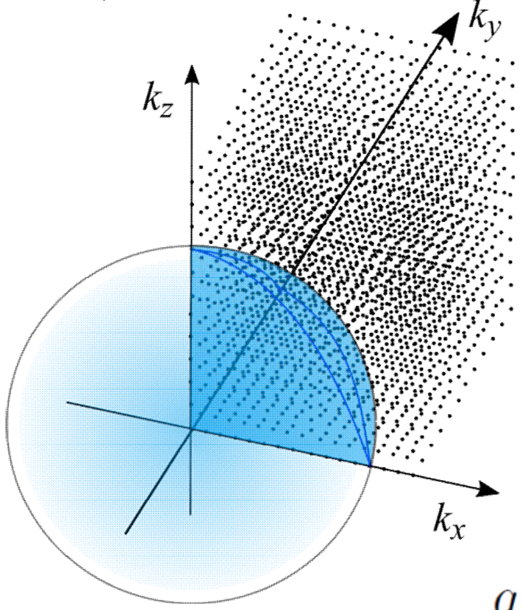
Modes of electromagnetic field

Standing waves in a box



$$\sin(k_x\, x)\sin(k_y\, y)\sin(k_z\, z)$$

$$\Delta k_x = \frac{\pi}{L}$$



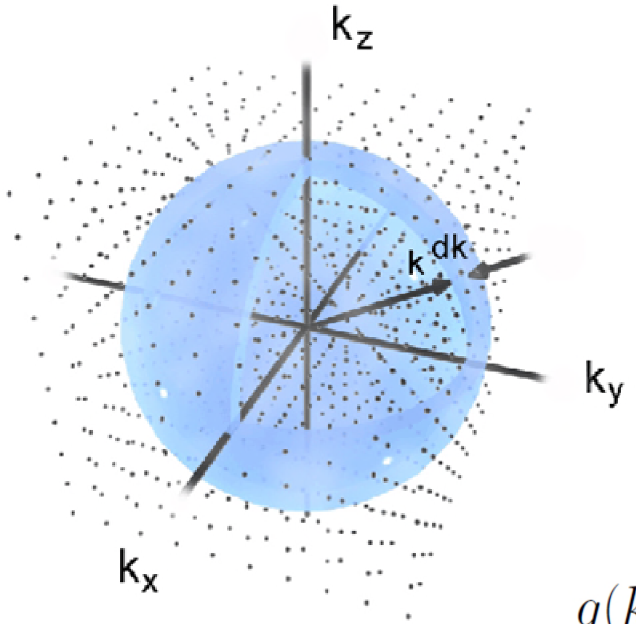
$$k_x, k_y, k_z > 0$$

$$g(k)\mathrm{dk} = \frac{1}{8} 4\pi \mathrm{k}^2 \frac{\mathrm{V}}{\pi^3} \mathrm{dk}$$

OR travelling waves with a *mathematical constraint*: must have period L.

Travelling waves,
$e^{ik_x x}\, e^{ik_y y}\, e^{ik_z z}$

$$\Delta k_x = \frac{2\pi}{L}$$



$$k_x, k_y, k_z$$

+ve or -ve

$$g(k)\mathrm{dk} = 4\pi \mathrm{k}^2 \frac{\mathrm{V}}{(2\pi)^3} \mathrm{dk}$$

Comparison between cavity radiation and an ideal gas

	Ideal gas			Cavity radiation
	U, N, V			U,V
N		N	$=$	bVT^3
p	$= nk_B T$	p	$=$	$0.9\, nk_B T$
U	$= \frac{1}{\gamma-1} Nk_B T,$	U	$=$	$2.7 Nk_B T,$
p	$= (\gamma - 1)u$	p	$=$	$\frac{1}{3} u$
S	$= Nk_B \left(\ln \left[a \frac{T^{1/(\gamma-1)}}{n} \right] \right)$	S	$=$	$3.6 Nk_B$
	$= \frac{U}{T} (\ln T - (\gamma - 1) \ln(n/a))$		$=$	$\frac{4}{3} \frac{U}{T}$
μ	$= (U - TS + pV)/N$	μ	$=$	0
	$= (\gamma u - Ts)/n$			

$$b \simeq 2.03 \times 10^7 \, \mathrm{m^{-3} K^{-3}}$$