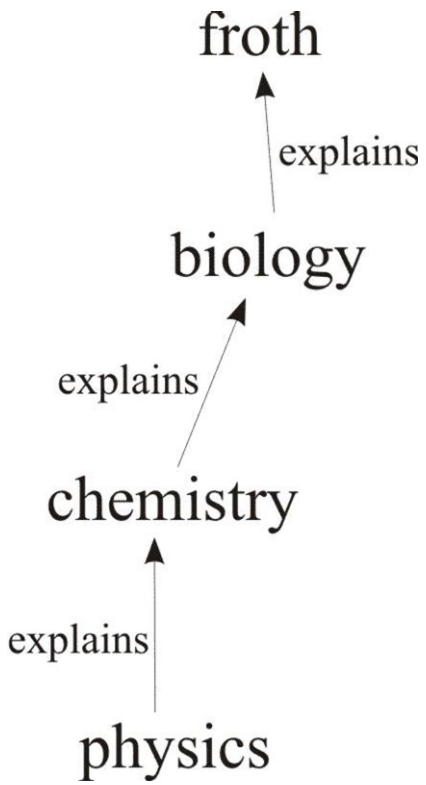
Thermodynamics lecture 14.

The structure of scientific reasoning and the nature of the physical world

Examples from thermal physics
Electromagnetism
…
Science more generally

The structure of science



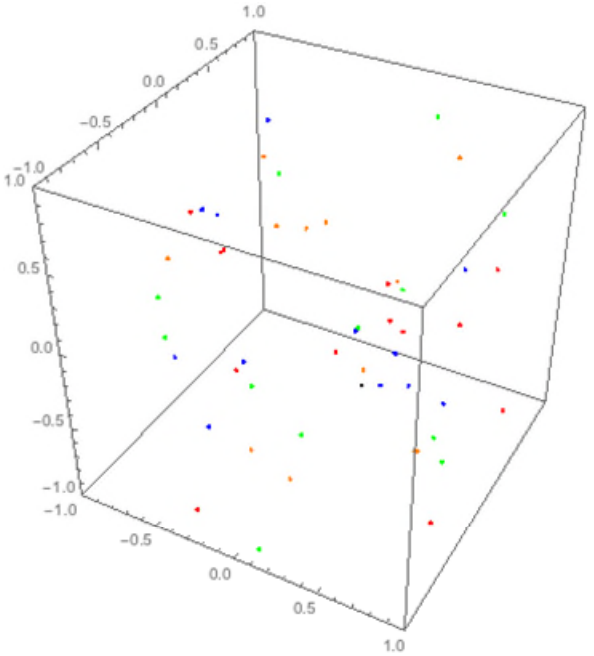
WRONG!

A gas

Kinetic theory:

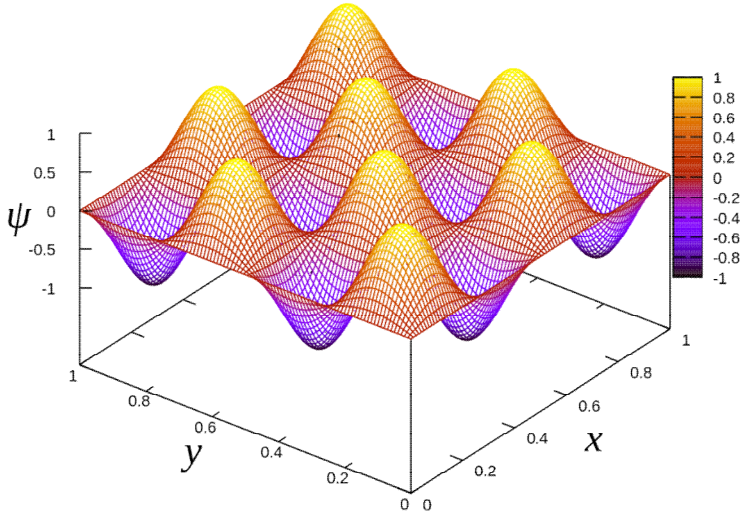
particles bouncing off the walls

$$P = \frac{1}{3} nm\langle v^2 \rangle \qquad \frac{1}{2} m\langle v^2 \rangle = \frac{3}{2} k_{\mathrm{B}} T$$



Statistical and quantum model:

waves extended throughout the chamber



Arguing from global principles

$$\mathrm{d}U = T\mathrm{d}S - p\mathrm{d}V + \mu\mathrm{d}N$$

1st and 2nd laws
(cons. of energy; nature of entropy)

$$F = U - TS$$

A convenient function (Helmholtz)

$$\mu = \left.\frac{\partial F}{\partial N}\right|_{T,V}$$

$$F = \mu N - pV$$

can be obtained from the above,

Hence

$$pV = N \left.\frac{\partial F}{\partial N}\right|_{T,V} - F.$$

For any system characterised by pressure and volume

How will a gas behave?

Now suppose:

1. A fixed, large number of parts: *N*
2. Which move independently
3. Which are all the same as each other (the system state does not change if two parts are swapped)
4. That have states of motion that can in principle be counted (but note, we don't know or care how the energy relates to the momentum, nor if the parts are particle-like or wave-like or something else).

4. → $F = -k_B T \ln Z_N$

Standard statistical reasoning (max entropy)

(2,3) → $Z_N = \frac{z_1^N}{N!}$

"single particle partition function"; it depends on things like energy, momentum and possible states of motion of each part

First recall

$$pV = N \left.\frac{\partial F}{\partial N}\right|_{T,V} - F.$$

Now use

$$\begin{aligned} F &= -k_{\mathrm{B}} T \ln(z_1^N / N!) \\ &= -N k_{\mathrm{B}} T \left(\ln z_1 - \ln N + 1\right) \end{aligned}$$

so

$$\left.\frac{\partial F}{\partial N}\right|_{T,V} = -k_{\mathrm{B}} T (\ln z_1 - \ln N).$$

Hence: Boyle's law!

$$pV = N k_{\mathrm{B}} T$$

See Steane, *Science and Humanity*, OUP 2018

So the answer to the question

“Is it particles bouncing in a box or quantum waves filling the box?”

is

“*it does not matter*”

(so far is the question concerns the equation of state).

Nor does it matter whether the parts are slow (Newtonian) or fast (relativistic) etc.

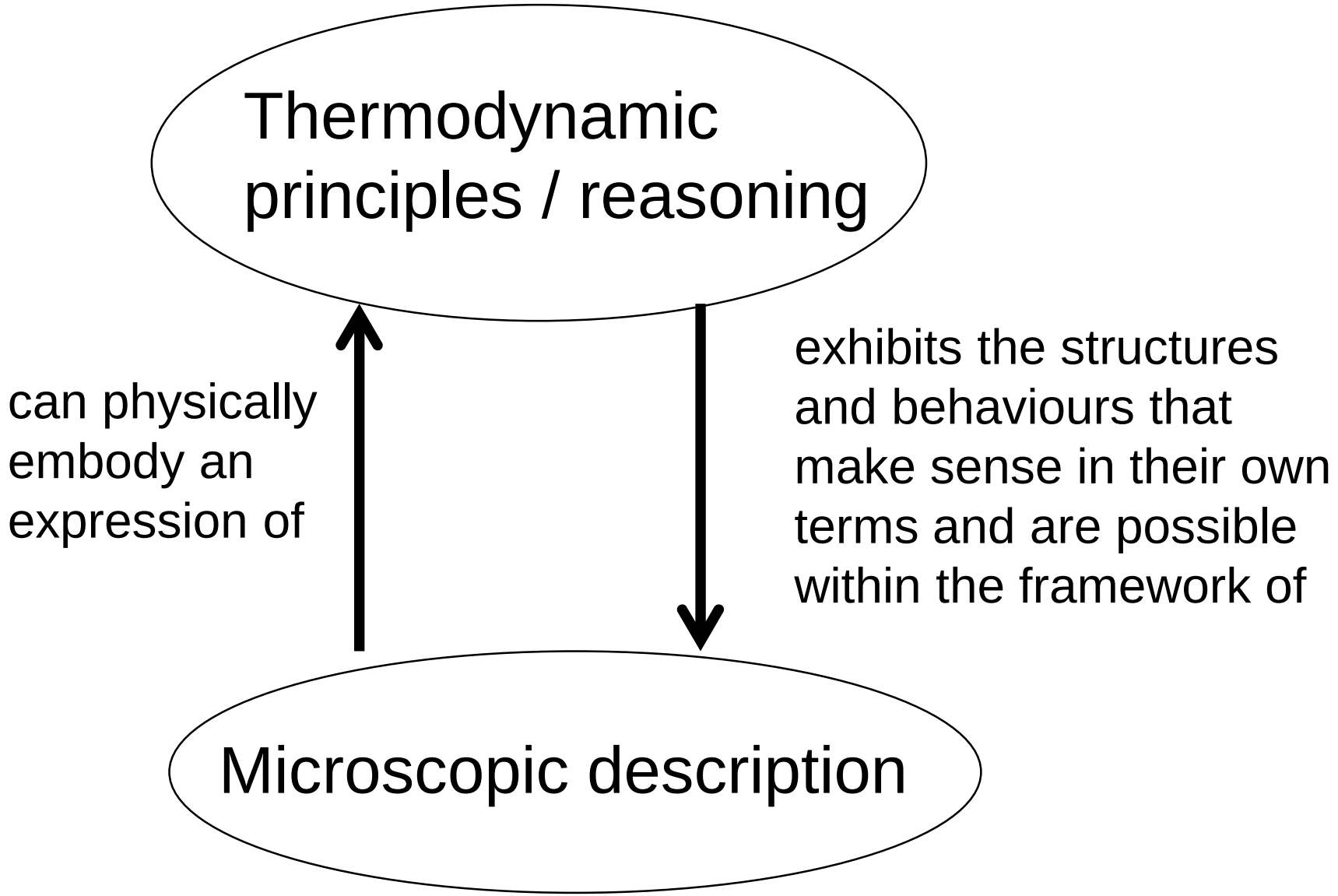
Further examples

- $$\frac{\kappa_T}{\kappa_S} = \frac{C_p}{C_V}$$
- $$C_p - C_V = -T \left.\frac{\partial V}{\partial T}\right|_p^2 \left.\frac{\partial p}{\partial V}\right|_T = \frac{TV\alpha^2}{\kappa_T}$$

- Clausius-Clapeyron equation $$\frac{\mathrm{d}p}{\mathrm{d}T} = \frac{L}{T\Delta V}$$
- Wien distribution law $$\rho_\omega(\omega, T) = T^3 \rho_\omega\left(\frac{\omega}{T}, 1\right)$$

- Van t' Hoff equation (concerns chemical equilibrium)
- Van der Waals theory of phase change …

The structure of explanation within thermal physics



Example from electro-magnetism

$$
\begin{aligned}
\operatorname{div}\,\mathbf{E} &= \rho/\epsilon_0 \\
\operatorname{div}\,\mathbf{B} &= 0 \\
\operatorname{curl}\,\mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
c^2 \operatorname{curl}\,\mathbf{B} &= \frac{\mathbf{j}}{\epsilon_0} + \frac{\partial \mathbf{E}}{\partial t}
\end{aligned}
$$

(Maxwell's equations for the electro-magnetic field)

Lorentz-FitzGerald contraction of moving objects

$$L = L_0\sqrt{1 - v^2/c^2}$$

1. Principle of relativity
2. Invariance of the speed of light

(Einstein 1905)

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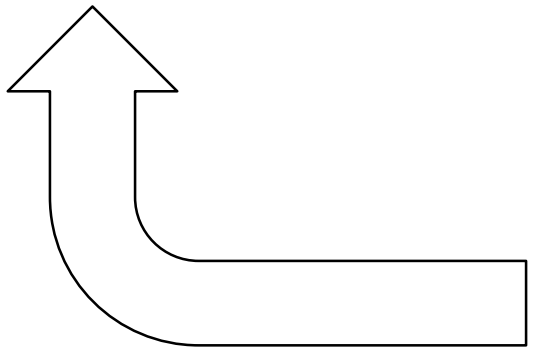


symmetry principle

(Principle of relativity and invariance of speed of light)

~~The symmetry principle "does not add anything."~~

It does not add anything because it has already added everything.

Further symmetry principles

Translational invariance

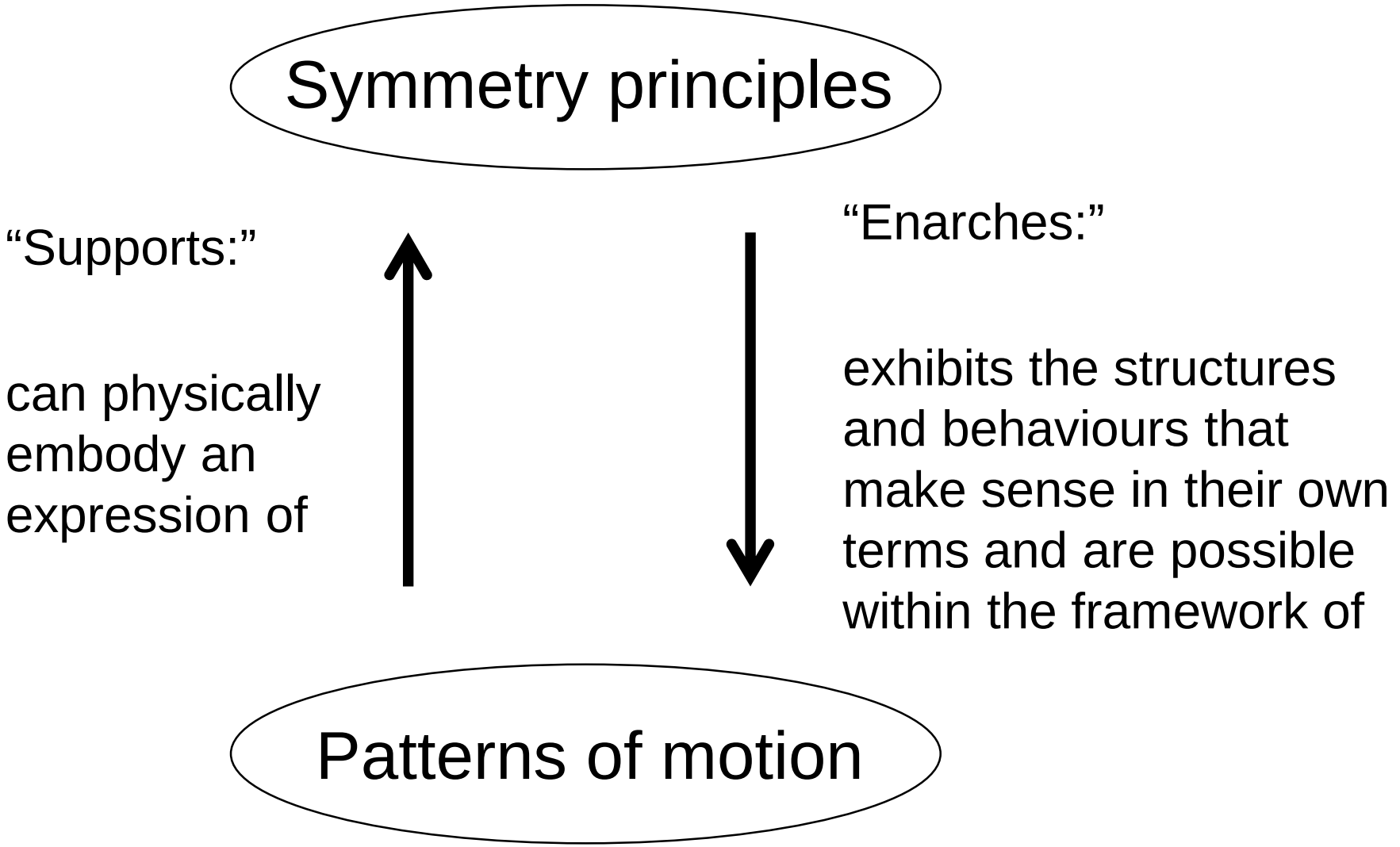
Relativity of inertial motion (Lorentz covariance)

General relativity

Charge conjugation, parity etc.

1st and 2nd laws of thermodynamics

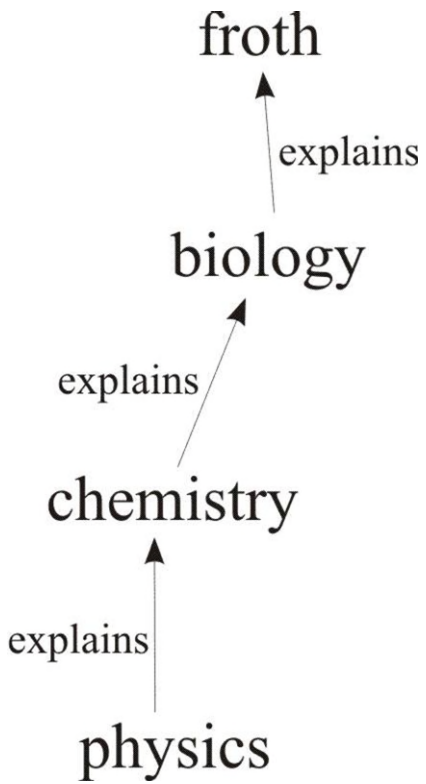
The structure of explanation within physics



Example: arches and stones

Even if all arches everywhere were made of stone, it would still be false to say that arches are explained by stones.

The structure of science



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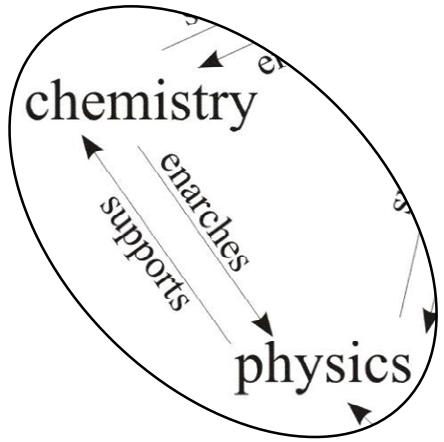

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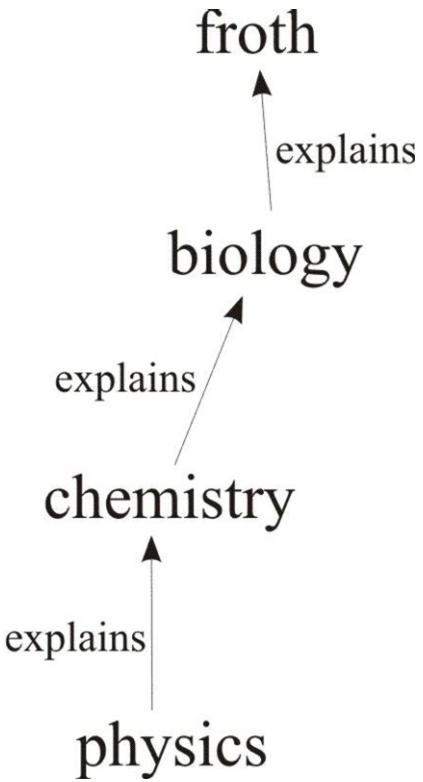

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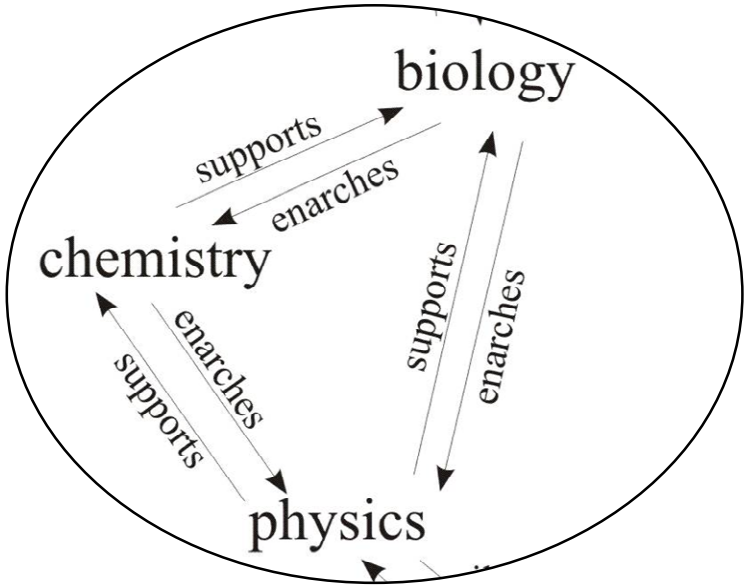

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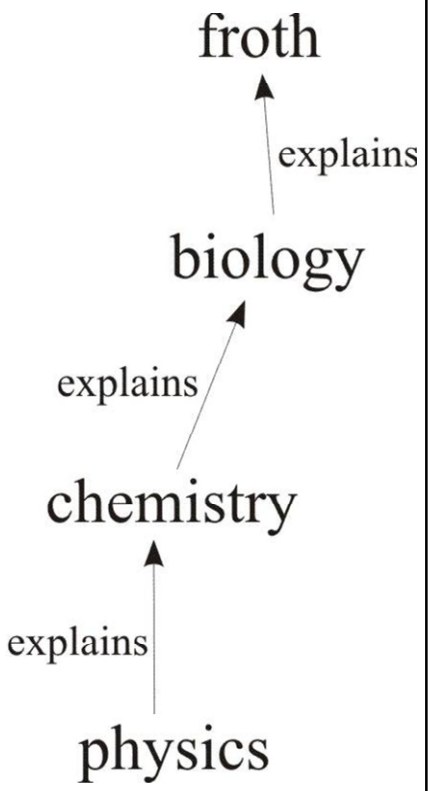

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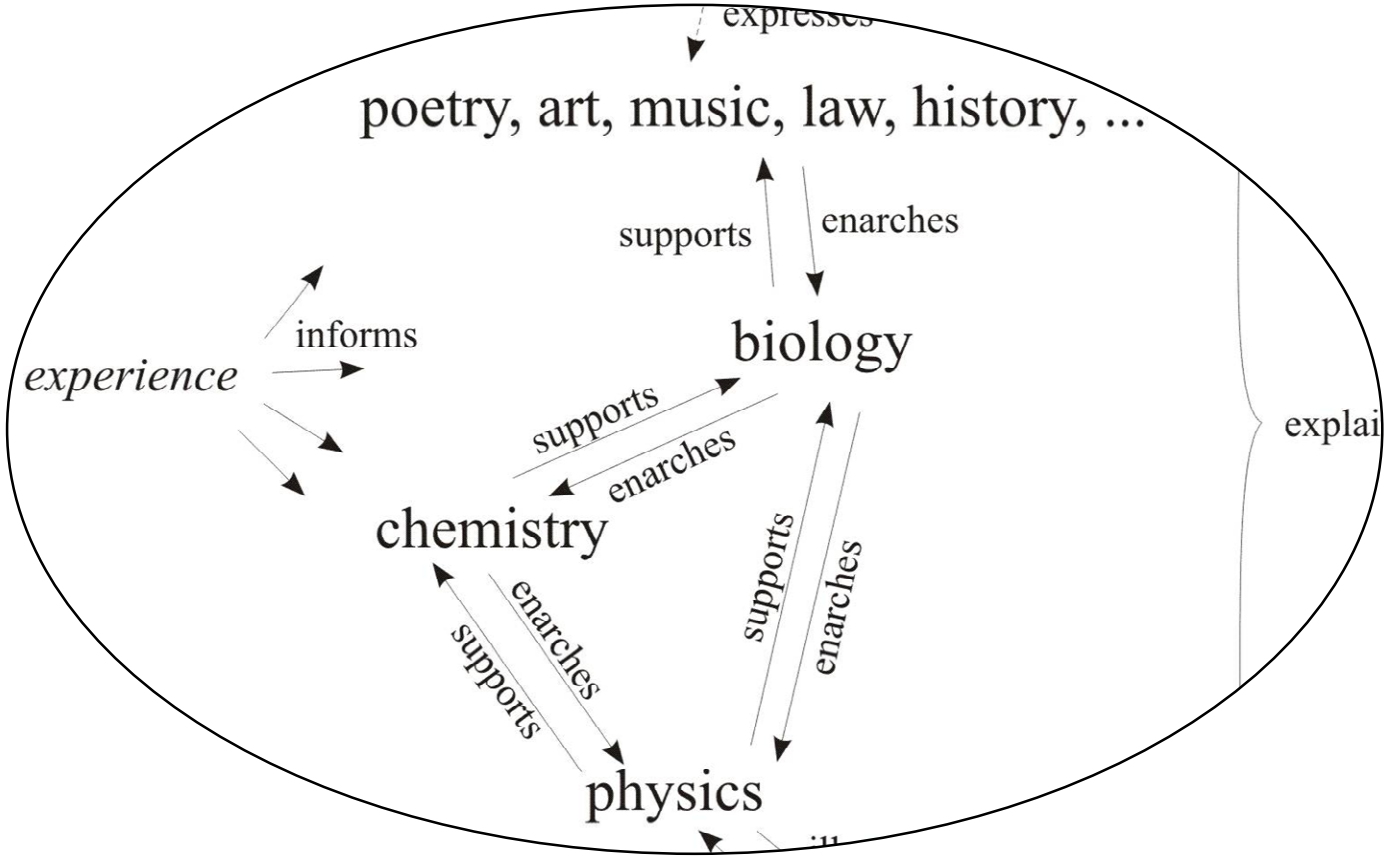

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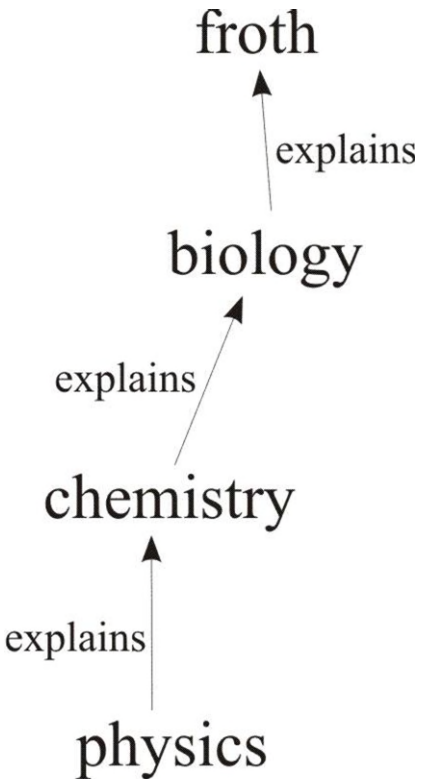

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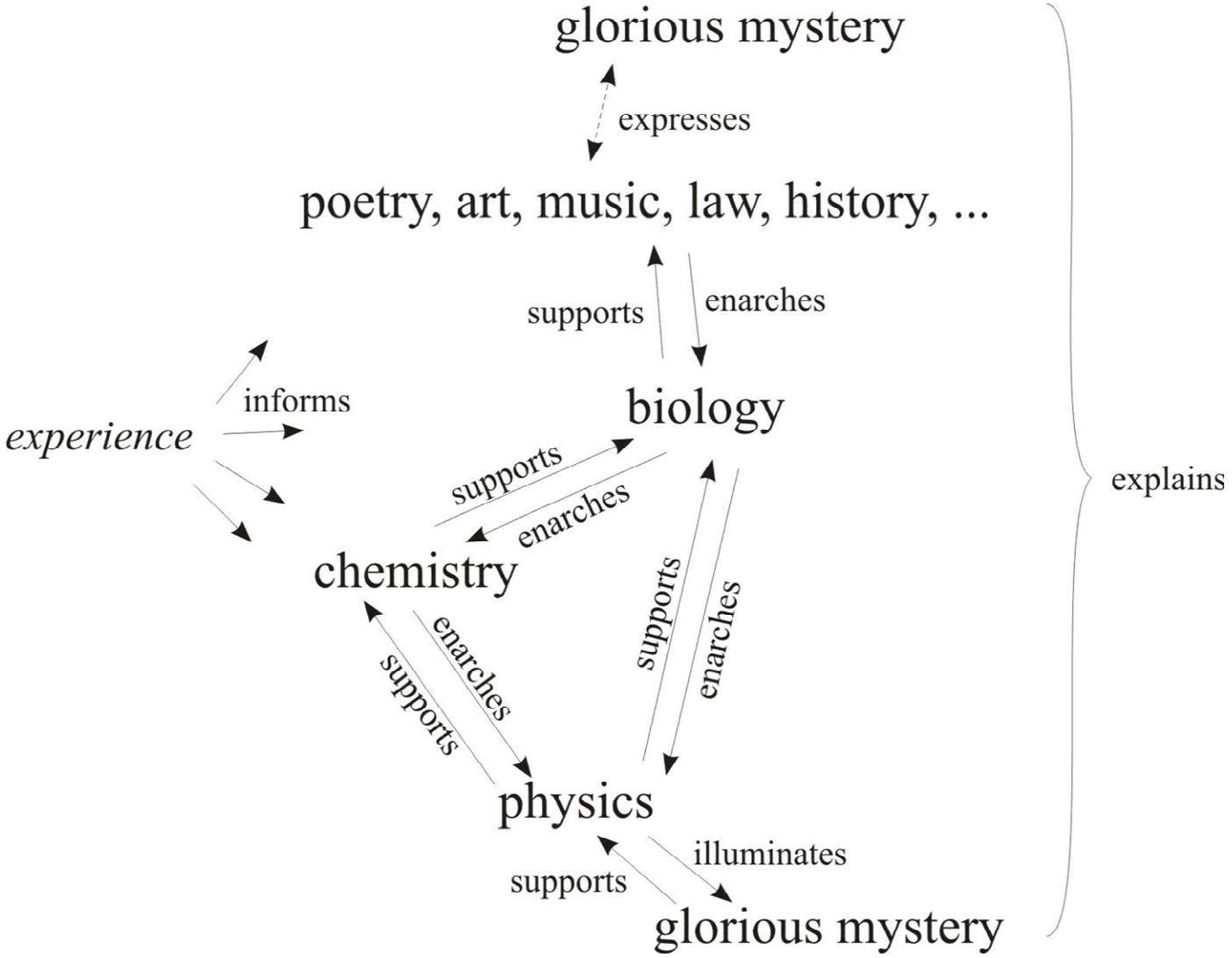

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The Embodiment Principle:

“Science is about building up an insightful picture in which the underlying microscopic dynamics do not replace, nor do they explain, the most significant larger principles, but rather they give examples of how those larger principles come to be physically embodied in particular cases. The lower level and higher level principles are in a reciprocal relationship of mutual consistency in which each illuminates the other.”

(A. M. Steane, *Science and Humanity*, OUP 2018)