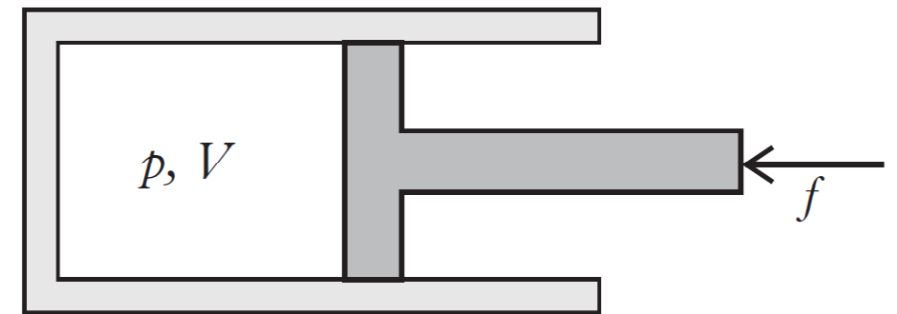
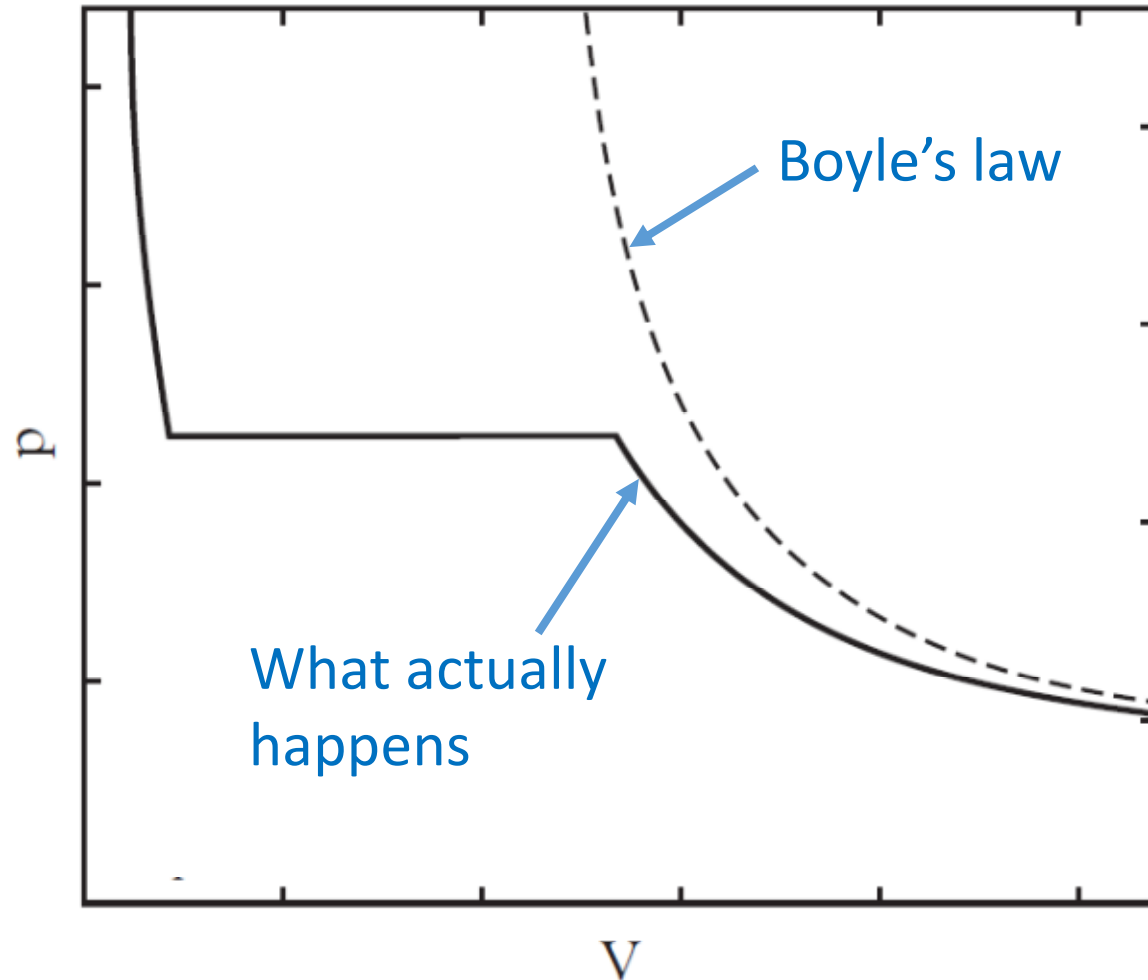
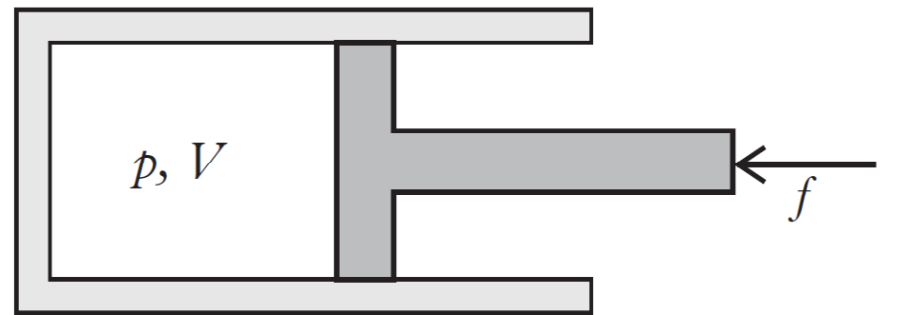
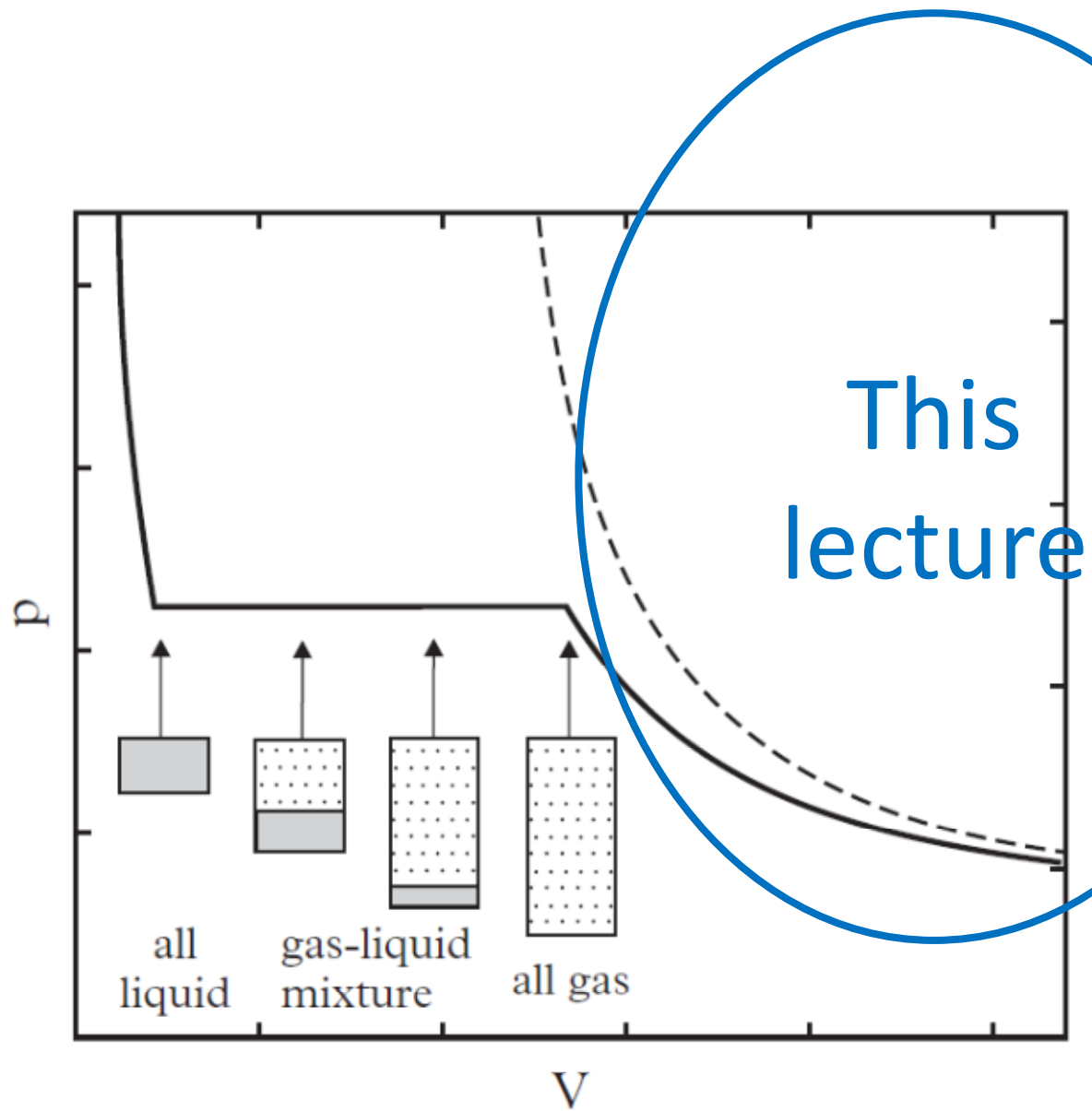


Thermodynamics lecture 10. Real gases and flow processes

1. Real (i.e. non-ideal) gases, critical point, critical parameters
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4. Linde process for liquefaction
5. General *flow process*

Compressing an ordinary substance at constant temperature





van der Waals' equation

$$\left(p + \frac{a}{V_m^2} \right) (V_m - b) = RT.$$

Redlich–Kwong equation

$$\left(p + \frac{a}{V_m(V_m + b)T^{1/2}} \right) (V_m - b) = RT.$$

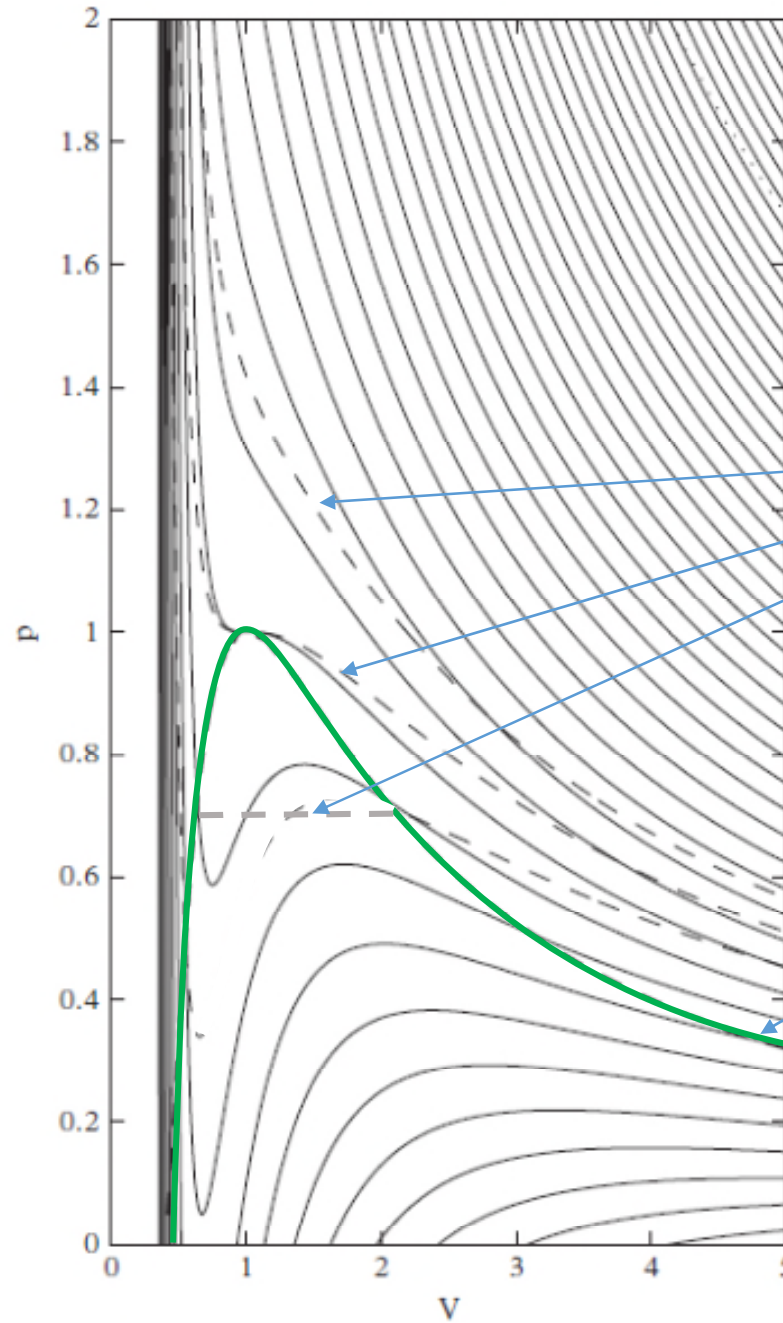
Dieterici's equation

$$p(V_m - b) e^{a/(RTV_m)} = RT.$$

Virial expansion

$$PV_m = RT \left(1 + \frac{B_2}{V_m} + \frac{B_3}{V_m^2} + \frac{B_4}{V_m^3} + \dots \right)$$

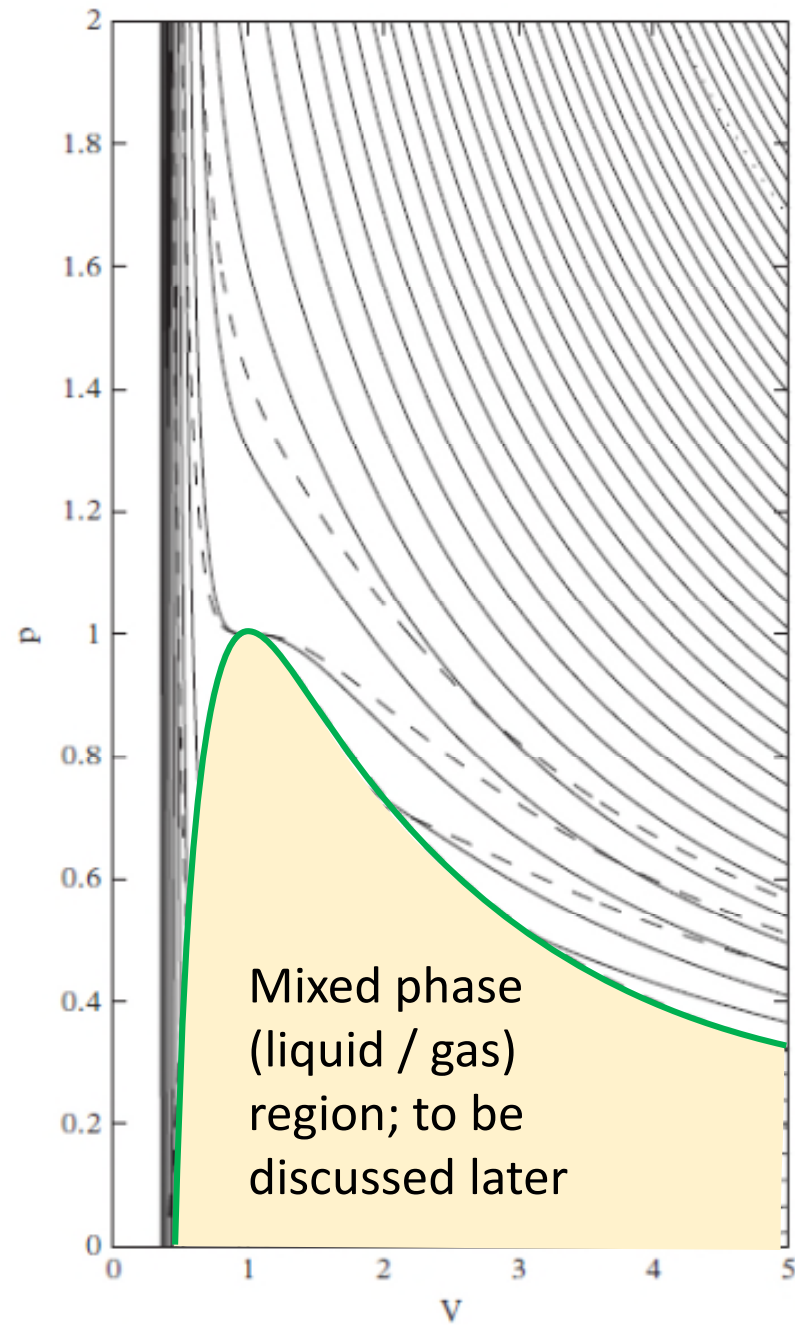
Isotherms
predicted by
van der Waals
equation



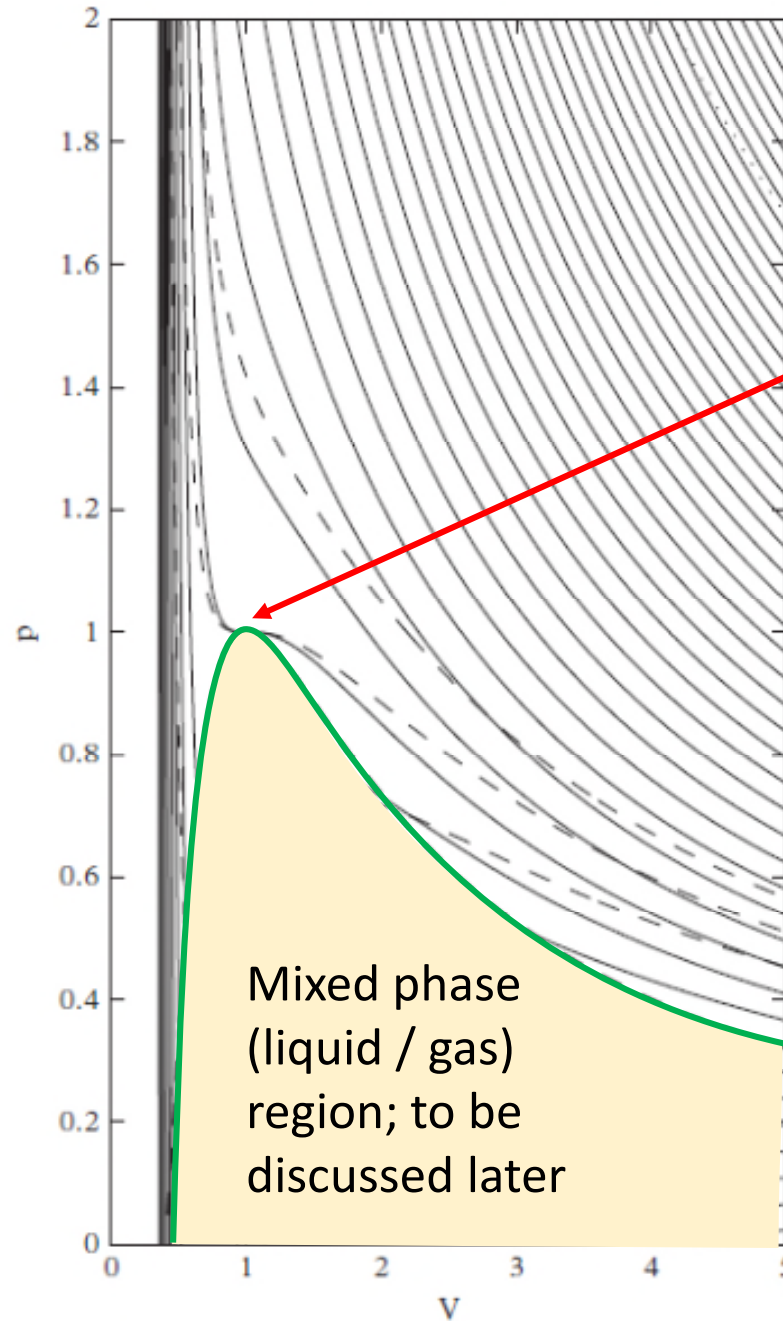
Dashed lines:
what a real
fluid does

Green: under here is
the dual phase
(liquid/vapour)
region

Isotherms
predicted by
van der Waals
equation



Isotherms
predicted by
van der Waals
equation



Critical point:

Isotherm has a
stationary point:

$$(\partial p / \partial V)_T = 0$$

which is also a point
of inflexion:

$$(\partial^2 p / \partial V^2)_T = 0$$

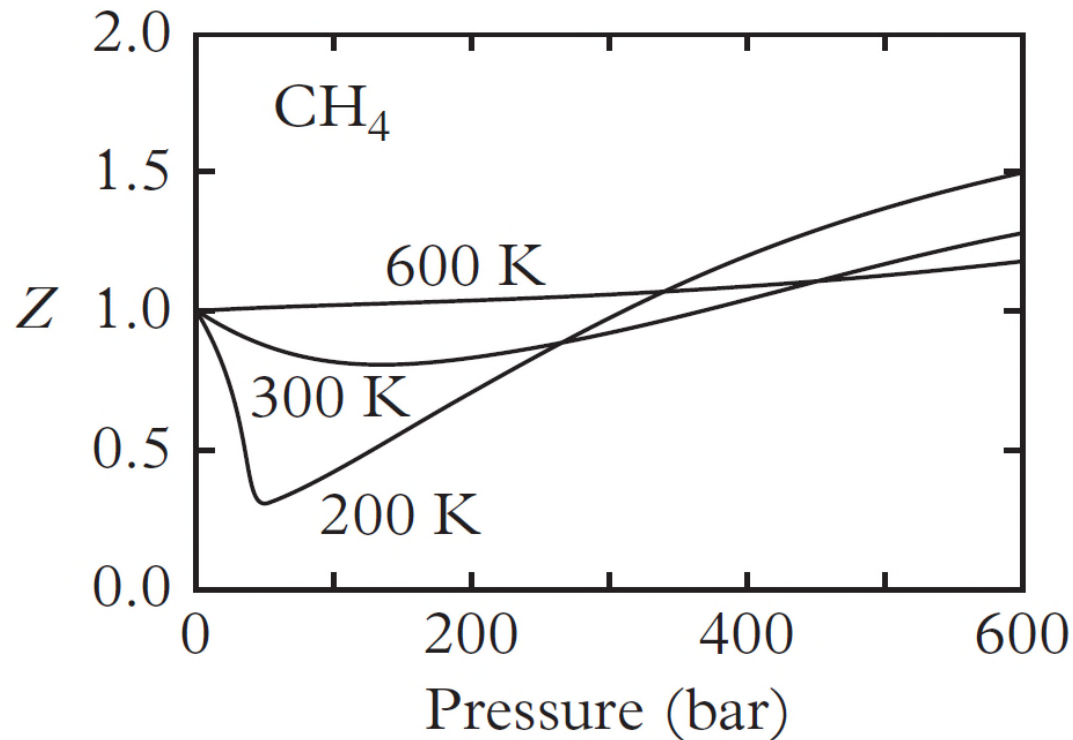
Critical point

van der Waals parameters

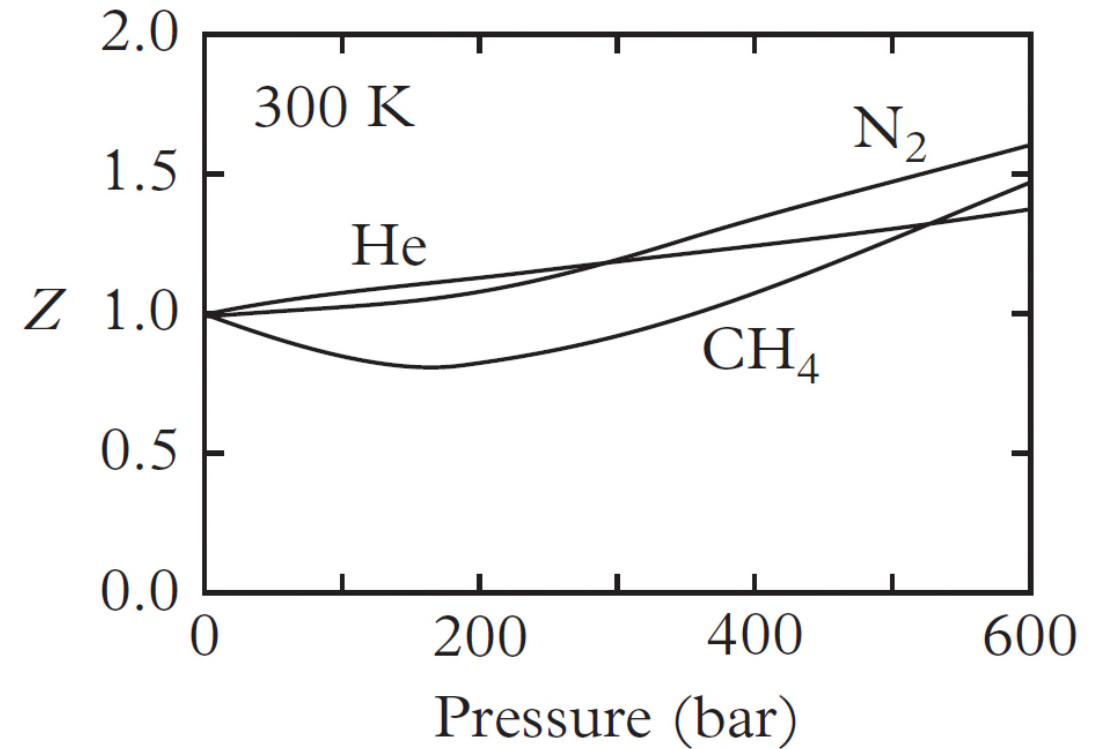
| Gas | T_c (K) | p_c (MPa) | Z_c | a (J m ³ mol ⁻²) | b (cm ³ mol ⁻¹) |
|------------------|-----------|-------------|-------|---|--|
| Ne | 44.4 | 2.76 | 0.311 | 0.0208 | 16.72 |
| Ar | 150.9 | 4.87 | 0.291 | 0.1363 | 32.19 |
| Kr | 209.4 | 5.50 | 0.288 | 0.233 | 39.57 |
| Xe | 289.7 | 5.84 | 0.287 | 0.419 | 51.56 |
| N ₂ | 126.2 | 3.39 | 0.289 | 0.1370 | 38.69 |
| CH ₄ | 190.6 | 4.60 | 0.286 | 0.230 | 43.06 |
| CO ₂ | 304.1 | 7.38 | 0.274 | 0.365 | 42.67 |
| H ₂ O | 647.1 | 22.06 | 0.229 | 0.5536 | 30.49 |
| H ₂ | 33.2 | 1.297 | 0.306 | 0.0248 | 26.60 |
| He | 5.19 | 0.227 | 0.301 | 0.00346 | 23.76 |

Compression factor (a way of saying how non-ideal the gas is)

$$Z = pV/RT$$



Methane at some example temperatures

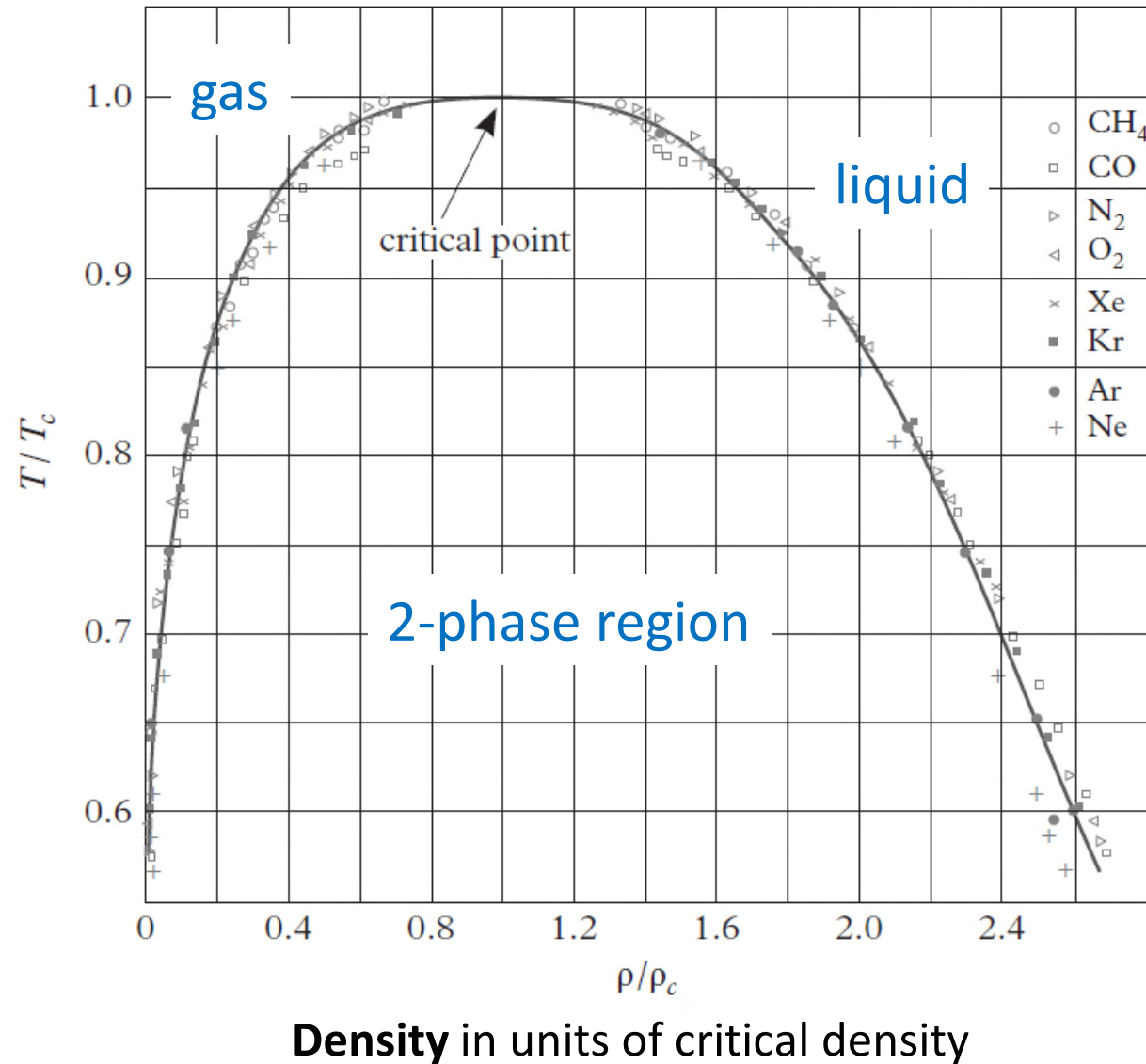


Some example gases at 300 K

Experimental evidence of the law of corresponding states

Plot the temperature vs density at the liquid/gas phase transition

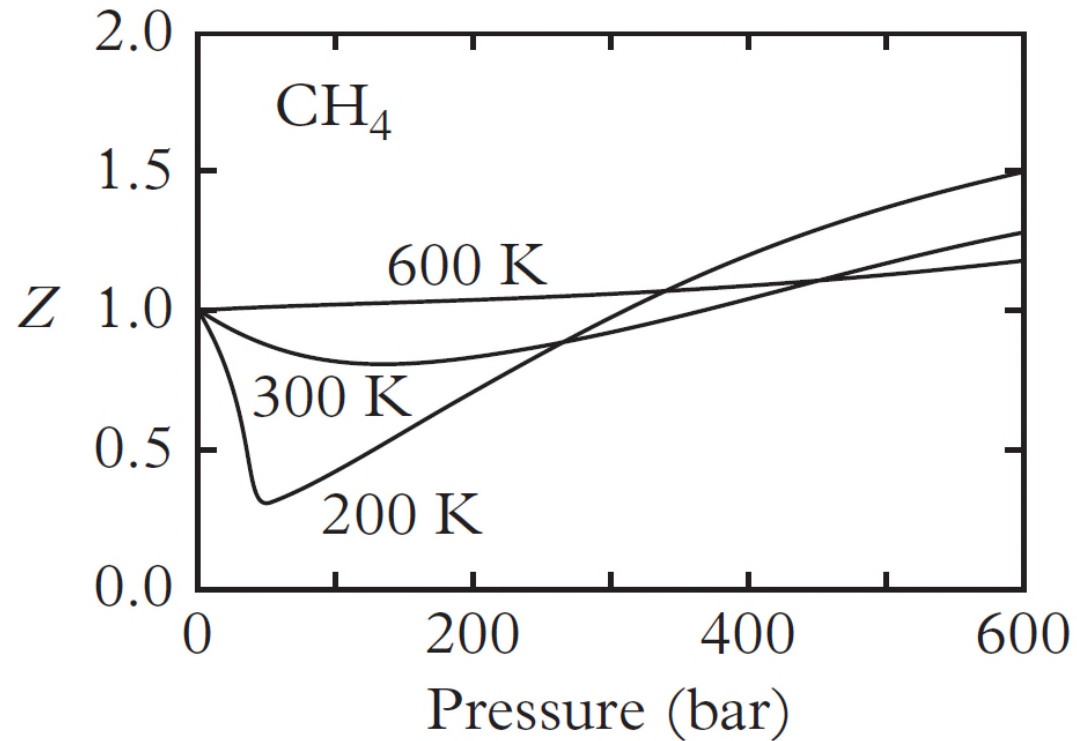
Temperature, in units of critical temperature



(Adapted from Guggenheim (1945))

Compression factor (a way of saying how non-ideal the gas is)

$$Z = pV/RT$$



~~Methane~~ at some example temperatures

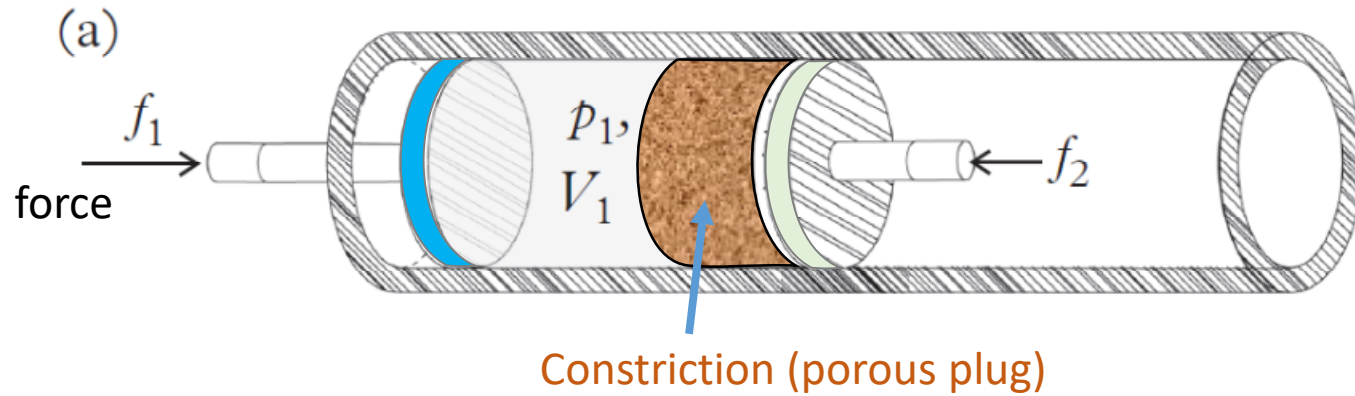
Any gas

(in suitably scaled units)

Thermodynamics lecture 10. Real gases and flow processes

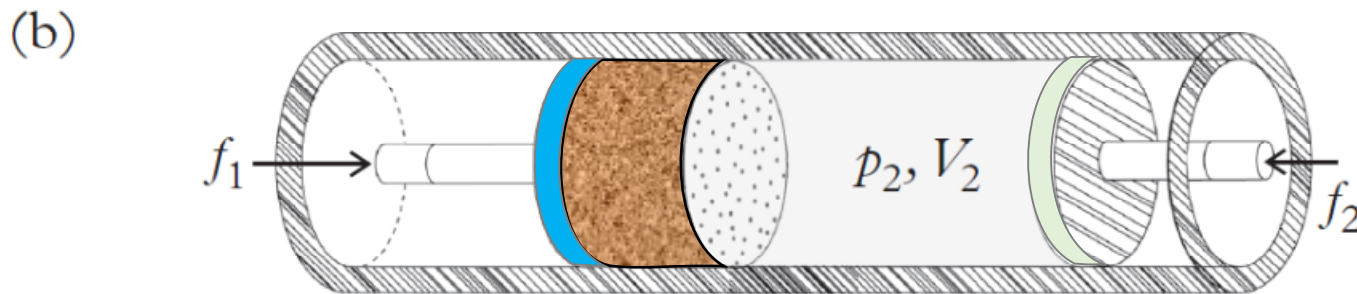
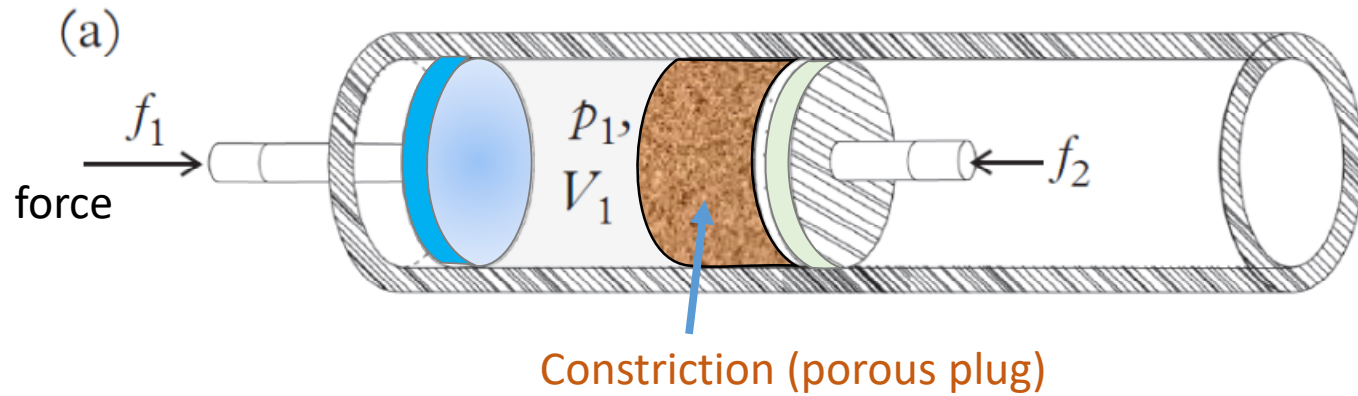
1. Real (i.e. non-ideal) gases, critical point, critical parameters
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Joule-Kelvin process (also called throttle process)



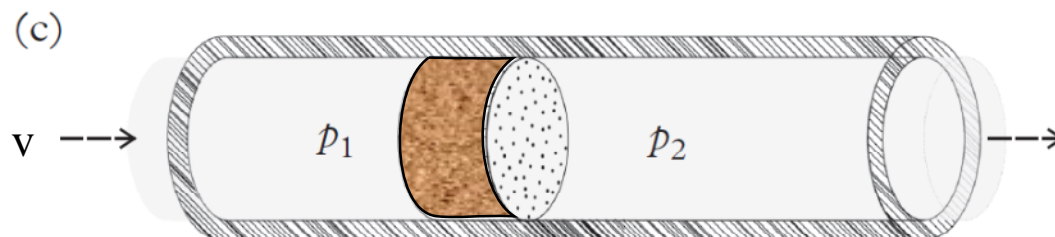
- Thermal isolation
 - Slow but non-zero rate of flow through a constriction (e.g. porous plug or small nozzle)
- $f_1 > f_2$

Joule-Kelvin process (also called throttle process)



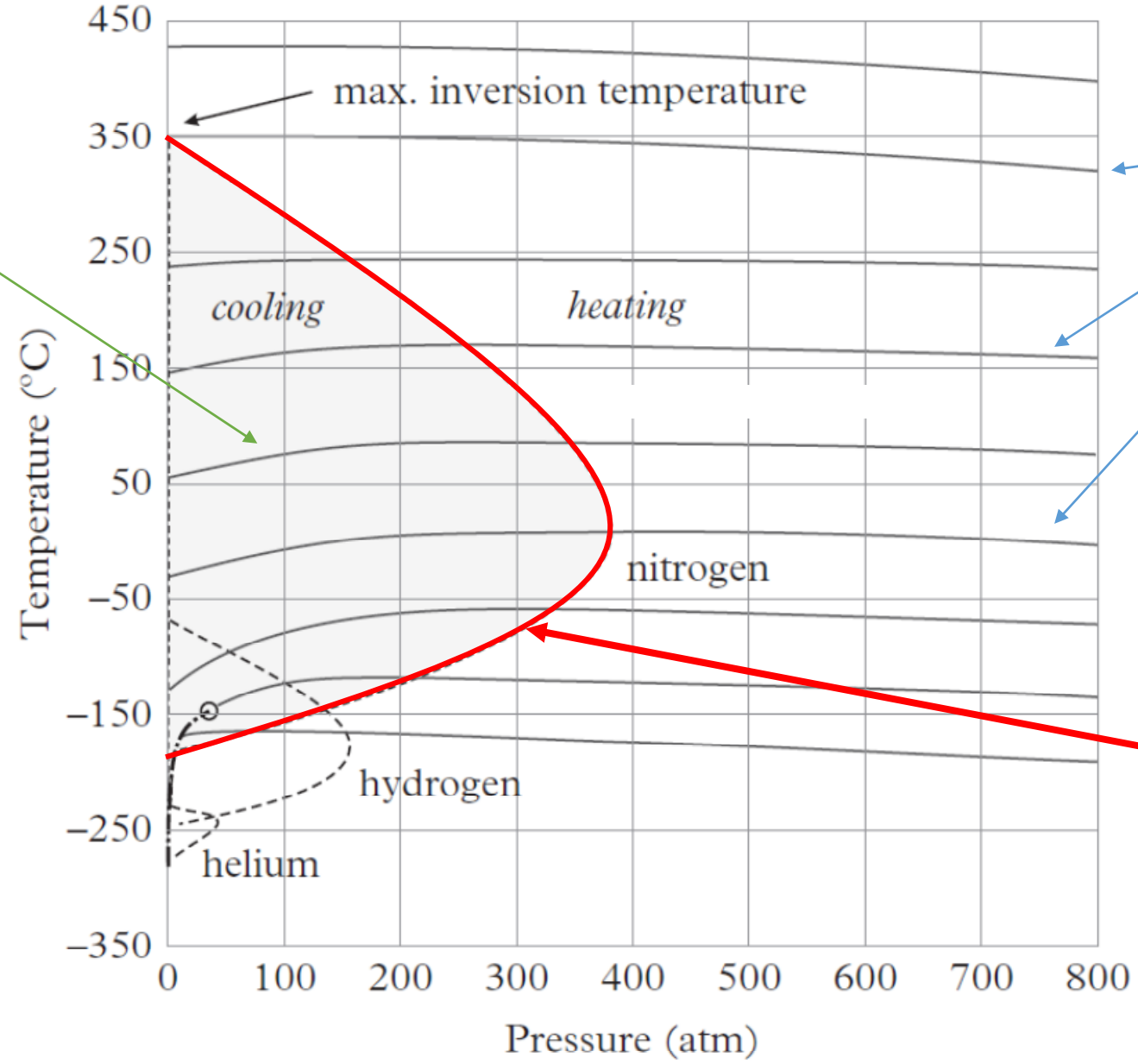
- Thermal isolation
 - Slow but non-zero rate of flow through a constriction (e.g. porous plug or small nozzle)
- $\rightarrow f_1 > f_2$

f_1 and f_2 are maintained constant, therefore so are p_1 and p_2



Often done in practice in a continuous flow

Get **cooling** when $dT/dp > 0$ (think: pressure falls and we want T to fall)



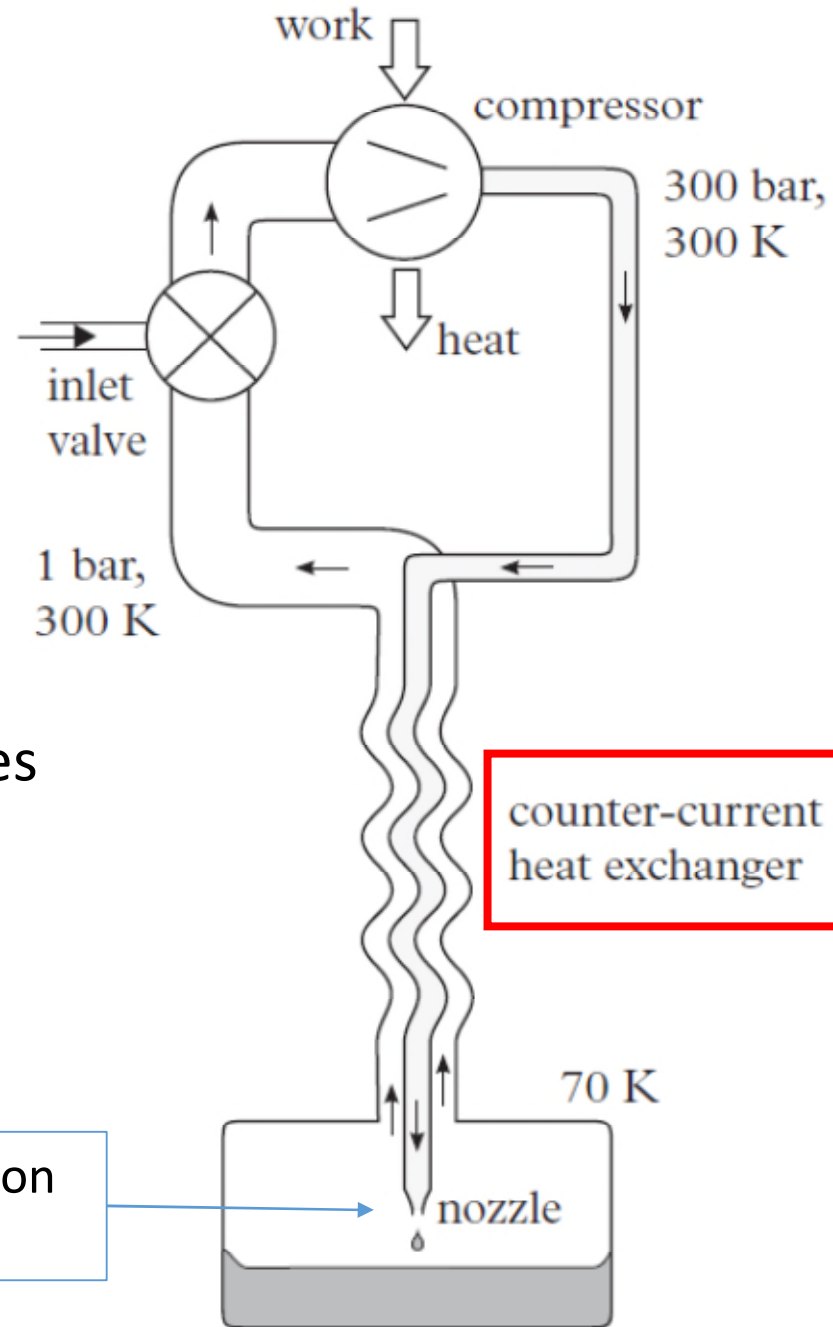
Curves of constant enthalpy (*isenthalps*)

Inversion curve:
where
$$\left. \frac{\partial T}{\partial p} \right|_H = 0$$

Thermodynamics lecture 8. Real gases and flow processes

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Linde process for liquefaction of gases



Example values
for nitrogen

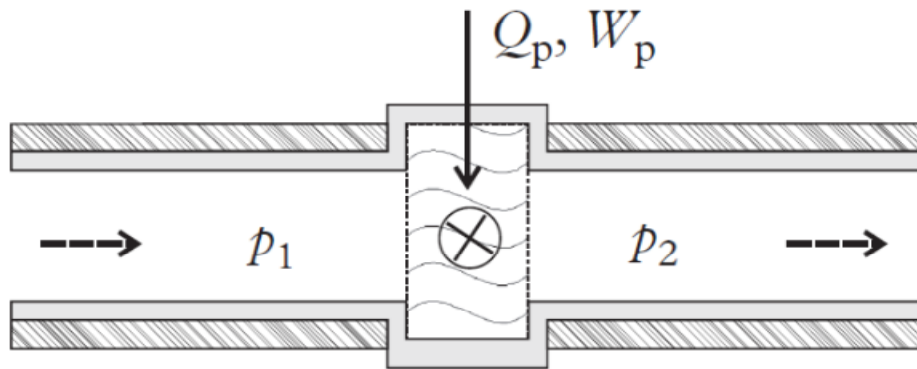
Joule-Kelvin expansion
happens here

Thermodynamics lecture 10. Real gases and flow processes

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5. *General flow process*

More general *flow process* (gas turbines, chemical process plants, etc.)

Process heat, process work



Conservation of energy:

$$Q_p + W_p = H_{\text{out}} - H_{\text{in}} \equiv \Delta H$$

e.g. adiabatic compressor:

$$Q_p = 0, \quad pV^\gamma = \text{const}$$

→ work required to compress a given amount of flowing fluid:

$$W_p = \Delta H = \int V dp$$

$$= V_1 \int_{p_1}^{p_2} \left(\frac{p_1}{p} \right)^{1/\gamma} dp = \frac{\gamma V_1 p_1}{\gamma - 1} \left[\left(\frac{p_2}{p_1} \right)^{1-1/\gamma} - 1 \right]$$