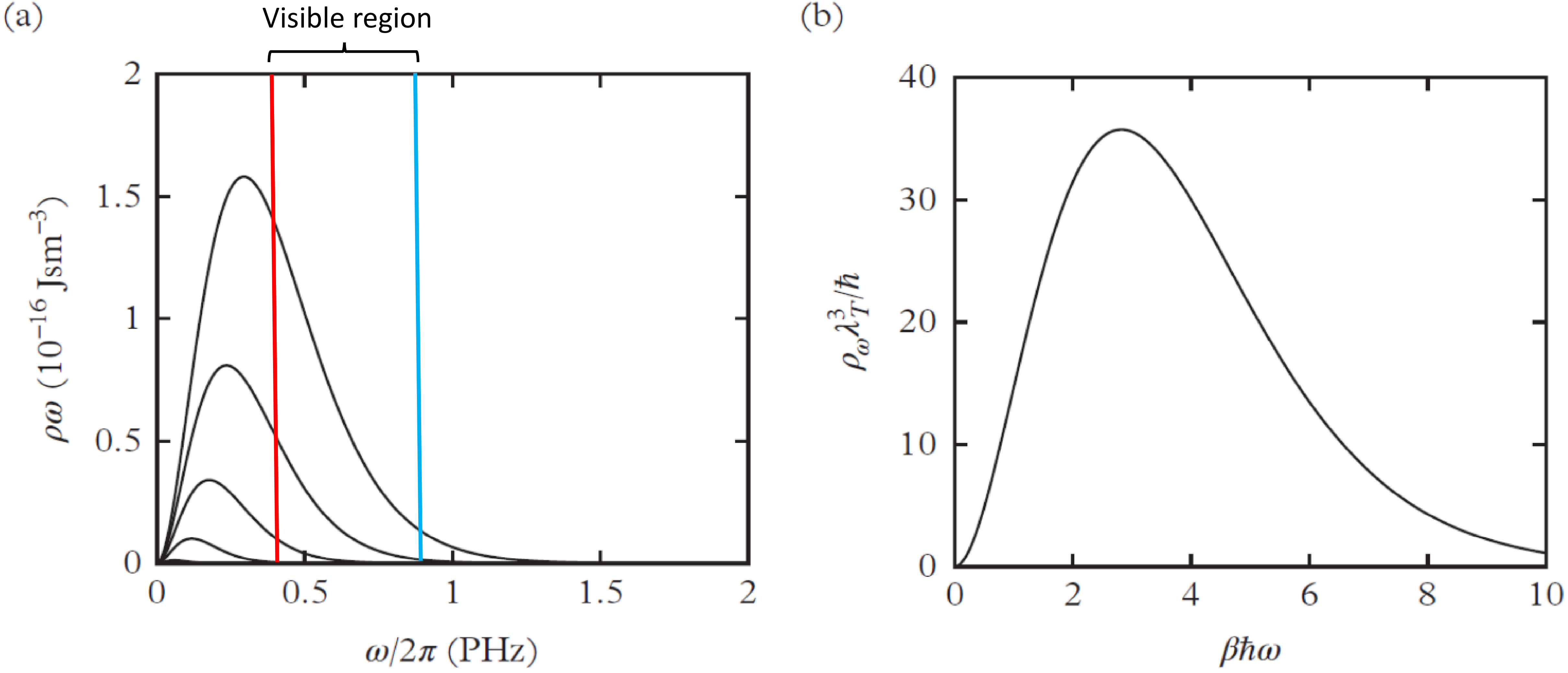
Thermodynamics lecture 9. Thermal radiation in more detail

1. Wien's argument to show adiabatic expansion of cavity radiation preserves its thermal character
2. Wien's laws:
   1. Wien's displacement law
   2. Wien's distribution law
3. Statistical mechanics of cavity radiation,
   Model 1: distinguishable modes (plane waves)
4. Energy, partition function, etc.
5. Model 2: indistinguishable photons
   (conceptually harder but equally important)

1. Wien's argument to show adiabatic expansion of cavity radiation preserves the thermal character
2. **Wien's laws:**
   1. **Wien's displacement law**
   2. **Wien's distribution law**
3. Statistical mechanics of cavity radiation,
   Model 1: distinguishable modes (plane waves)
4. Energy, partition function, etc.
5. Model 2: indistinguishable photons
   (conceptually harder but equally important)



Spectral energy density at
1000, 2000, 3000, 4000, 5000 K

It’s the same function each time,
scaled by λ_T^3

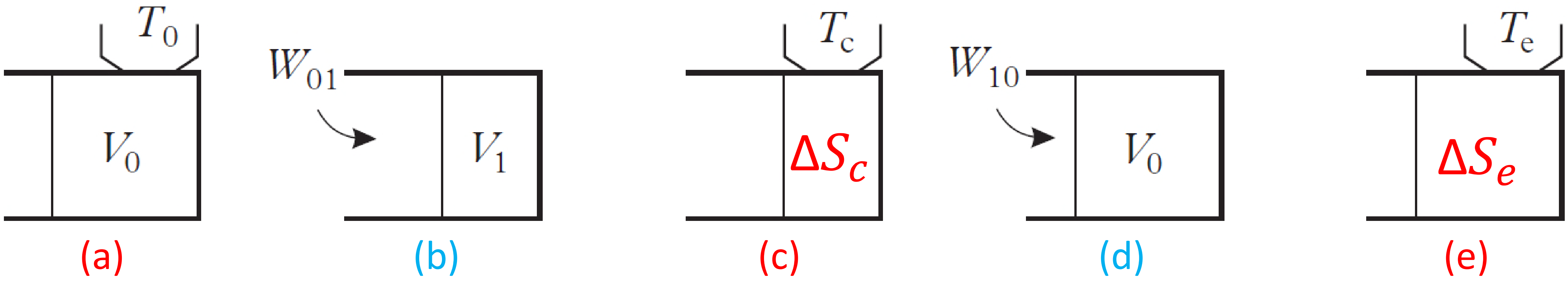
$$\lambda_T = 2\pi\hbar c/k_B T.$$

Thermodynamics lecture 9. **Thermal radiation in more detail**

1. **Wien's argument to show adiabatic expansion of cavity radiation preserves its thermal character**
2. Wien's laws:
   1. Wien's displacement law
   2. Wien's distribution law
3. Statistical mechanics of cavity radiation,
   Model 1: distinguishable modes (plane waves)
4. Energy, partition function, etc.
5. Model 2: indistinguishable photons
   (conceptually harder but equally important)

Slow expansion of cavity radiation in a reflecting cavity: does it remain in thermal equilibrium state?

Wien's argument to show the answer is *yes*:



Stages (c) and (e) : we pick Tc and Te such that the radiation comes to equilibrium with no net ΔU

Stages (b) and (d):
Adiabatic change of volume (so work is done and U changes and changes back)

Argument:

ΔU = 0 → ΔS_C **>= 0,** ΔS_e **>= 0**
But no net change in state →
$\Delta S_{tot} = 0$
→ $\Delta S_C = \Delta S_e = 0$

Thermodynamics lecture 9. Thermal radiation in more detail

1. Wien's argument to show adiabatic expansion of cavity radiation preserves its thermal character
2. **Wien's laws:**
   1. **Wien's displacement law**
   2. **Wien's distribution law**
3. Statistical mechanics of cavity radiation,
   Model 1: distinguishable modes (plane waves)
4. Energy, partition function, etc.
5. Model 2: indistinguishable photons
   (conceptually harder but equally important)

Thermodynamics lecture 9. Thermal radiation in more detail

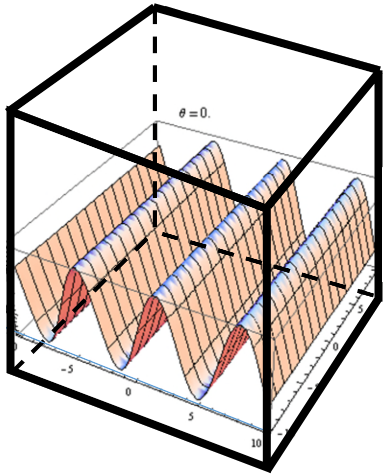
1. Wien's laws:
   1. Wien's displacement law
   2. Wien's distribution law
2. **Statistical mechanics of cavity radiation, Model 1: distinguishable modes (plane waves)**
3. Energy, partition function, etc.
4. Model 2: indistinguishable photons (conceptually harder but equally important)

Thermodynamics lecture 9. Thermal radiation in more detail

1. Wien's argument to show adiabatic expansion of cavity radiation preserves its thermal character
2. Wien's laws:
   1. Wien's displacement law
   2. Wien's distribution law
3. **Statistical mechanics of cavity radiation, Model 1: distinguishable modes (plane waves)**
4. Energy, partition function, etc.
5. Model 2: indistinguishable photons (conceptually harder but equally important)

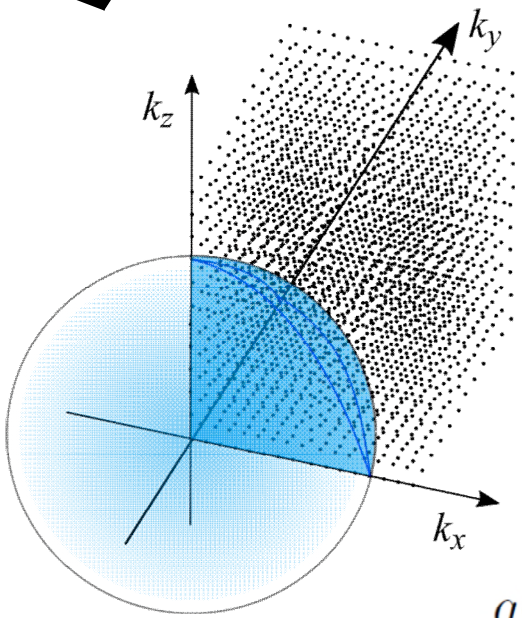
Modes of electromagnetic field

Standing waves in a box



$$\sin(k_x\, x)\sin(k_y\, y)\sin(k_z\, z)$$

$$\Delta k_x = \frac{\pi}{L}$$



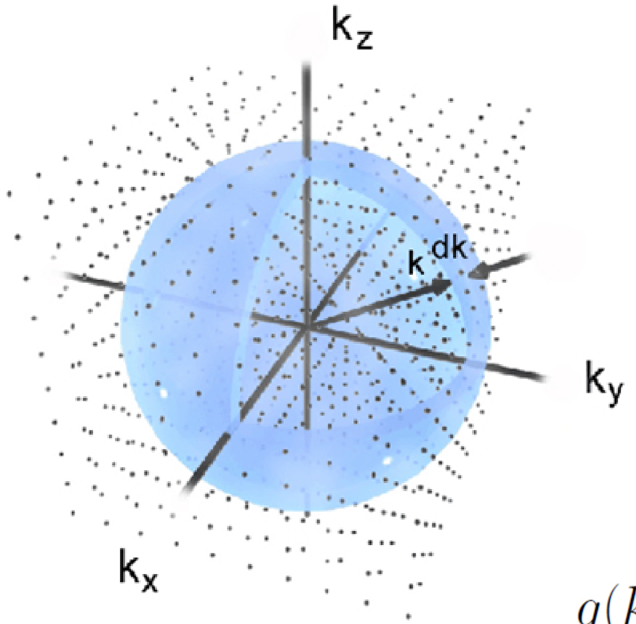
$$k_x, k_y, k_z > 0$$

$$g(k)\mathrm{dk} = \frac{1}{8}4\pi \mathrm{k}^2\frac{\mathrm{V}}{\pi^3}\mathrm{dk}$$

OR travelling waves with a *mathematical constraint*: must have period L.

Travelling waves,
$e^{ik_x x}\, e^{ik_y y}\, e^{ik_z z}$

$$\Delta k_x = \frac{2\pi}{L}$$



$$k_x, k_y, k_z$$

+ve or -ve

$$g(k)\mathrm{dk} = 4\pi \mathrm{k}^2\frac{\mathrm{V}}{(2\pi)^3}\mathrm{dk}$$

Comparison between cavity radiation and an ideal gas

	Ideal gas			Cavity radiation		
Independent variables	U, N, V			U,V		
Number of particles	N			N	$=$	bVT^3
Equation of state	p	$=$	$nk_B T$	p	$=$	$0.9\, nk_B T$
Energy	U	$=$	$\frac{1}{\gamma-1} Nk_B T,$	U	$=$	$2.7 Nk_B T,$
u and p	p	$=$	$(\gamma - 1)u$	p	$=$	$\frac{1}{3}u$
Entropy	S	$=$	$Nk_B \left(\ln\left[a\frac{T^{1/(\gamma-1)}}{n}\right]\right)$	S	$=$	$3.6 Nk_B$
		$=$	$\frac{U}{T}(\ln T - (\gamma - 1)\ln(n/a))$		$=$	$\frac{4}{3}\frac{U}{T}$
Chemical potential	μ	$=$	$(U - TS + pV)/N$	μ	$=$	0
		$=$	$(\gamma u - Ts)/n$			

$$b \simeq 2.03 \times 10^7\ \mathrm{m^{-3}K^{-3}}$$