

## Thermodynamics lecture 2.

### W.A.L.T. (we are learning today)

- Some mathematical methods with partial differentiation and functions of state.
- “improper” or “inexact” differential
- Zeroth law  $\rightarrow$  temperature and equation of state
  - We learn both a method of reasoning
  - and the result: temperature can be defined, and every system has an equation of state for temperature
- Some examples equations of state
  - Including, for example, gas and one kind of magnet

Function of state  $\leftrightarrow$  proper differential

Pressure  $p$

Volume  $V$

Temperature  $T$

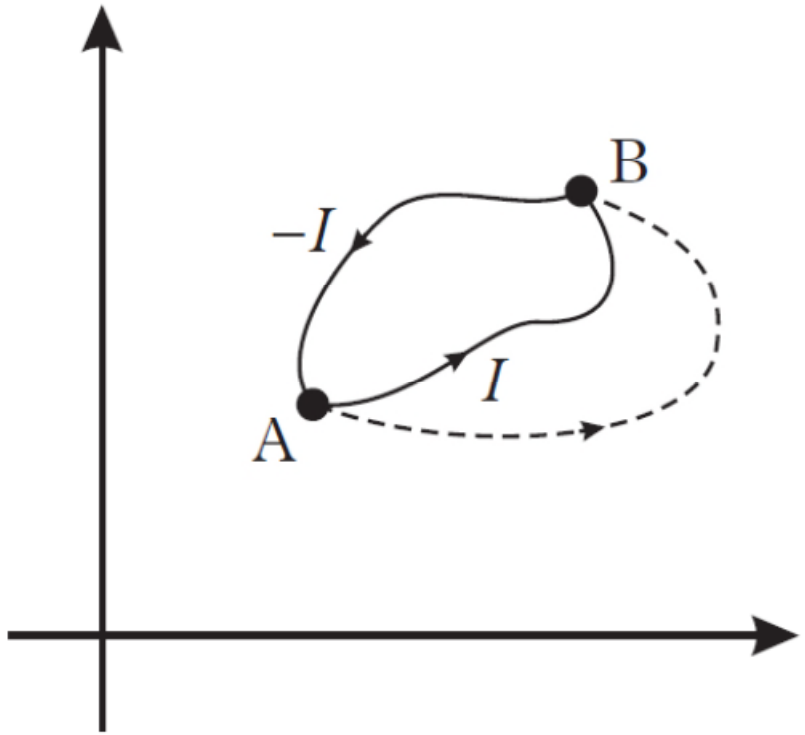
$dp, dV, dT$

Not function of state  $\leftrightarrow$  improper differential

Work  $W$

Heat  $Q$

$\delta W, \delta Q$



Theorem:

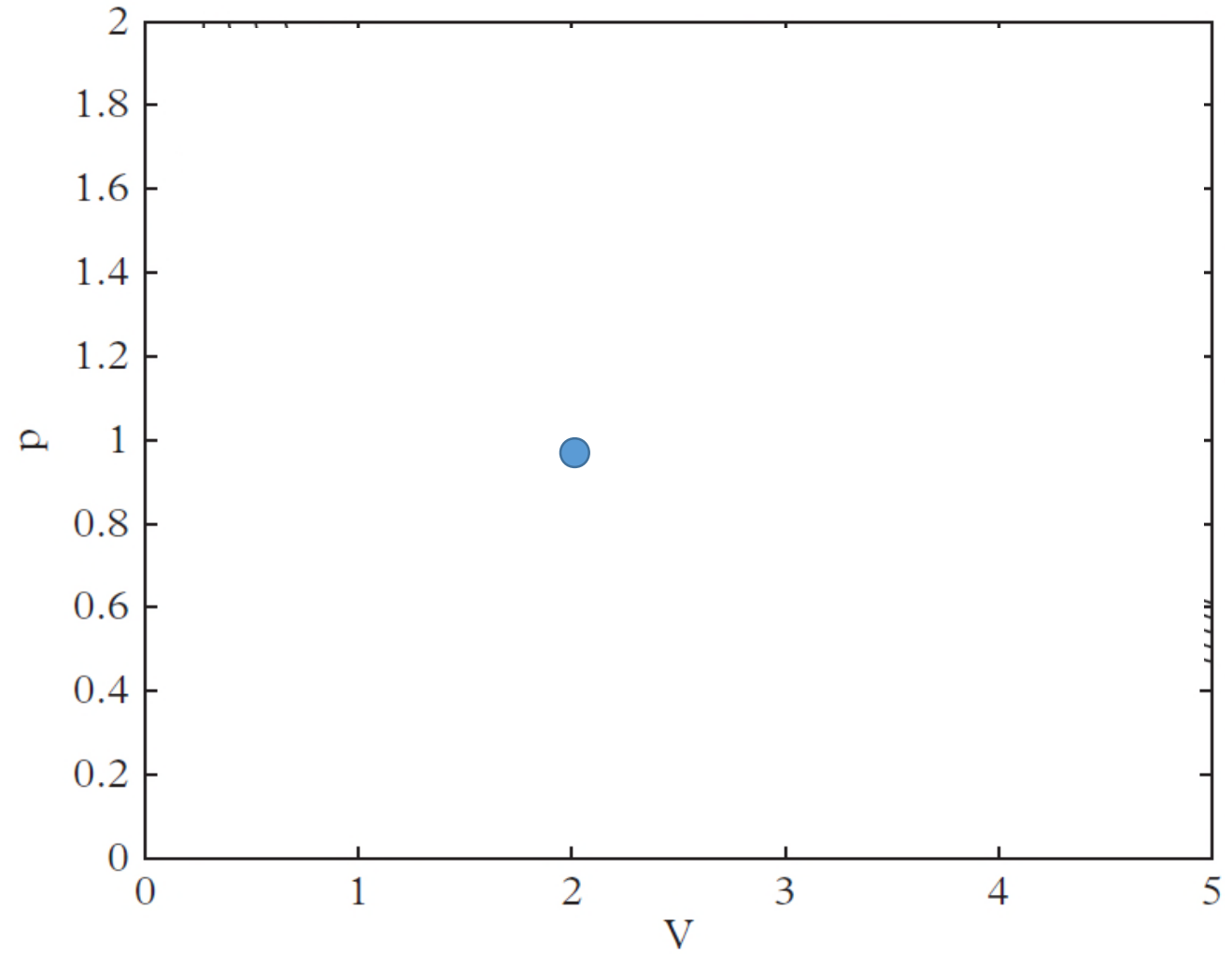
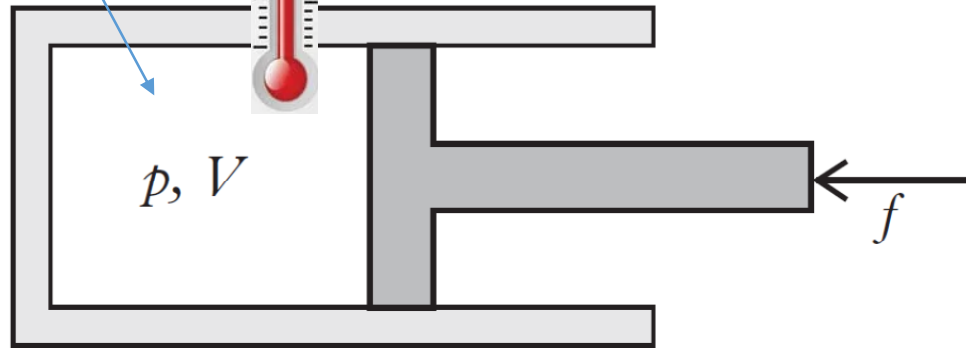
If the integral of a function  $f$  is zero around all closed paths, then  $f$  is a function of state.

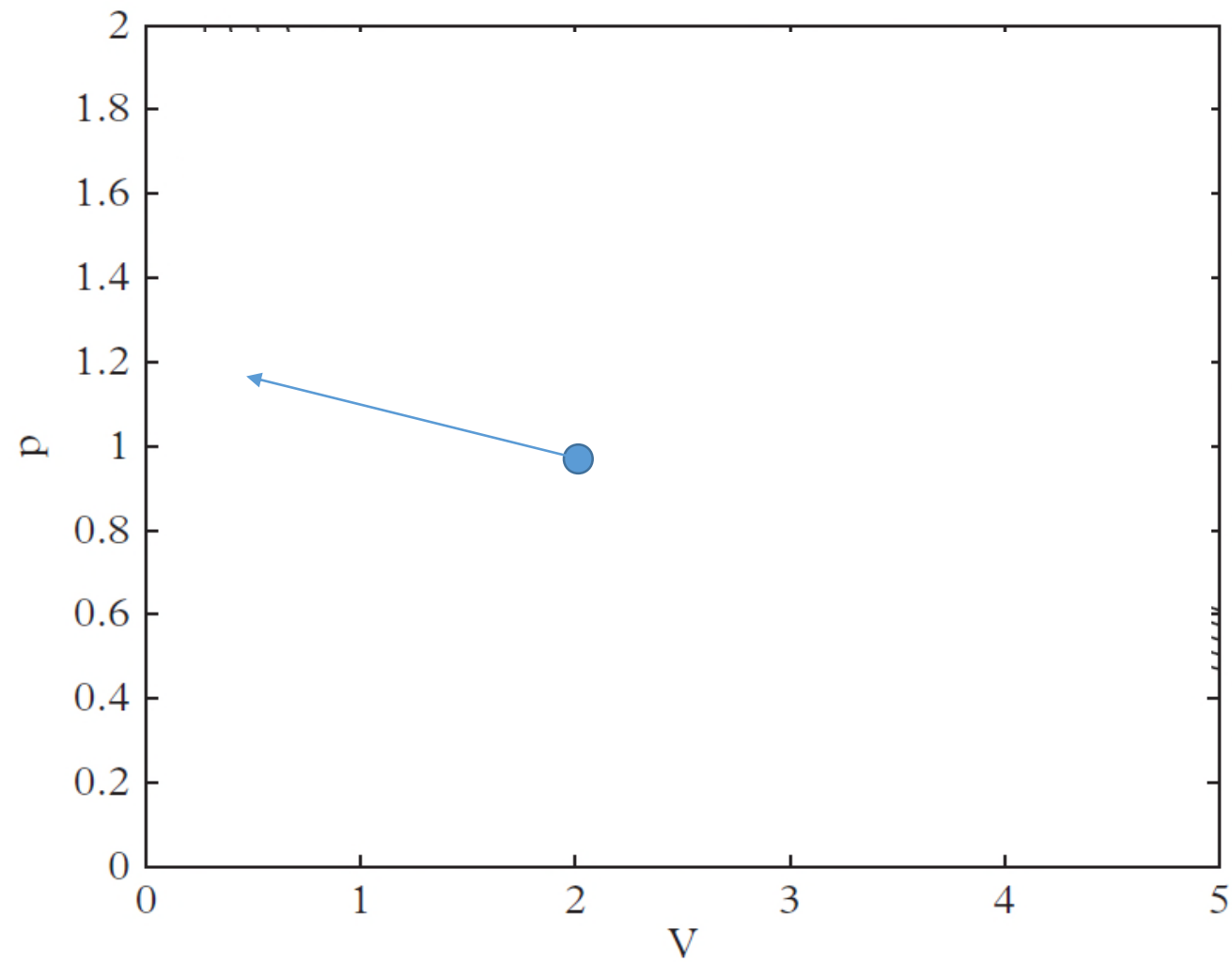
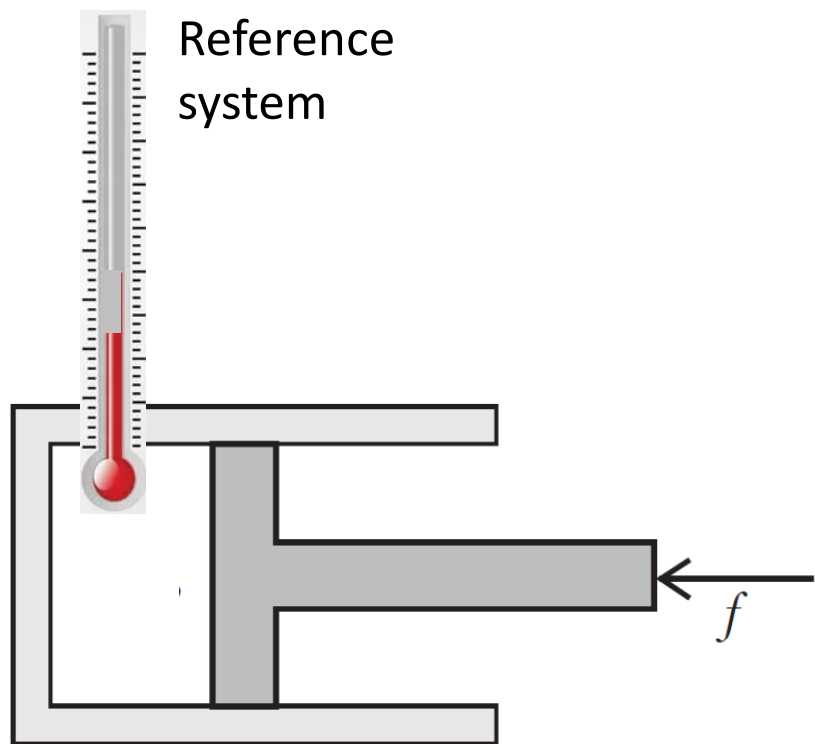
Zeroth law,  
empirical temperature  
and equation of state

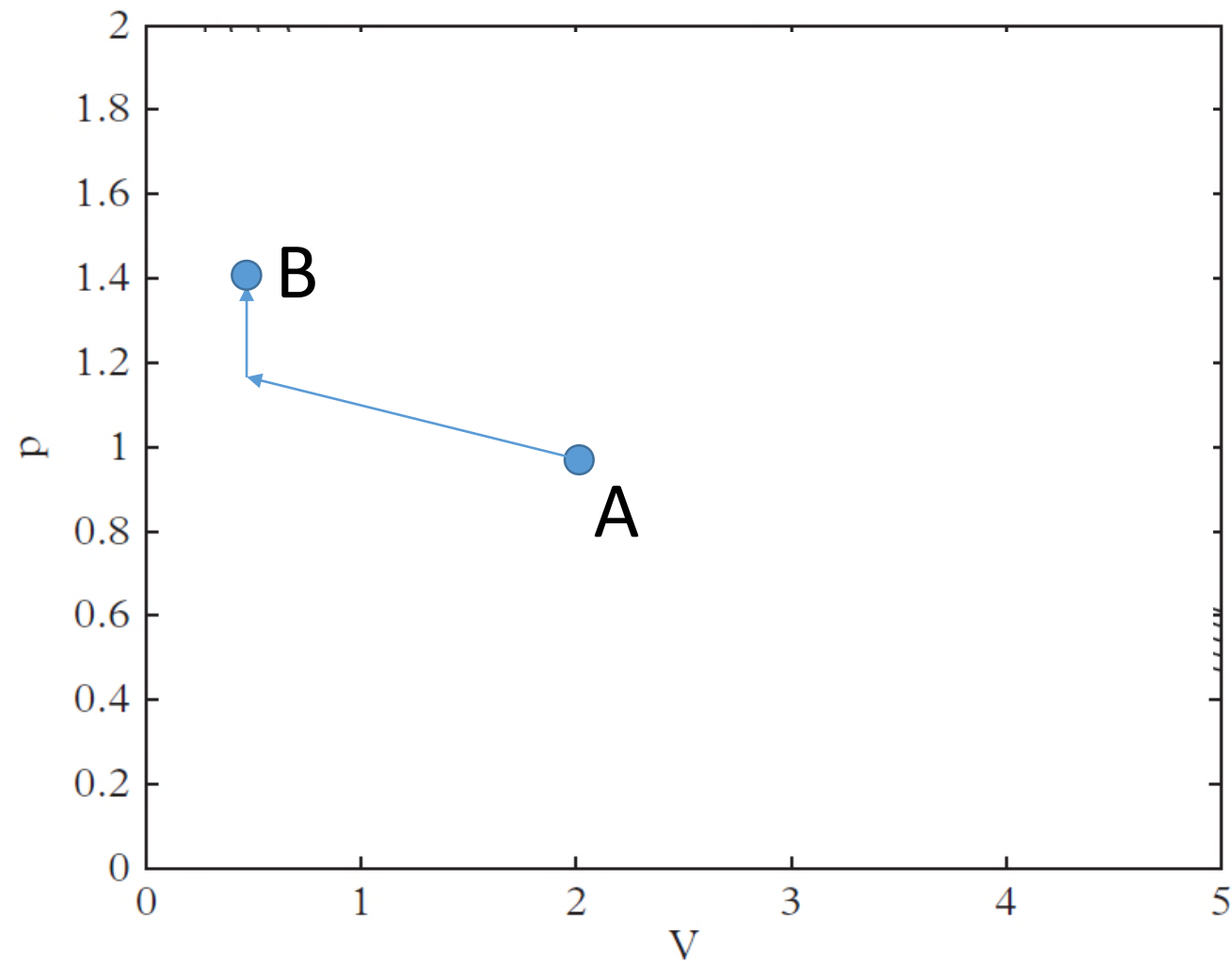
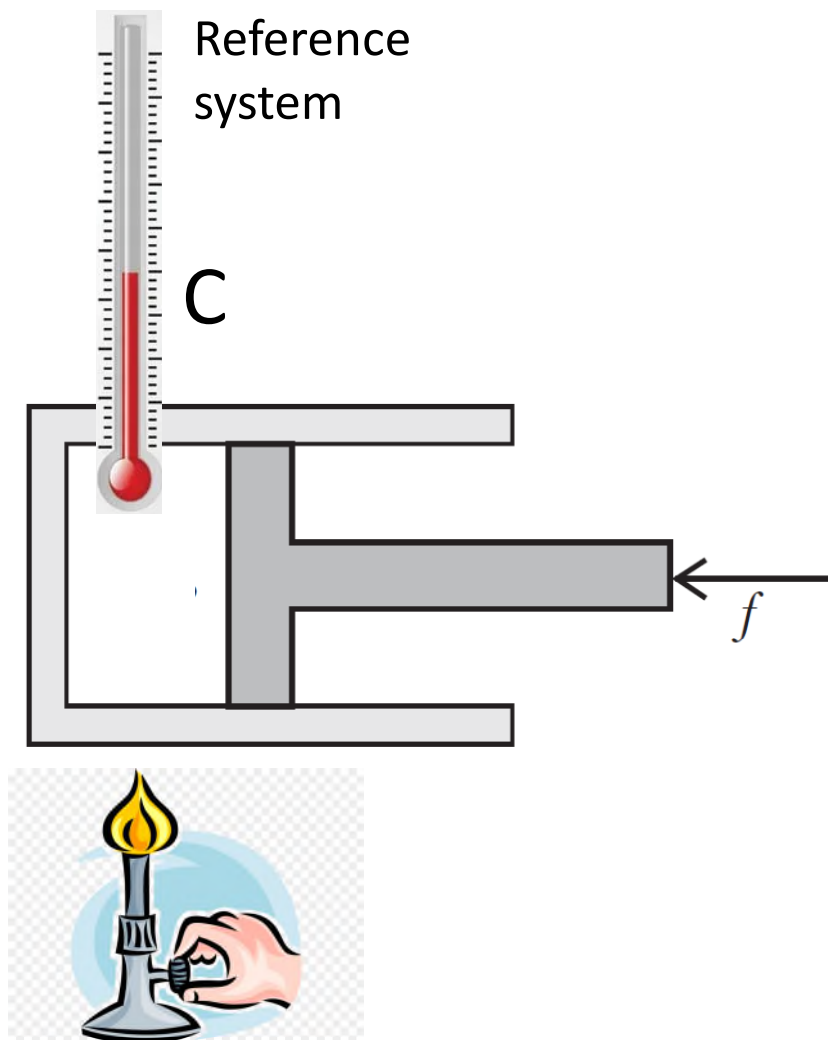
Reference  
system

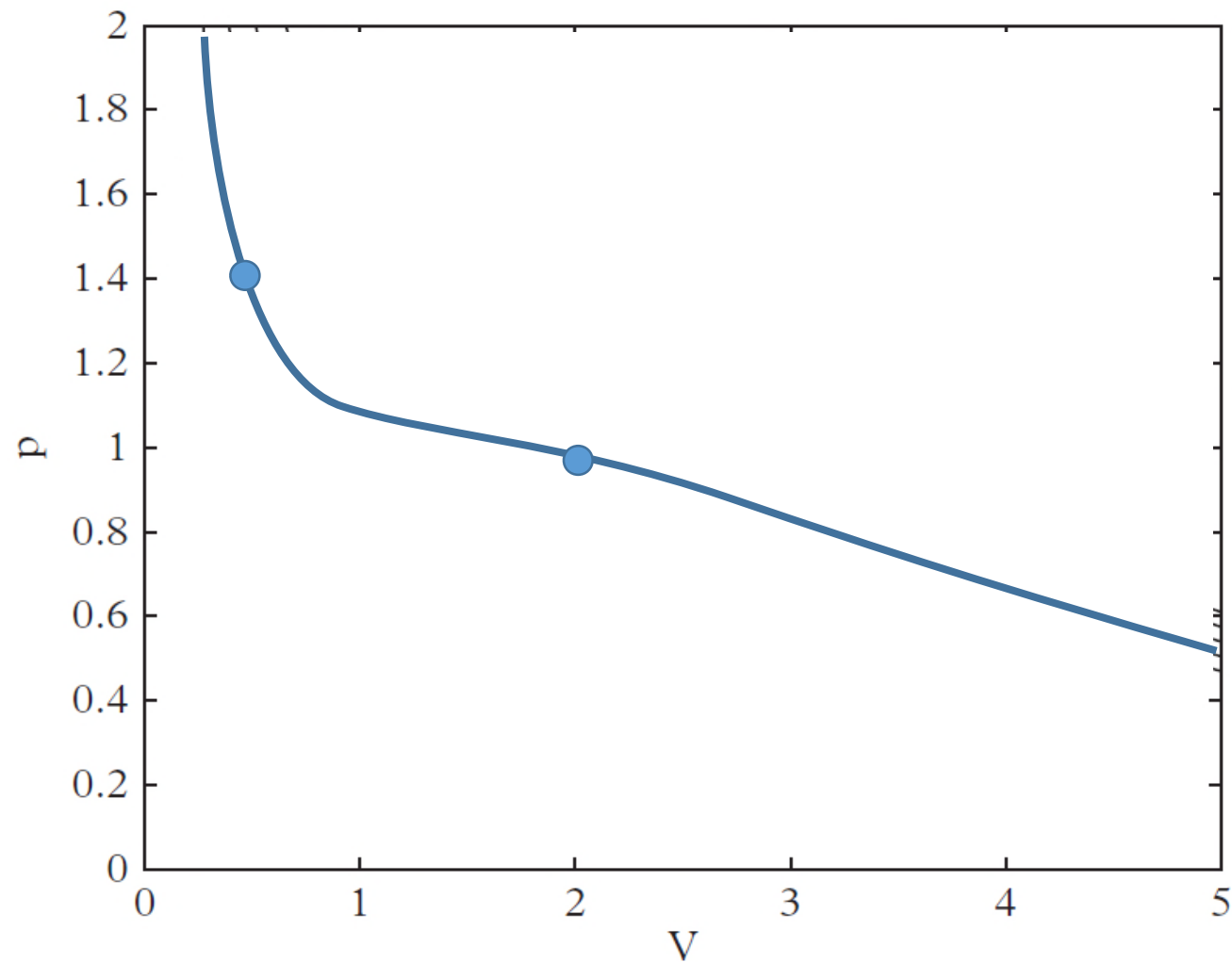
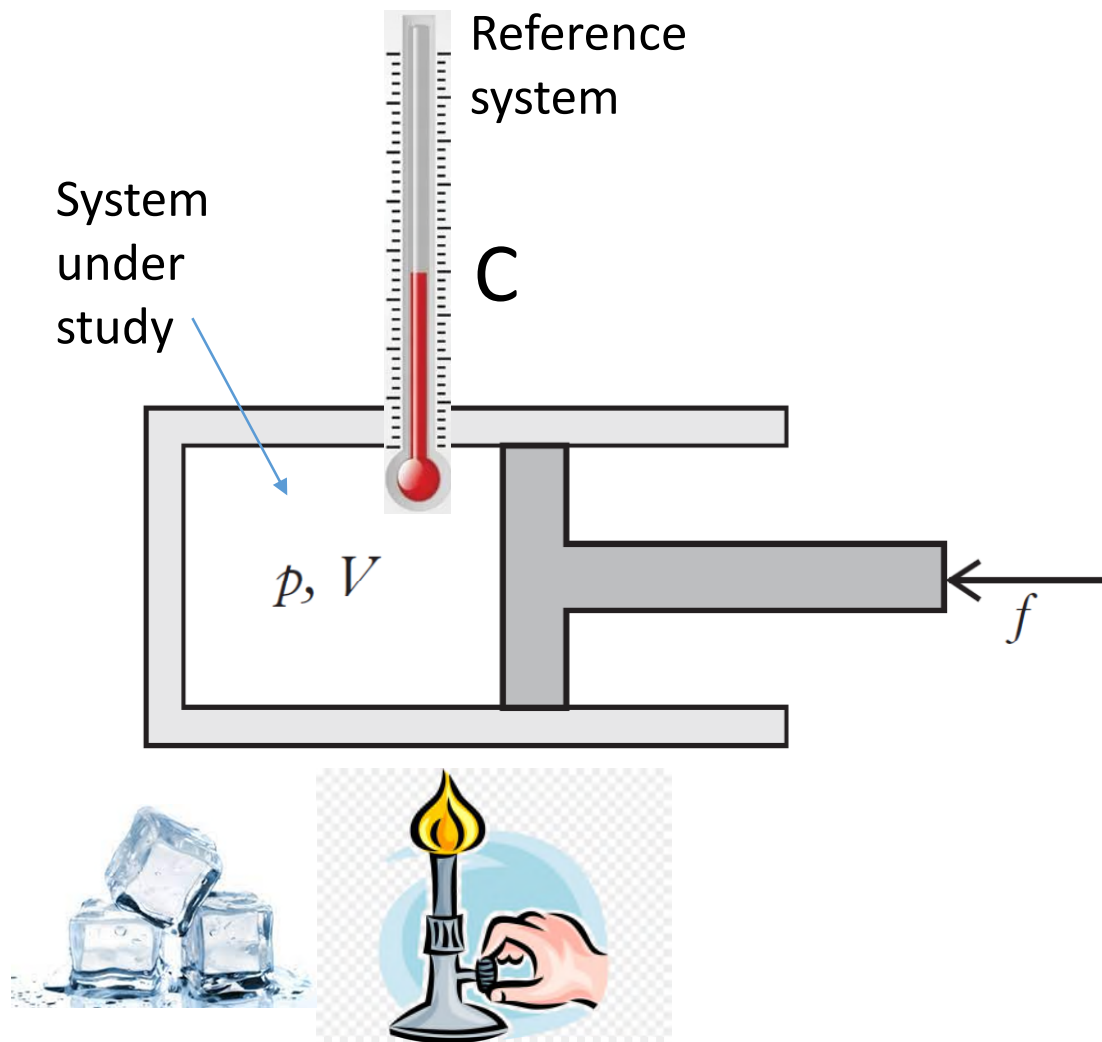
Let  $\theta$  indicate the state of the  
reference system;  
e.g. the length of a column of  
mercury (at the vapour pressure)

System  
under  
study

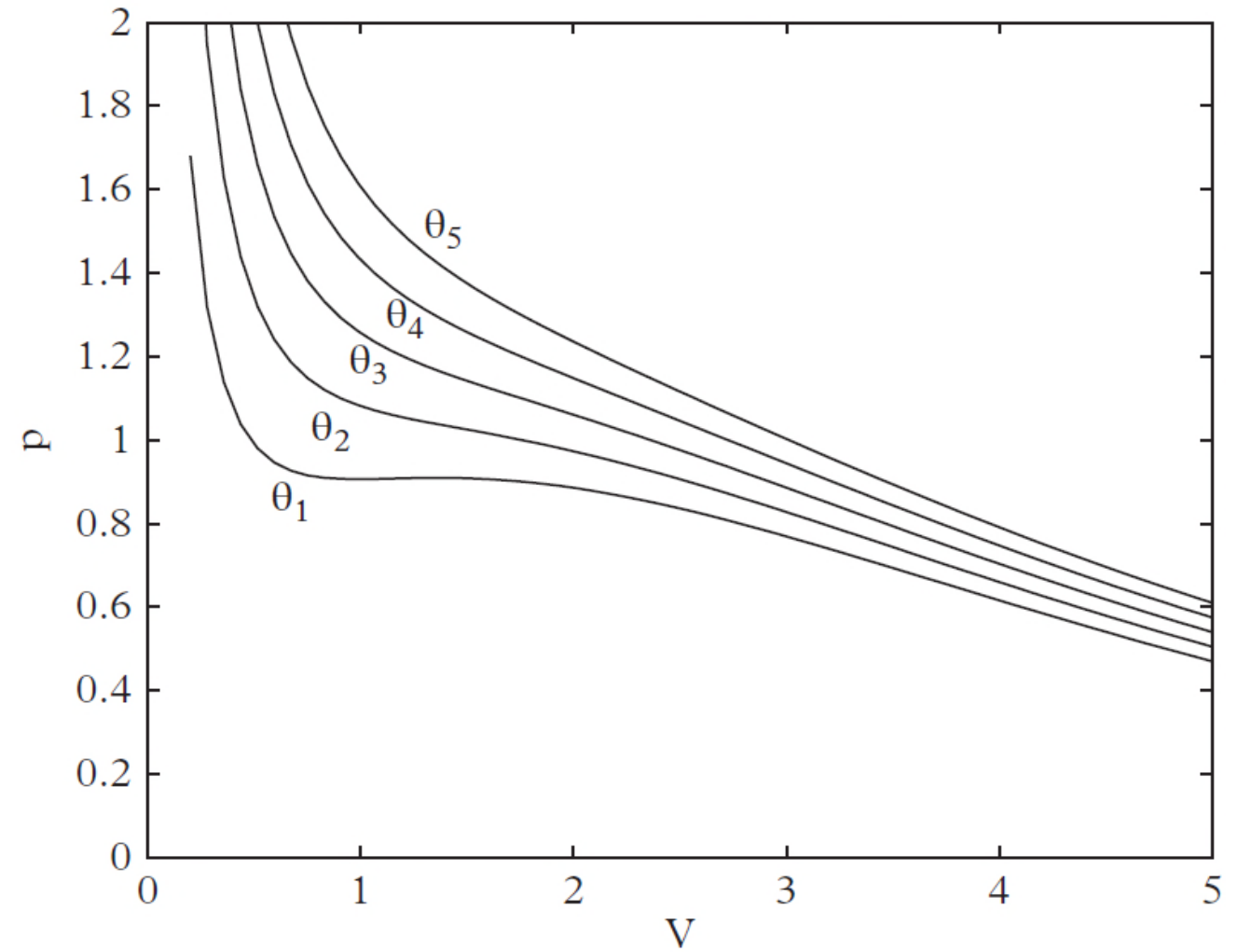
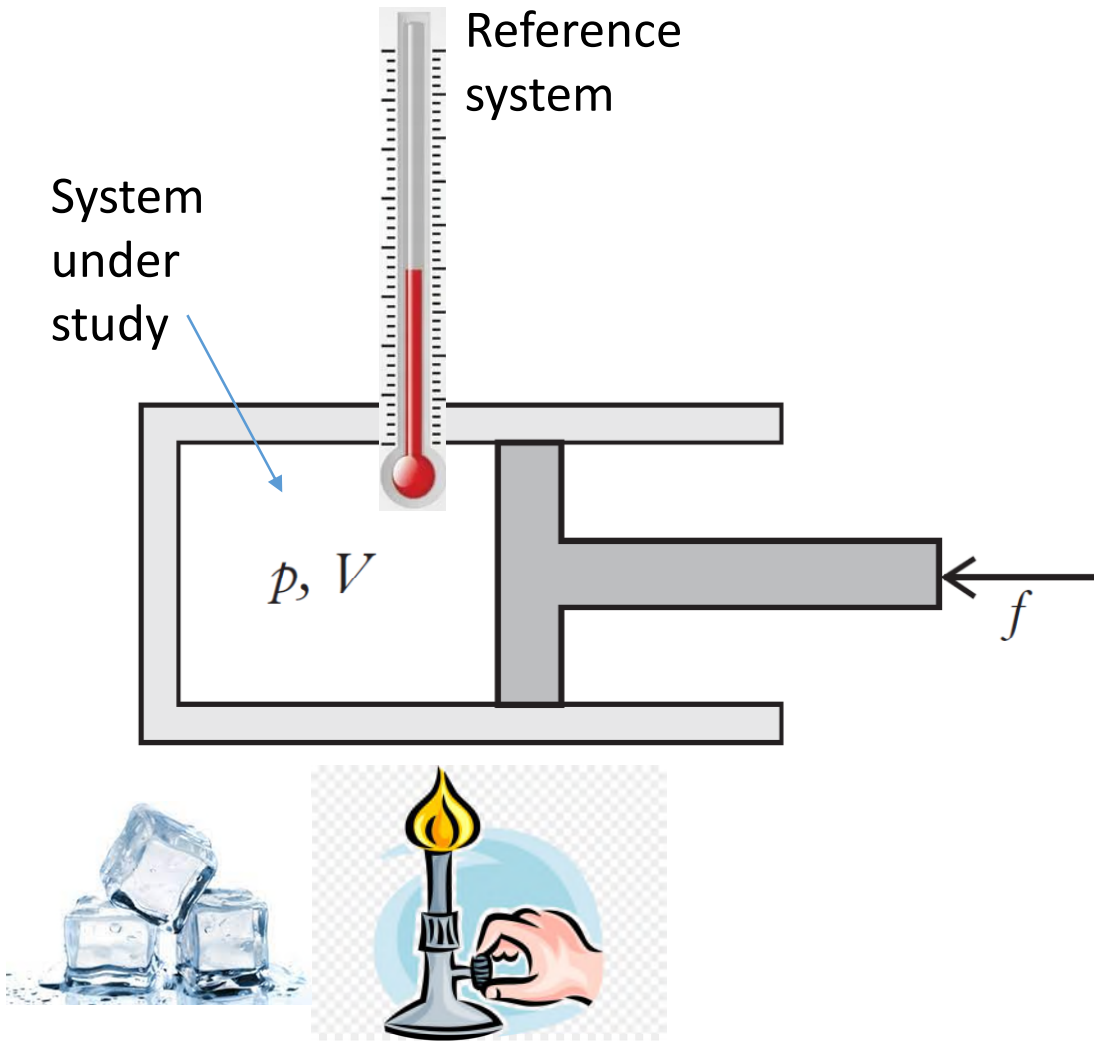










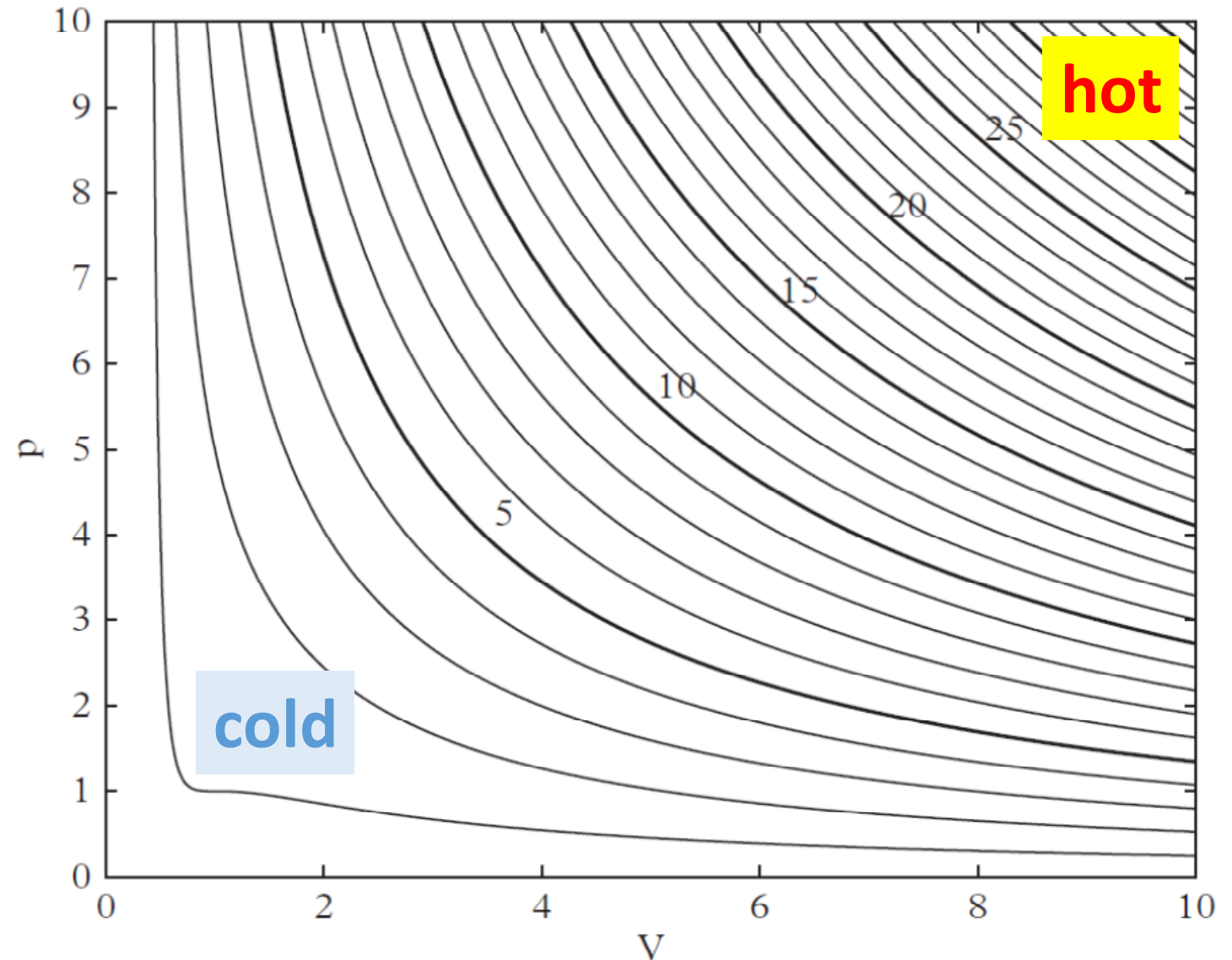


Here  $\theta$  indicates the state of the reference system;  
e.g. the length of a column of mercury (at the vapour pressure)

**van der Waals** equation  
of state  
(reasonably accurate for  
most gases if the  
pressure is not too high)

$$\left(p + \frac{N^2 a}{V^2}\right) (V - Nb) = Nk_B T$$

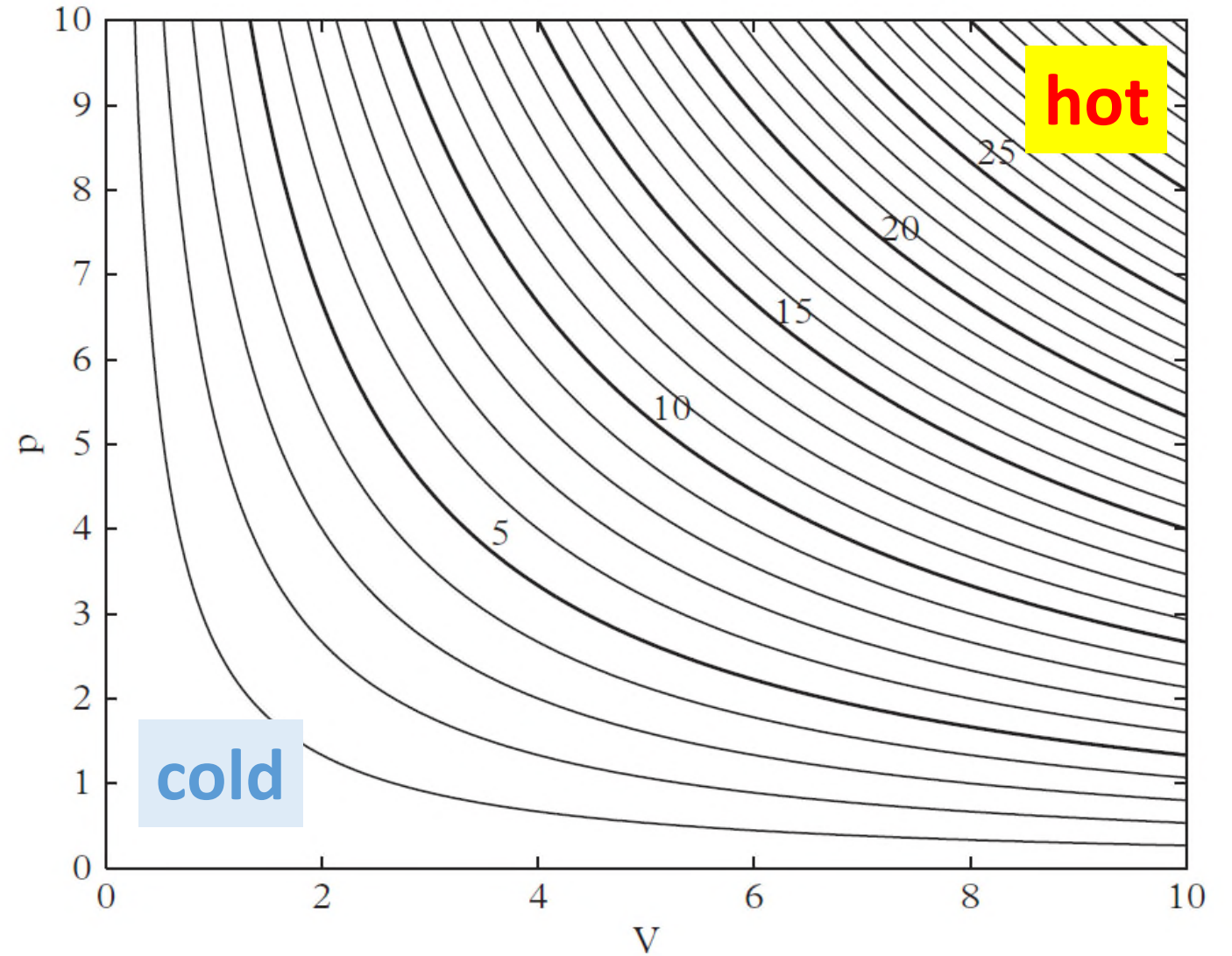
N = number of molecules  
a,b = constants which depend on the type of gas



Isotherms of **van der Waals** gas, in the gaseous (not liquid) region

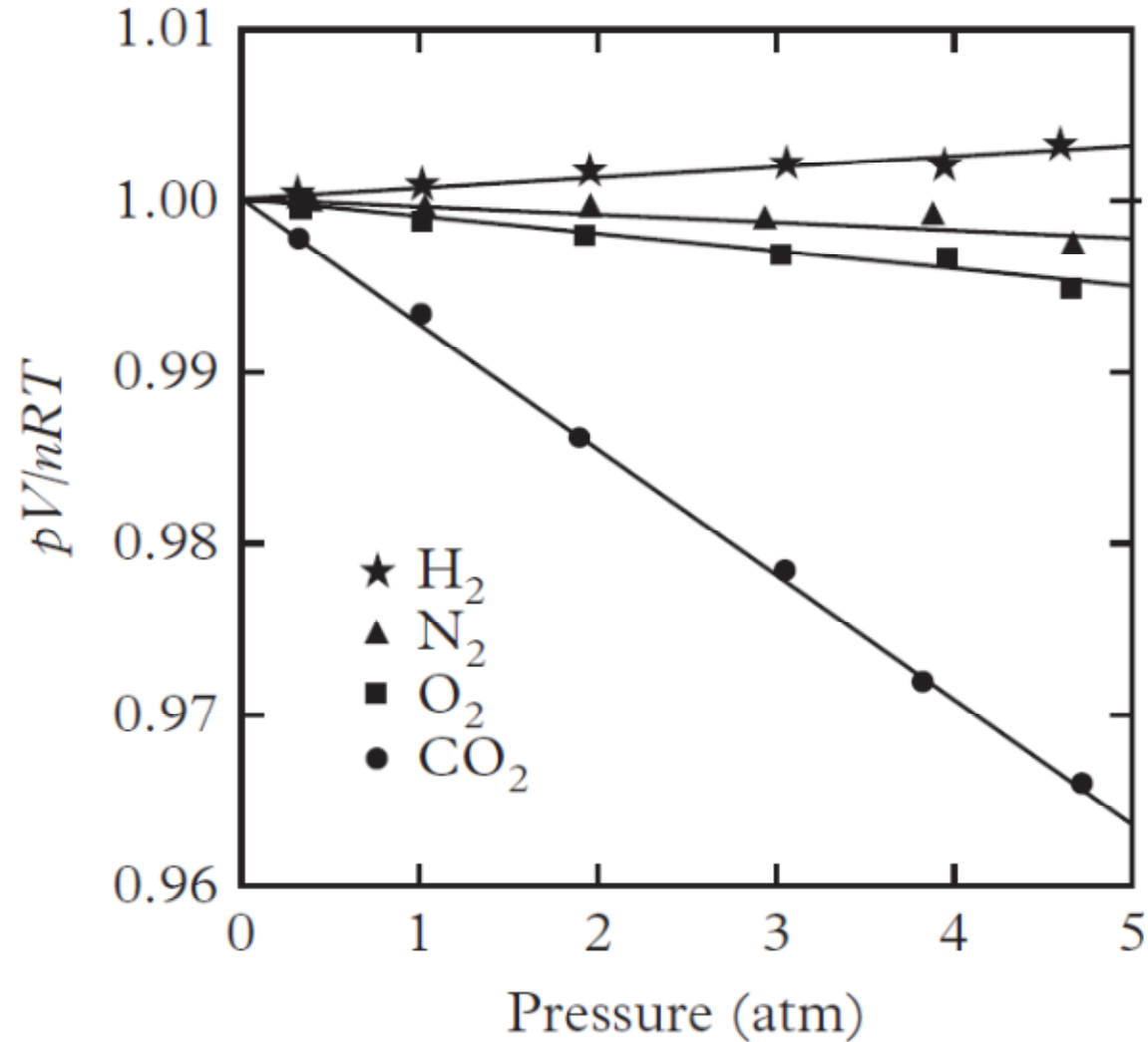
Ideal gas

$$pV = Nk_B T$$



Isotherms of ideal gas

Real gases tend to ideal at low pressure



## *Definition of an ideal gas*

- (1) Boyle's law:  $pV$  is constant at fixed temperature.*
- (2) Joule's law: The internal energy is independent of pressure at fixed temperature.*

(1) Implies  $pV = f(T)$  for some function  $f$ .

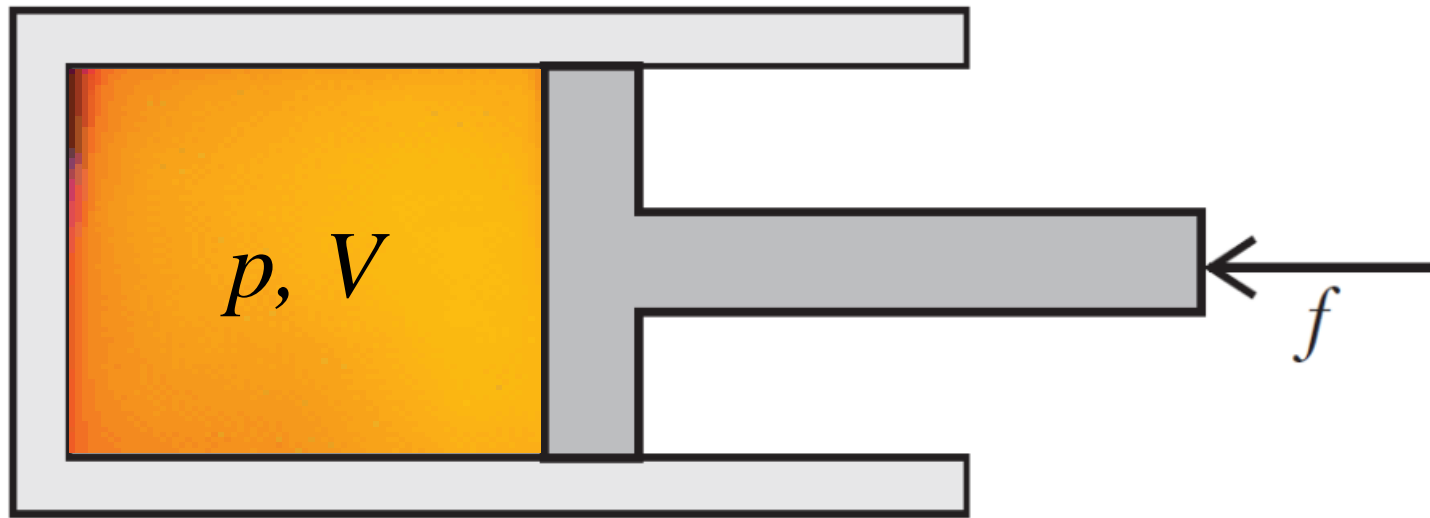
(2) Is often written  $U = U(T)$ .

We will show later that (1) and (2) together imply  $f = \text{const} \times T$ ,  
for a fixed amount of gas,

hence  $pV = Nk_B T$

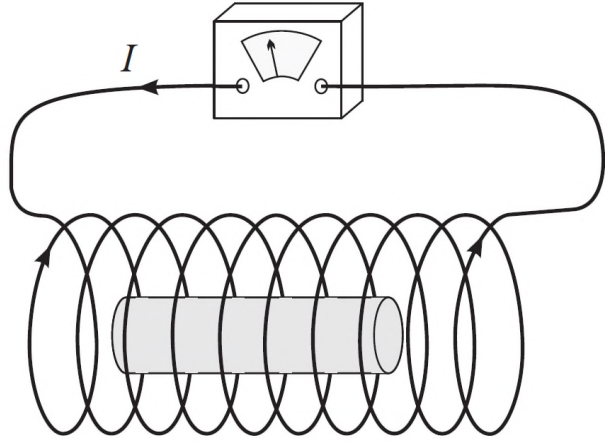
# Thermal radiation

electromagnetic waves in a cavity in equilibrium at temperature  $T$ .



Equation of state:  $p = aT^4$

# Paramagnetism

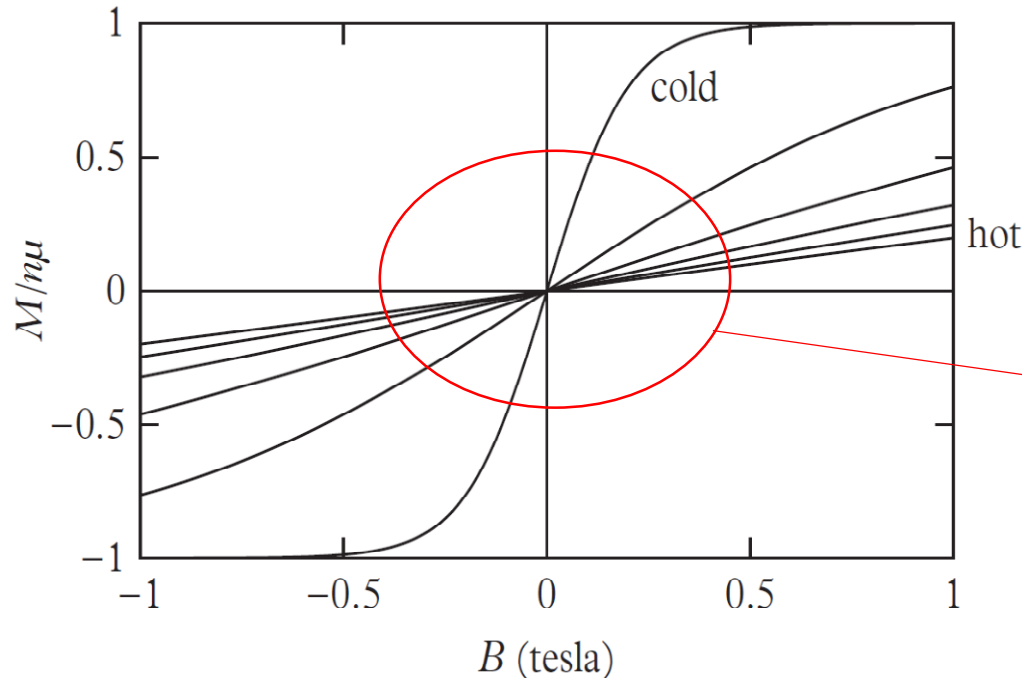


State can be specified by  $\mathbf{M}$ ,  $\mathbf{B}$ , where  
*magnetization*  $\mathbf{M}$  = magnetic dipole moment  
per unit volume.

Let 
$$\mathbf{H} \equiv \frac{1}{\mu_0} \mathbf{B} - \mathbf{M}$$

Then for paramagnet,  $\mathbf{M} = \chi \mathbf{H}$

where  $\chi$  is called *magnetic susceptibility*.



Example equation of state:

Curie's law 
$$\mathbf{M} = \frac{a}{T} \mathbf{H} \simeq \frac{a}{\mu_0 T} \mathbf{B}$$