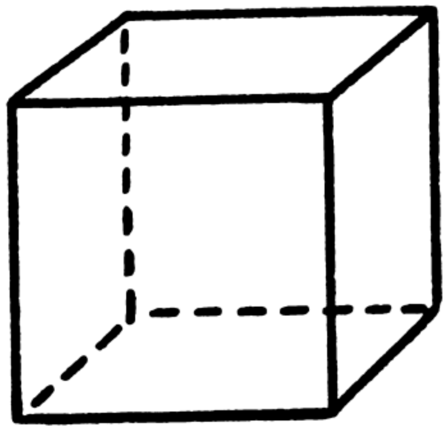
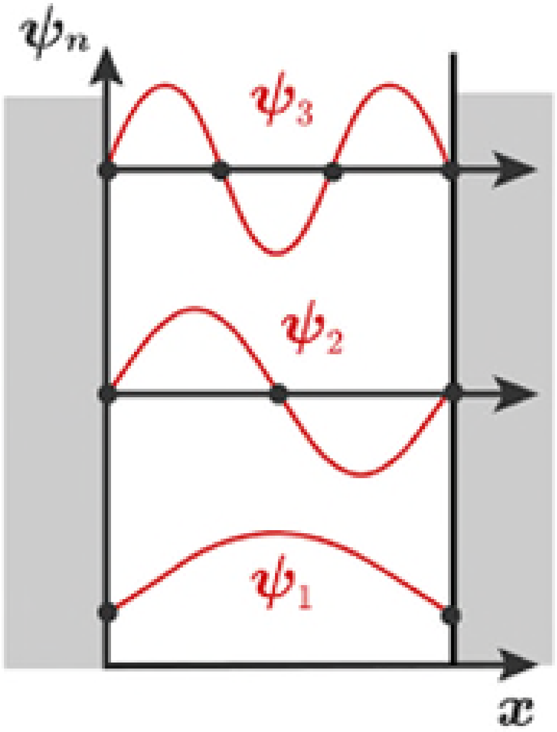
Some further points on statistical mechanics

1. **The method of periodic boundary conditions**
2. **Gas in 2, 1 (and 0) dimensions**
3. Intrinsic spin of atoms and nuclei → (2J+1) factor
4. identical nuclei → reduced set of states
5. **Stability of thermal equilibrium**
6. Negative temperature in spin system
7. Ferro-magnetic phase transition (brief remarks)

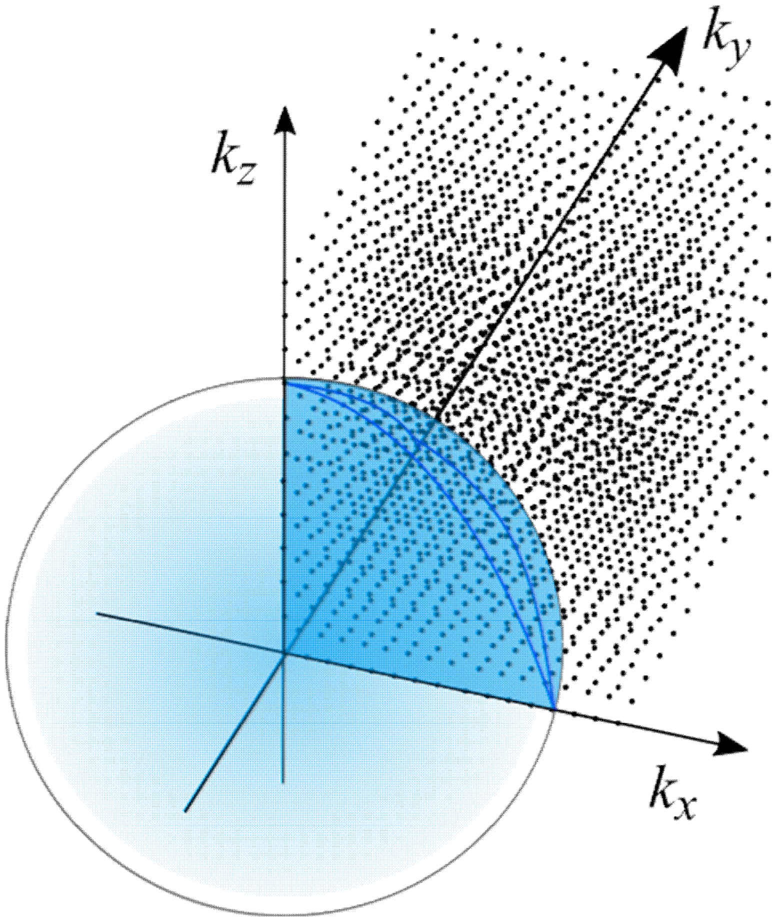
Gas in a box





Potential well with

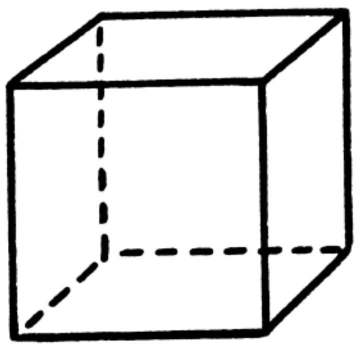
energy
eigenstates
(in one dimension)



States in k space

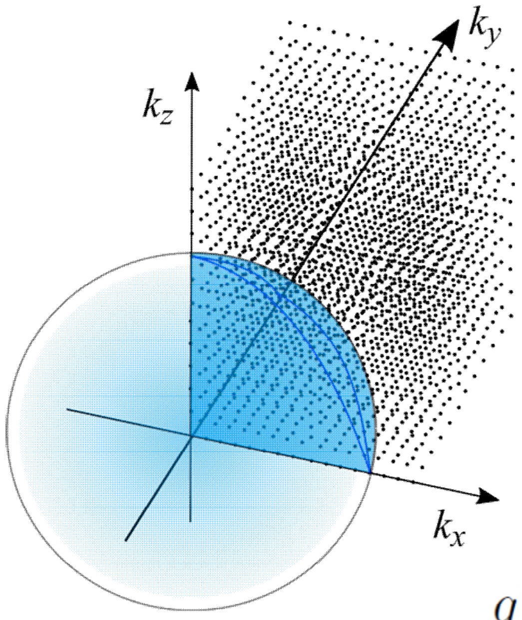
The method of periodic boundary conditions

Particles confined in a box



Standing waves,
$\sin(k_x\, x)\sin(k_y\, y)\sin(k_z\, z)$

$$\Delta k_x = \frac{\pi}{L}$$



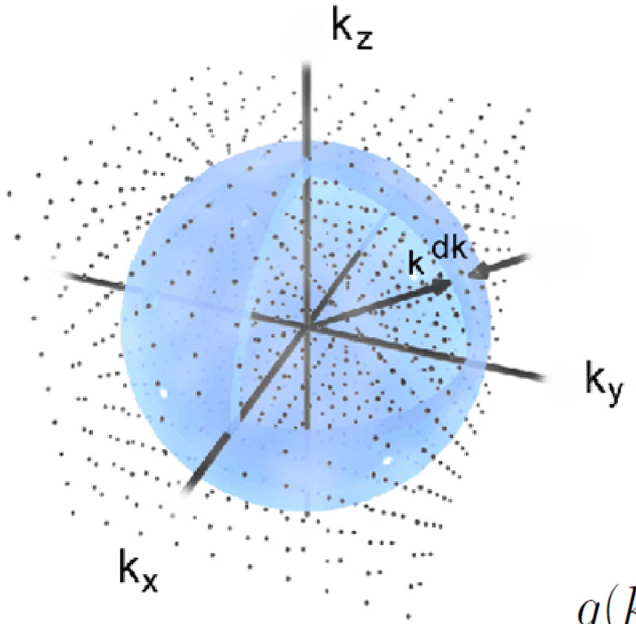
$$k_x, k_y, k_z > 0$$

$$g(k)\mathrm{dk} = \frac{1}{8}4\pi \mathrm{k}^2 \frac{\mathrm{V}}{\pi^3}\mathrm{dk}$$

Free particles with a *mathematical constraint*: wavefunctions must have period L.

Travelling waves,
$e^{ik_x x}\, e^{ik_y y}\, e^{ik_z z}$

$$\Delta k_x = \frac{2\pi}{L}$$

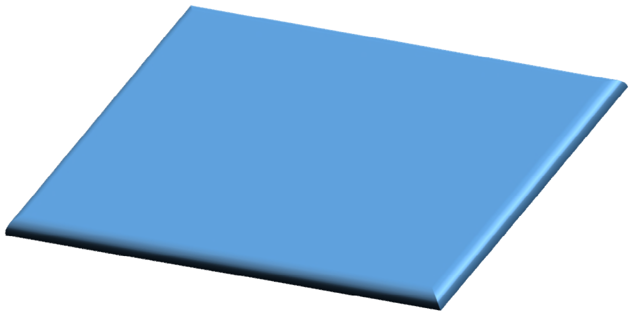


$$k_x, k_y, k_z$$

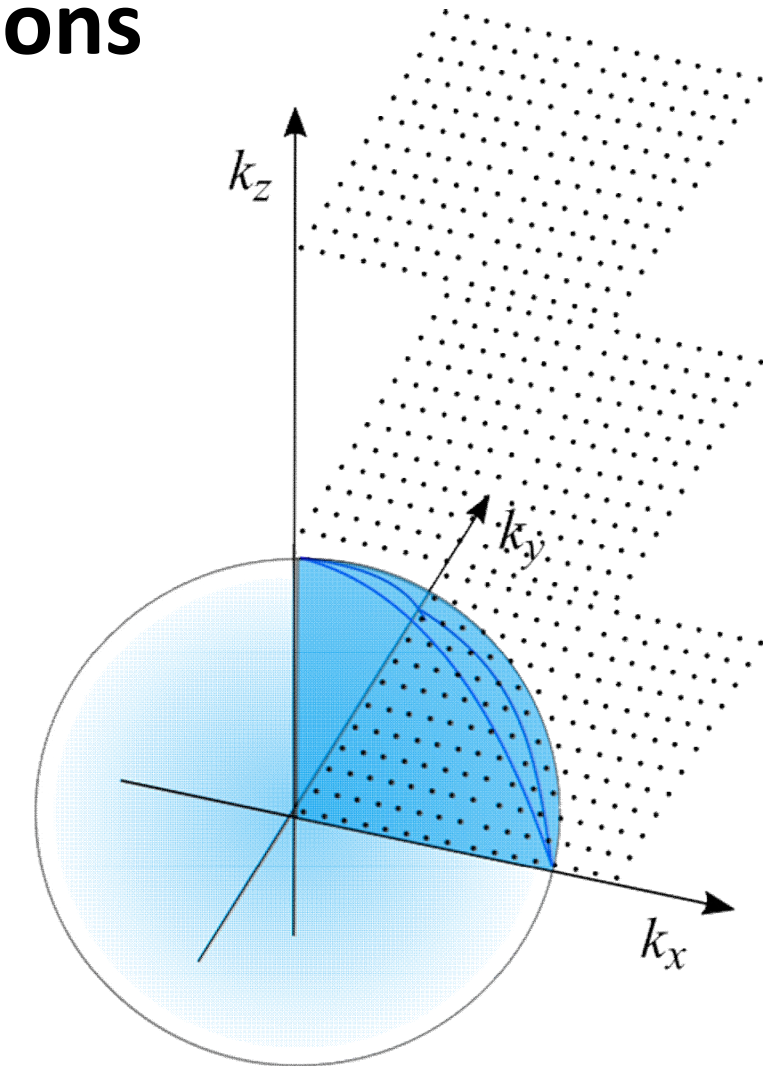
+ve or -ve

$$g(k)\mathrm{dk} = 4\pi \mathrm{k}^2 \frac{\mathrm{V}}{(2\pi)^3}\mathrm{dk}$$

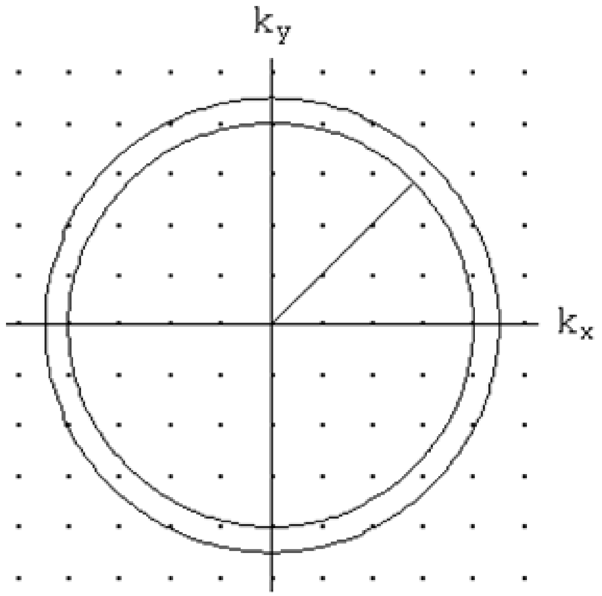
Gas in 2 dimensions



Thin box,
area A



k space: for lowish T, only 1st layer of states are excited → z part of the motion is in its ground state and stays there

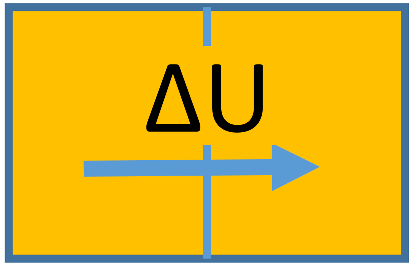


Hence we say we have a "2-dimensional gas".

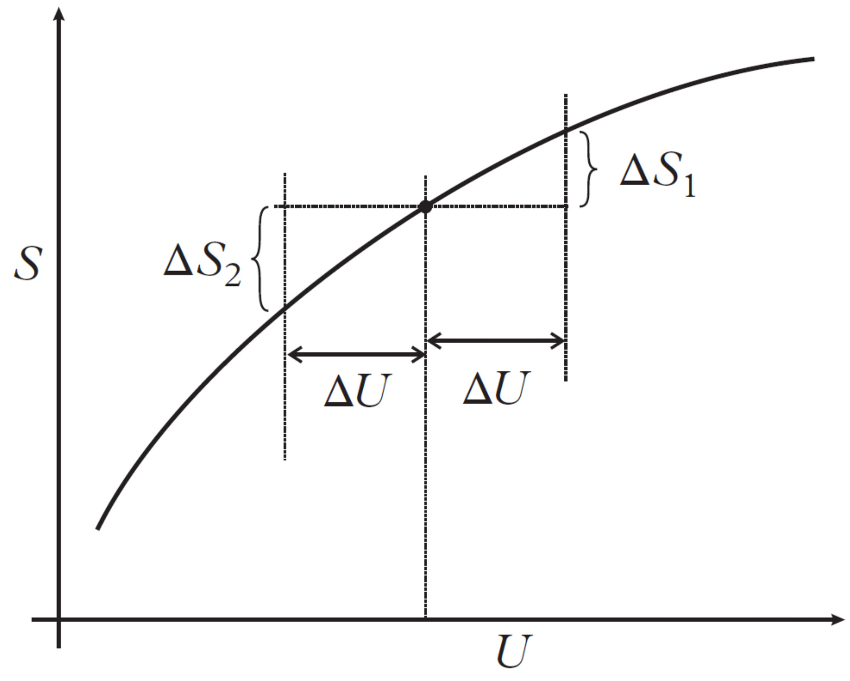
Adopt $\boldsymbol{k} \equiv (k_x, k_y)$

$$g(k)\mathrm{d}k = \frac{A}{(2\pi)^2} 2\pi k \mathrm{d}k$$

Stability of thermal equilibrium

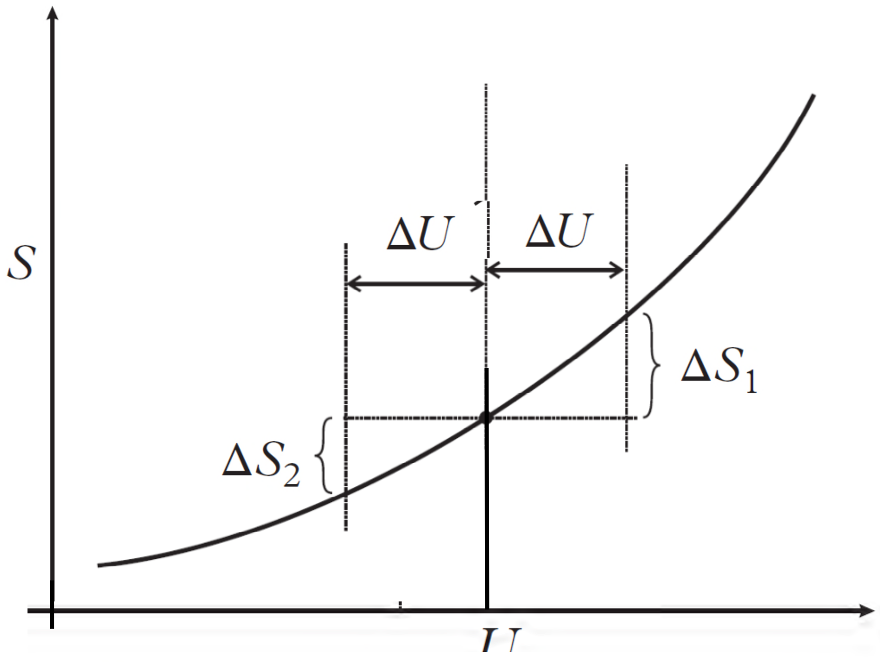


Thermodynamic system with an internal movement of energy



$$\left(\frac{\partial^2 S}{\partial U^2}\right)_{V,N} < 0$$

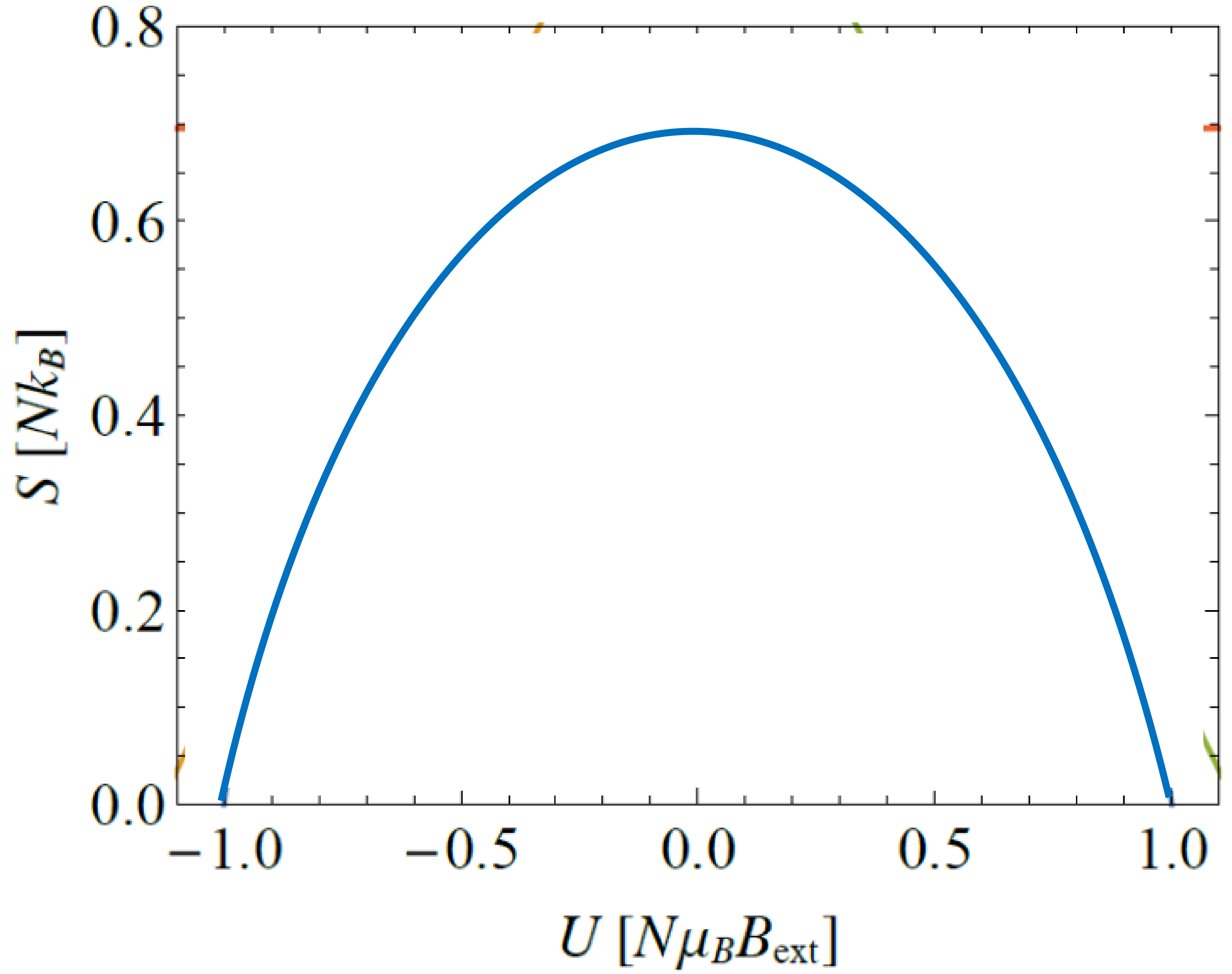
STABLE



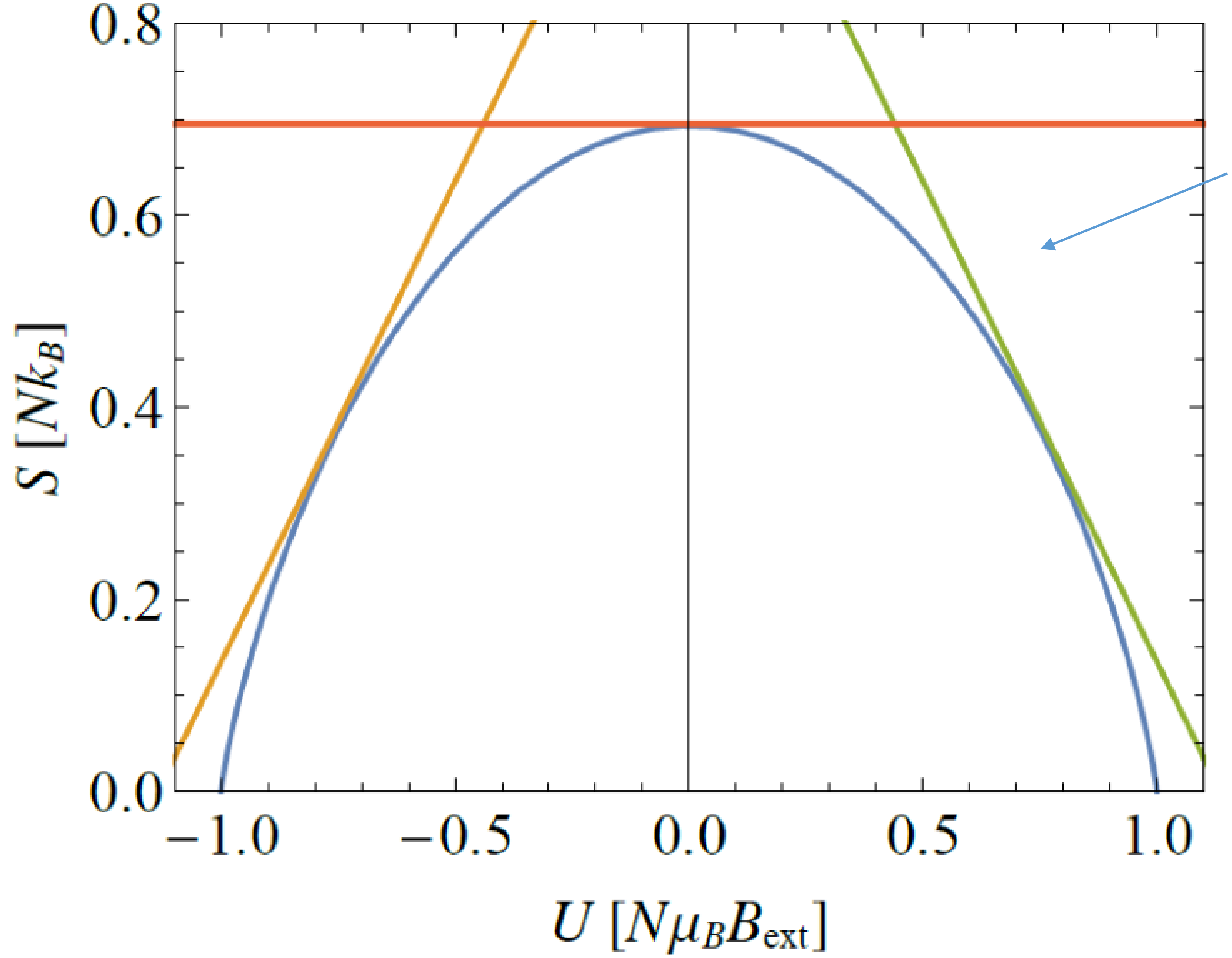
$$\left(\frac{\partial^2 S}{\partial U^2}\right)_{V,N} > 0$$

UNSTABLE

Entropy of a paramagnet as a function of internal energy.



Entropy of a paramagnet as a function of internal energy.

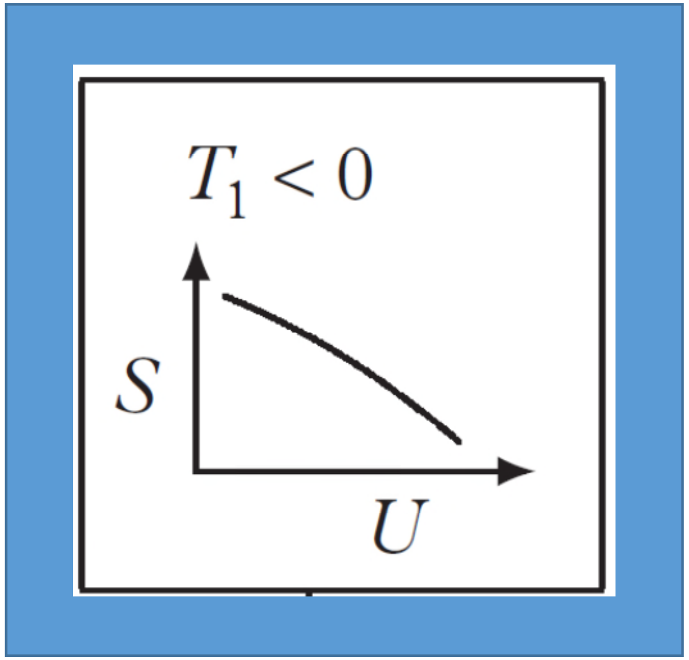


Negative temperature regime.

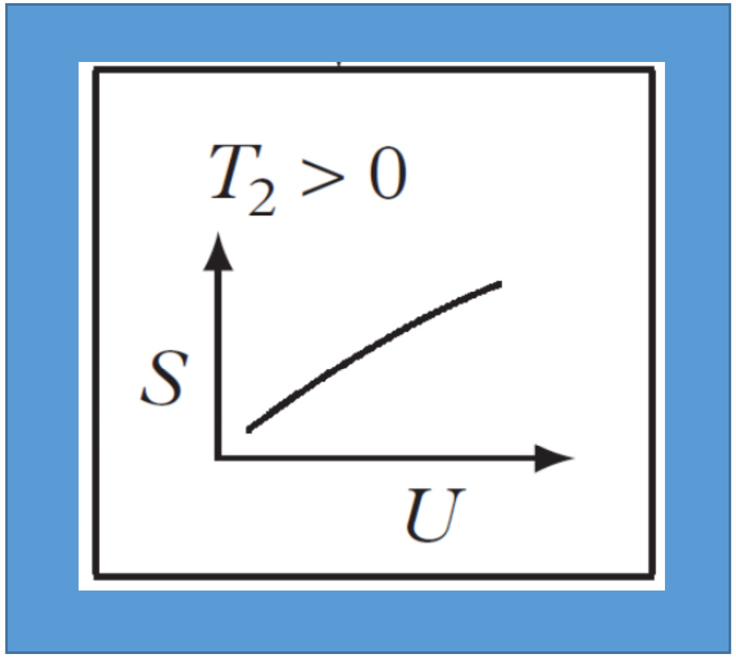
It can be accessed by abruptly reversing the B field.

The straight lines give three examples of the slope (dS/dU).

Which way will the heat flow?

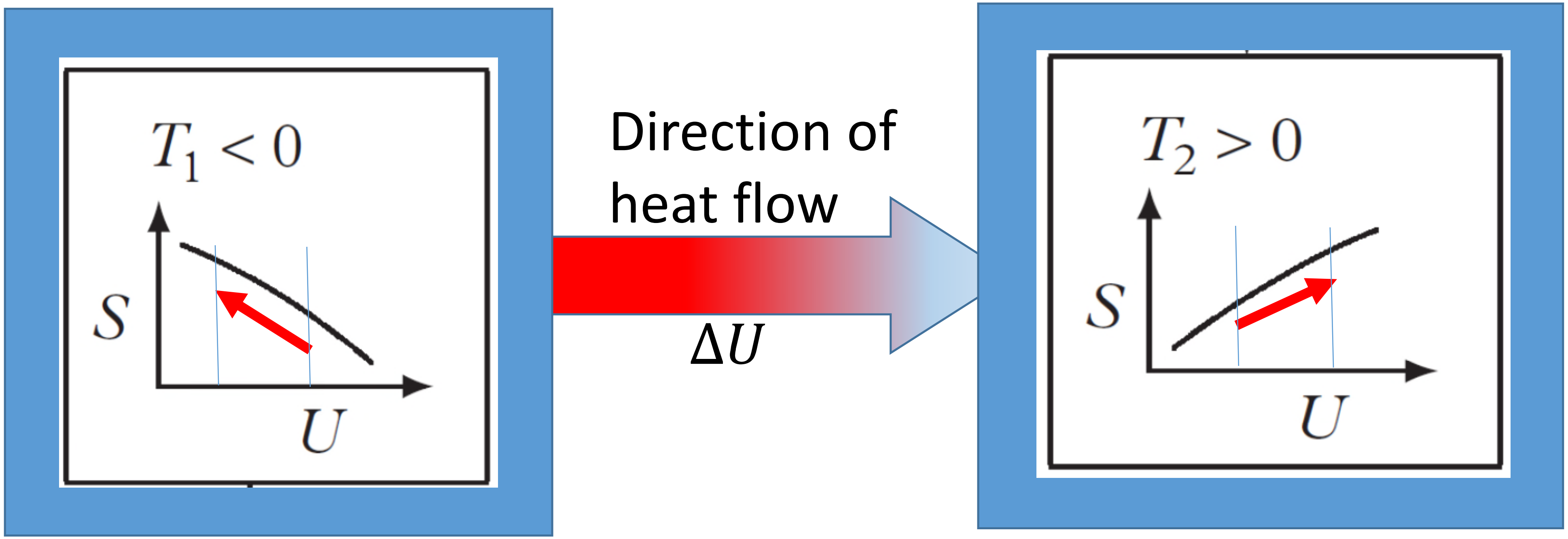


A system at negative temperature



A system at positive temperature

Negative temperature means the system is extremely hot



A system at negative temperature

A system at positive temperature

It follows that negative temperature is always at most metastable, not fully stable.